ADAPTIVE ROBUST TRACKING CONTROL OF AN UNDERWATER VEHICLE-MANIPULATOR SYSTEM WITH SUB-REGION AND SELF-MOTION CRITERIA

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Abstract

This paper proposes an adaptive robust control scheme for an Underwater-Vehicle Manipulator System (UVMS). The proposed controller enables the tracking of the intersection of multiple local sub-regions that are assigned for each subsystem of a UVMS under the influence of modelling uncertainties as well as additive disturbances. The presence of variable ocean currents creates hydrodynamic forces and moments that are not well-known or predictable, even though they are bounded. Therefore, the control task of tracking a prescribed sub-region trajectory is challenging due to these additive bounded disturbances. In the presented adaptive control law, a least-squares estimation algorithm is utilized rather than gradient-type approach. The use of the self-motion due to the kinematically redundant system allows performance of multiple sub-tasks (e.g., maintaining manipulability and avoidance of mechanical joint limits). The asymptotically sub-region and sub-task tracking are ensured using the proposed control law despite the parametric uncertainty of the UVMS and external additive disturbances. The stability analysis is carried out using the Lyapunov-type approach. The simulation results illustrate the validity of the proposed control scheme.

Key Words

Adaptive robust control, sub-region and sub-task tracking control, least-squares estimation, underwater vehicle-manipulator system

1. Introduction

Over the past few decades, the development of tracking controllers has been extensively studied for an Underwater Vehicle-Manipulator System (UVMS). The tracking control of this system is relatively challenging due to the highly coupled dynamics of the sub-systems, the nonlinearities and uncertainties in the system models. External disturbances also act on the system where they cannot be sensed and estimated accurately. Therefore, the UVMS controller has to be robust to dynamical parametric uncertainties as well as the additive disturbance terms.

For the kinematically redundant UVMS which consists of an underwater vehicle and one or more manipulators (i.e., \( n \) dimensional), there is no unique solution between the vehicle/joint variables and the end-effector position/orientation. Thus, the UVMS has more degrees of freedom (DOF) than is required to perform a task in operational space (i.e., \( m \) dimensional). These extra DOFs, due to DOFs provided by the vehicle and onboard manipulator, allow the system to achieve increased flexibility for execution of sophisticated underwater tasks. In other words, any link velocity in the null space (i.e., \( n - m \) ) of the UVMS Jacobian will not affect the operational space velocity. This fact is commonly referred to as self-motion since it is not observed in the operational space. Thus, there are generally an infinite number of solutions for the inverse kinematics of a UVMS system; i.e., manipulability in [1] and distance of mechanical joint limits in [1] and [2].

To achieve accurate end-effector tracking while allowing the self-motion of the system to be available for performance augmentation Zergeroglu et al. presented a model-based controller that achieves exponential tracking for a fixed-based manipulator [3]. The extensions for adaptive and model-based output feedback type controllers were also presented, however the controller proposed either required exact knowledge of the dynamics or the uncertain robot dynamics to be linearly parameterizable and did not take into account the external disturbances. To overcome these, a robust controller with multiple sub-task objectives was proposed in [4] which guarantee uniformly ultimately bounded task-space end-effector position and
sub-task tracking. It is interesting to note that the results in [3] and [4] focus on set-point control where the desired target is specified as a point. On the other hand, the desired target can also be defined as a region instead of a point. Recently, the region control laws in [5] and [6] are focused on reaching a stationary region. In many underwater tracking problems, the system is required to follow the moving target in a particular time, i.e., a pipeline maintenance task, rather than reaching a static target. Based on these problems, a region tracking scheme was introduced for a fixed-base robot manipulator in [7], where an adaptive inertia-related approach is utilized such that the desired target is specified as a moving target. Both set-point and region tracking control objectives of an Autonomous Underwater Vehicle (AUV) are illustrated in Fig. 1.

In the underwater robot control literature, e.g., Antonelli [1] and Sun and Cheah [6], researchers typically prefer to utilize gradient-type algorithms for parameter estimation due to their simplicity. It should be noted that the gradient-type update laws often exhibit slow parameter convergence [8], hence a control methodology that provides flexibility in the design of the parameter estimation update law is highly desirable.

In this paper, based on a generalized/pseudo-inverse formulation, an adaptive robust tracking control scheme is proposed for a kinematically redundant UVMS. Using the proposed controller, the UVMS tracking control of multiple sub-regions and sub-tasks objectives is guaranteed in spite of the parametric uncertainties and additive external disturbances. The intersection of multiple local sub-regions is used as an absolute sub-region and the multiple sub-task objectives are formulated using the weighted-sum approach. In addition, a least-squares estimation gain is used in the proposed control concept that represents a novel departure from conventional adaptive robust control of a UVMS.

The rest of the paper is organized as follows. Section 2 briefs unit quaternion representation. Section 3 gives the kinematic and dynamic model of a redundant UVMS used in the theoretical development. This model includes the effects of the bounded disturbance forces and moments. Section 4 discusses the control objective and the error system development. The formulation of multiple sub-task objectives is presented in Section 5. Section 6 shows the simulation results of the proposed control law. Finally, Section 7 concludes the paper with a summary.

2. Unit Quaternion Representation

In 3D underwater operational space, a 3-parameter representation such as roll, pitch, and yaw forms only a local parameterization and exhibits singularities. Otherwise, the unit quaternion or Euler parameters can be used to represent attitude without singularities with one constraint equation. Moreover, the quaternion provides a global non-singular parameterization with desirable computational properties. Consider two orthonormal right-handed coordinate frames: the inertial reference frame, \( \Sigma_i \), and body-fixed frame, \( \Sigma_v \). Define the matrix \( R \), as a \( 3 \times 3 \) rotational matrix from the body-fixed frame to the inertial-fixed frame. The unit quaternion representation of the rotational matrix, \( R \), can be defined by:

\[
\mathbf{\epsilon} = \begin{bmatrix} \epsilon_0 & \epsilon_1 & \epsilon_2 & \epsilon_3 \end{bmatrix}^T = \begin{bmatrix} \epsilon_0 & \epsilon_2^T \end{bmatrix}^T \in \mathbb{R}^4
\]  

with:

\[
\epsilon_0 = \cos(\vartheta/2); \quad \epsilon_\vartheta = k \sin(\vartheta/2)
\]  

where \( \vartheta \) is the angle and \( k(t) \in \mathbb{R}^3 \) are the Euler angle/axis parameters subject to the constraint \( \mathbf{\epsilon}^T \mathbf{\epsilon} = 1 \). The rotational matrix can be determined through:

\[
R = (\epsilon_0^2 - \epsilon_\vartheta^T \epsilon_\vartheta) I_3 + 2 \epsilon_\vartheta \epsilon_\vartheta^T - 2 \epsilon_0 \epsilon_\vartheta^T \epsilon_\vartheta
\]

where for any vector \( \mathbf{a} = [a_1, a_2, a_3]^T \), the notation \( \mathbf{a}^\times \) denotes the skew-symmetric matrix of the form:

![Figure 1. Tracking control of an autonomous underwater vehicle. (a) set-point tracking control and (b) region tracking control.](image-url)
\[ a^\times \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \]

where the product \( a^T a^\times \) satisfies the following property:

\[ a^T a^\times = [0 \ 0 \ 0]^T \] (5)

Generally, the relationship between unit quaternion and angular velocity is in the body-fixed frame, \( \omega \) can be obtained using the quaternion propagation equation:

\[ \dot{\epsilon} = \frac{1}{2} E(\dot{\epsilon}) \omega \] (6)

with:

\[ E(\epsilon) = \begin{bmatrix} -\epsilon_2^T \\ \epsilon_0 I_3 + \epsilon_3^T \end{bmatrix} \] (7)

where the Jacobian \( E(\epsilon) \) satisfies the following important properties:

\[ E^T(\epsilon) E(\epsilon) = I_{3\times3}; \quad E^T(\epsilon) \epsilon = 0 \] (8)

Consequently, from (6) and (7), the inverse kinematics can be computed as:

\[ \omega = 2E^T(\epsilon) \dot{\epsilon} \] (9)

### 3. Kinematic and Dynamic Model of an UVMS

#### 3.1 Vehicle Kinematic Model

The relationship between the inertial- and body-fixed vehicle velocity can be described using the Jacobian matrix \( J_v(\epsilon_v) \) in the following form:

\[ \begin{bmatrix} \dot{p}_v \\ \dot{\epsilon}_v \end{bmatrix} = \begin{bmatrix} R(\epsilon_v) & 0_{3\times3} \\ 0_{4\times3} & \frac{1}{2} E(\epsilon_v) \end{bmatrix} \begin{bmatrix} v_v \\ \omega_v \end{bmatrix} \Leftrightarrow \dot{\epsilon}_v = J_v(\epsilon_v) v \] (10)

where \( p_v \in \mathbb{R}^3 \) and \( \epsilon_v \in \mathbb{R}^4 \) denote the position and the unit quaternion representation of the vehicle, respectively, expressed in the inertial-fixed frame. \( R \) and \( E \) are defined as in (3) and (7) respectively. The linear and angular velocities of the vehicle, \( v_v \in \mathbb{R}^3 \) and \( \omega_v \in \mathbb{R}^3 \), respectively, are described in terms of the body-fixed frame.

### 3.2 Manipulator Kinematic Model

For an \( n \)-link manipulator, the joint position state vector is defined by \( q = [q_1 \ q_2 \ldots q_n]^T \in \mathbb{R}^n \) and the end-effector composition vector is described by \( \zeta_{v,m} = [\nu_{v,m} \ \nu_{e,m}]^T \in \mathbb{R}^7 \), where \( \nu_{v,m} \in \mathbb{R}^3 \) and \( \nu_{e,m} \in \mathbb{R}^4 \) are the position and unit quaternion orientation representations respectively. The superscript \( v \) indicates that the vectors are expressed in the body-fixed frame. The relationship between body-fixed manipulator velocity \( \dot{\zeta}_{v,m} \) and joint velocity \( \dot{q} \) can be represented using the analytical Jacobian of the manipulator as [9]:

\[ \begin{bmatrix} \nu_{\dot{p},v,m} \\ \nu_{\dot{e},v,m} \end{bmatrix} = \begin{bmatrix} J_{mp}(q) \\ J_{mo}(q) \end{bmatrix} \dot{q} \Leftrightarrow \dot{\zeta}_{v,m} = J_m^T(\nu_{v,m}) \dot{q} \] (11)

where \( J_{mp} \in \mathbb{R}^{3\times n} \) and \( J_{mo} \in \mathbb{R}^{4\times n} \) denote the position and orientation Jacobian matrices from the manipulator base to end-effector, respectively. The end-effector angular velocity expressed in the manipulator base frame \( \nu_{\omega,v,m} \) is related to \( \nu_{\dot{e},v,m} \) through (9). Taking into account of (6) and (8) and (11) the following expression relating the generalized end-effector velocity vector can be obtained:

\[ v_M = \begin{bmatrix} \nu_{\dot{p},v,m} \\ \nu_{\omega,v,m} \end{bmatrix} = J_M(\nu_{v,m},q) \dot{q} \] (12)

where \( J_M(\nu_{v,m},q) \in \mathbb{R}^{6\times n} \) is the end-effector Jacobian matrix and is explicitly defined as:

\[ J_M(\nu_{v,m},q) \triangleq \begin{bmatrix} J_{mp}(q) \\ 2E^T(\nu_{v,m}) J_{mo}(q) \end{bmatrix} \] (13)

where \( E \) is defined as in (7). For a redundant onboard manipulator (i.e., \( n > 6 \)), the inverse kinematics at the velocity level can be obtained using (12) as follows:

\[ \dot{q} = J_M^T v_M + (I_n - J_M^T J_M) z \] (14)

where \( I_n \) denotes the \( n \times n \) identity matrix, \( (I_n - J_M^T J_M) \) is the projection matrix into the null space of \( J_M \). \( (I_n - J_M^T J_M) z \) is the homogeneous solution of (14) orthogonal to its particular solution, \( J_M^T v_M \). \( J_M^T \in \mathbb{R}^{n\times m} \) is the Moore-Penrose pseudo-inverse of the manipulator Jacobian that can be defined as:

\[ J_M^T = J_M^T (J_M J_M^T)^{-1} \] (15)

which is also termed as the right pseudo-inverse since \( J_M J_M^T = I_n \). From (14), the vector \( z \in \mathbb{R}^n \) denotes an auxiliary velocity which can be constructed to improve the performance of the manipulator according to an additional
control objective. This possible performance enhancement is achieved by optimizing a proper performance criterion function, instead of $z$. Let $H(q) \in \mathbb{R}$ be the optimal positive function, then $z$ is defined as:

$$z = \kappa \nabla H(q)$$

where $\nabla H(q)$ is the gradient of $H(q)$ and $\kappa$ is a real valued scalar. Note that, the pseudo-inverse defined by (15) satisfies the Moore-Penrose Conditions and the properties of the null space matrix $(I_n - J_M^T J_M)$ [10].

### 3.3 Dynamic Model of an UVMS

The dynamic equations of motion for a manipulator mounted on a mobile platform were analysed in detail in [11]. In [12] and [13], a similar recursive concept was utilized, where the hydrodynamic effects (added mass, drag, lift, and buoyancy) were included. Consider the velocity vector as $\xi = [v \quad \dot{q}]^T$, thus the equation of motion of the UVMS is given in the body-fixed frame [1], [12]:

$$M(q) \ddot{\xi} + C(q, \xi) \dot{\xi} + D(q, \xi) \dot{\xi} + g(\eta_v, q) = \tau$$

where $M(q) \in \mathbb{R}^{(6+n) \times (6+n)}$ is the inertia matrix including added mass, $C(q, \xi) \in \mathbb{R}^{(6+n)}$ is the vector of Coriolis and centripetal terms, $D(q, \xi) \in \mathbb{R}^{(6+n)}$ is the vector of hydrodynamic damping, $g(\eta_v, q) \in \mathbb{R}^{(6+n)}$ is the vector of gravity and buoyancy forces, and $\tau \in \mathbb{R}^{(6+n)}$ is the vector of generalized forces acting on the vehicle and joint torque. Properties of the dynamic equation described by (17) are given as [1]:

**Property 1:** The inertia matrix $M(q)$ is symmetric and positive definite, i.e., $M(q) = M^T(q) > 0$.

**Property 2:** $\dot{M}(q) - 2C(q, \xi)$ is skew-symmetric.

**Property 3:** The hydrodynamic damping matrix $D(q, \xi)$ is positive definite such that $D(q, \xi) = D^T(q, \xi) > 0$.

The UVMS dynamic model as described in (17) is linear in a set of dynamic parameters $\Phi \in \mathbb{R}^{p_r}$ and can be written as:

$$M(q) \ddot{\xi} + C(q, \xi) \dot{\xi} + D(q, \xi) \dot{\xi} + g(\eta_v, q, \xi) = Y(q, \eta_v, \xi, \dot{\xi}) \Phi$$

where $Y(q, \eta_v, \xi, \dot{\xi}) \in \mathbb{R}^{(6+n) \times np}$ is the UVMS regression matrix; $np$ is the total number of physical parameters. It is assumed that if the arguments of $Y(\cdot)$ are bounded then $Y(\cdot)$ is bounded.

**Remark 1.** The dynamic and kinematic terms for the general UVMS, denoted above by $M(q), C(q, \xi), D(q, \xi), g(\eta_v, q, J_M(q))$, and $J_M(q)$ are assumed to depend on $q(t)$ as arguments of trigonometric functions and hence remain bounded for all possible $q(t)$. Using the fact that $\epsilon(t)$ is always bounded as $\epsilon(t) = 1$, hence $J_v$ and $J_M$ are bounded. If $p_v(t)$ is bounded then $\eta_v(t)$ is also bounded. During the control development, the assumption is made that if $\eta_v(t)$ is bounded then $\nu(t)$ is a bounded signal.

**Remark 2.** During the subsequent control development, the assumption is made that the minimum singular value of the onboard manipulator Jacobian matrix, denoted by $\sigma_m$, is greater than a known small positive constant $\delta > 0$, such that $\max\{||J_{vM}(q)||\}$ is known a priori and all kinematic singularities are always avoided.

### 4. Control Objective and Error Formulation

In this section, the control objective and the error system development are presented. The primary control objective of a UVMS is to design the input command signal such that the vehicle and the end-effector can follow their individual sub-region and orientation signal as close as possible. The individual sub-region is obtained from the intersection of multiple local sub-regions. Figure 2 shows an intersection of two local sub-regions that is sub-region 1 and sub-region 2 for tracking control of an onboard manipulator. The similar approach also can be extended for the underwater vehicle. The local sub-region intersection can be exploited in various ways; spherical intersection or cubic intersection as discussed in [5] and [14].

Additionally, the secondary objective is to design the control input which allows the redundancy of the system to perform the sub-task defined by at least one motion optimization index such as maintaining manipulability and/or avoidance of mechanical joint limits. Defining the vehicle region tracking error $\tilde{e}_{pv}$ and end-effector region tracking error $\tilde{e}_{pm}$ as follows:

$$\tilde{e}_{pv} \triangleq \sum_{j=1}^{N_1} \frac{W_{v_j}}{k_{v_j}} \max(0, f_{v_j}(\delta p_{v_j})) \left( \frac{\partial f_{v_j}(\delta p_{v_j})}{\delta p_{v_j}} \right)^T$$

$$\tilde{e}_{pm} \triangleq \sum_{i=1}^{N_2} \frac{W_{m_i}}{k_{m_i}} \max(0, f_{m_i}(\delta p_{v,m_i})) \left( \frac{\partial f_{m_i}(\delta p_{v,m_i})}{\delta p_{v,m_i}} \right)^T$$

Figure 2. An end-effector tracking control with intersection of two local sub-regions.
where $w_{v,ij}$ and $w_{mi}$ are the weighted gains. The subscripts $v$ and $m$ denote the vehicle and manipulator end-effector, respectively. After defining the proper intersection of multiple local individual regions, the overall desired vehicle and end-effector sub-regions, represented by the scalar function $f_{v,j}(\delta p_{v,j}) \in \mathbb{R}$ and $f_{m,i}(\delta \gamma_{v,mi}) \in \mathbb{R}$, respectively are delineated as follows [7]:

$$f_{v,j}(\delta p_{v,j}) = \begin{bmatrix} f_{v1}(\delta p_{v1}) \\ f_{v2}(\delta p_{v2}) \\ \vdots \\ f_{vNv}(\delta p_{vNv}) \end{bmatrix} \leq 0 \iff f_{v}(\delta p_{v}) \leq 0 \quad (21)$$

$$f_{m,i}(\delta \gamma_{v,mi}) = \begin{bmatrix} f_{m1}(\delta \gamma_{v,m1}) \\ f_{m2}(\delta \gamma_{v,m2}) \\ \vdots \\ f_{mNv}(\delta \gamma_{v,mNv}) \end{bmatrix} \leq 0 \iff f_{m}(\delta \gamma_{v,m}) \leq 0 \quad (22)$$

where $\delta p_{v,j} = (p_{v,dj} - p_{v}) \in \mathbb{R}^3$ and $\delta \gamma_{v,mi} = (\gamma_{v,mdi} - \gamma_{v,m}) \in \mathbb{R}^3$ denote the continuous first partial derivatives. $p_{v,di}(t) \in \mathbb{R}^3$ is the reference point inside the $j$th desired vehicle sub-region, $j = 1, 2, \ldots, N_v$ and $\gamma_{v,mdi}(t) \in \mathbb{R}^3$ is the reference point inside the $i$th desired end-effector sub-region, $i = 1, 2, \ldots, N_d$; $N_v$ and $N_d$ are the maximum number of vehicle and end-effector local sub-regions, respectively. $\dot{p}_{v,dj}(t)$ and $\dot{\gamma}_{v,m,di}(t)$ can be obtained using the similar method used in [15] such that end-effector tracking as the primary task is achieved. To ensure simultaneous region tracking of the vehicle and end-effector, the functions of (21) and (22) are bounded as follows:

$$f_{v,j}(\delta p_{v,j}) \leq f_{m,i}(\delta \gamma_{v,mi}) \quad (23)$$

Next, the two vectors $\dot{p}_{v}$ and $\dot{\gamma}_{v,mi}$ are defined as follows:

$$\dot{p}_{v} = \dot{p}_{v,d} + \lambda_{v1} \dot{e}_{pv} \quad (24)$$

$$\dot{\gamma}_{v,mi} = \dot{\gamma}_{v,m,di} + \lambda_{m1} \dot{e}_{pm} \quad (25)$$

where each local individual regions are assumed to have the identical speed and acceleration. $\dot{e}_{pv}$ and $\dot{e}_{pm}$ are defined in (19) and (20), respectively. $\lambda_{v1}$ and $\lambda_{m1}$ are diagonal and positive definite gain matrices. It assumed that $p_{v,d}(t)$, $\dot{p}_{v,d}(t)$, $\ddot{p}_{v,d}(t)$, $\gamma_{v,m,di}(t)$, $\dot{\gamma}_{v,m,di}(t)$, $\ddot{\gamma}_{v,m,di}(t)$, are all bounded functions of time. Note that, for a boundedness of $f_{v,j}(\delta p_{v,j})$, $\delta p_{v,j}$ is bounded. Likewise, if $f_{m,i}(\delta \gamma_{v,mi})$ is bounded, then $\delta \gamma_{v,mi}$ is also bounded.

On the other hand, the error between the actual and desired orientation for the vehicle and manipulator can be generalized as follows [15]:

$$\tilde{R} \triangleq RR_d^T = (\tilde{e}^2_0 - \tilde{e}_x^2 \tilde{e}_z)I_3 + 2\tilde{e}_x \tilde{e}_z^T - 2\tilde{e}_0 \tilde{e}_x^T \quad (26)$$

where $R$ is defined in (3) and $R_d$ is the rotational matrix of $R$ expressing the desired orientation which also is described by the quaternion $\epsilon \triangleq [\epsilon_0, \epsilon_x^T]$. The corresponding unit quaternion representation is denoted by $\hat{\epsilon} \triangleq [\epsilon_0, \epsilon_x^T]$. Thus, the quaternion propagation equation can be considered as:

$$\dot{\epsilon}_0 = -\frac{1}{2} \hat{\epsilon}_x \tilde{w}$$

$$\dot{\epsilon}_x = (\epsilon_0 \hat{I}_3 + \hat{\epsilon}_x^T)\tilde{w} \quad (27)$$

where $\tilde{w} = \omega_d - \omega$; $\omega$ is defined in (10) for the vehicle and in (12) for the manipulator. $\omega_d$ is the desired angular velocity of the vehicle/manipulator.

**Remark 3.** Equation (27) is invertible provided $\epsilon_0(t) \neq 0$ for any time. To ensure that $\epsilon_0(t) \neq 0$ for all the time, the desired trajectory must be initialized to guaranteed that $\epsilon_0(t) \neq 0$, and the subsequent control design must ensure that $\epsilon_0(t) \neq 0$ after the initial time.

The general filtered tracking error vector is defined as $r(t) = [r_v(t) \quad r_m(t)]^T \in \mathbb{R}^{(6+n)}$. Based on the subsequent stability analysis, an auxiliary signal for the vehicle, denoted by $r_v(t) = [r_{v1} \quad r_{v2}]^T \in \mathbb{R}^3$, is defined as:

$$r_v \triangleq \begin{bmatrix} R^T \dot{p}_{v,d} + R^T \lambda_{v1} \hat{e}_{pv} \\ \omega_{v,d} + \lambda_{v2} \hat{e}_{pv} \end{bmatrix} - v \quad (28)$$

where $v(t)$ is defined in (10) and $\lambda_{v2}$ is a diagonal gain matrix. On the other hand, the filtered tracking error for the redundant manipulator can be obtained as:

$$r_m \triangleq J_M^T \begin{bmatrix} \dot{v}_{v,m,d} + \lambda_{m1} \hat{e}_{pm} \\ \omega_{v,m,d} + \lambda_{m2} \hat{e}_{pm} \end{bmatrix} + (I_n - J_M^T J_M) z - \hat{q} \quad (29)$$

with $\lambda_{m2}$ being diagonal and positive definite gain matrices. The vector $z$ is defined in (16) and $I_n$ is the $n \times n$ identity matrix. Define the sub-task tracking error as follows [16]:

$$\hat{e}_{sub} = (I_n - J_M^T J_M)(z - \hat{q}) \quad (30)$$

The properties of full space of the pseudo-inverse can be used to show that the sub-task tracking error defined in (30) is also regulated when $r_m(t)$ is regulated to obtain:

$$\hat{e}_{sub} = (I_n - J_M^T J_M)r_m \quad (31)$$

where (29) is pre-multiplied by $(I_n - J_M^T J_M)$. Therefore, the sub-task control is also achieved.

In general, the development of the open-loop error system for $r(t)$ can be obtained by pre-multiplying the inertial matrix with the time derivative of $r(t)$ to yield:

$$M \ddot{r} = Y(\cdot) \Phi - \tau - Cr - Dr \quad (32)$$

where $Y(\cdot) \in \mathbb{R}^{(6+n) \times n_p}$ denotes a measurable regression matrix and $\Phi$ is defined in (18). Based on the error
system development and the subsequent stability analysis, an adaptive robust control law for a UVMS can be proposed as:

\[
\tau = K_0K_p \begin{bmatrix} \dot{e}_{pu} \\ \dot{e}_{ev} \\ \dot{e}_{pm} \\ \dot{e}_{em} \end{bmatrix} + K_r r + v_R \quad (33)
\]

with:

\[
K_0 = \text{diag}\{R^T, I_{3 \times 3}, J_M^T\} \in \mathbb{R}^{(6+n) \times 12};
\]

\[
K_p = \text{diag}\{K_{v1}, K_{v2}, K_m1, K_m2\} \in \mathbb{R}^{12 \times 12};
\]

where \(K_r \in \mathbb{R}^{(6+n) \times (6+n)}\) is a positive definite and diagonal gain matrix and \(v_R \in \mathbb{R}^{(6+n)}\) is a vector representing an auxiliary robust controller defined by:

\[
v_R = \frac{r\dot{p}^2}{\dot{p}} \left| \dot{r} \right| + \varepsilon \quad \text{with} \quad \dot{e} = -k_z \varepsilon; \quad \varepsilon(0) > 0 \quad (34)
\]

where \(k_z\) is a positive scalar constant and the positive scalar function \(\dot{p}\) is bounded as follows:

\[
\dot{p} \geq \left| Y \Phi \right| \quad (35)
\]

where \(Y\) is given in (32) and the bounding estimate vector \(\Phi\) is updated online by:

\[
\dot{\Phi} = \Gamma Y^T ||r|| + \Gamma Y^T (\cdot) Y (\cdot) \dot{\Phi} \quad (36)
\]

where \(\Gamma(t) \in \mathbb{R}^{n_p \times p}\) denotes a least-squares estimation gain matrix designed as follows:

\[
\frac{d}{dt} (\Gamma^{-1}) = Y^T (\cdot) Y (\cdot); \quad \Gamma(0) = \Gamma^T (0) > 0; \quad (37)
\]

**Remark 4.** When \(\Gamma^{-1}(t_0)\) is chosen to be positive definite and symmetric, then it is clear that \(\Gamma(t_0)\) is also positive definite and symmetric. Hence, it follows that both \(\Gamma^{-1}(t)\) and \(\Gamma(t)\) will remain positive definite and symmetric \(\forall t\). From (37), the following expression can be obtained:

\[
\dot{\Gamma} = -\Gamma Y^T (\cdot) Y (\cdot) \Gamma \quad (38)
\]

In (38), it is shown that \(\dot{\Gamma}(t)\) is negative semidefinite; therefore, the estimation gain matrix \(\Gamma(t)\) is always constant or decreasing, and leads to the boundedness of \(\Gamma(t)\) as delineated in de Queiroz et al. [8] and Krstic et al. [17]. To this end, the closed-loop equation can be obtained by substituting (33) into (32) to yield:

\[
M \dot{e} = Y (\cdot) \Phi - K_r r - K_0 K_p \begin{bmatrix} \dot{e}_{pu} \\ \dot{e}_{ev} \\ \dot{e}_{pm} \\ \dot{e}_{em} \end{bmatrix} - Cr - Dr - v_R \quad (39)
\]

The stability of the proposed tracking control is specified by the following theorem:

**Theorem 1.** The adaptive robust control law described by (33), (34), (35), and (36) along with a least-squares estimation gain (37) ensures asymptotic sub-region, orientation, and sub-tasks tracking for the kinematically redundant underwater robot given by (17) in the sense that:

\[
\lim_{t \to \infty} \dot{e}_{pu}(t), \dot{e}_{pm}(t), \dot{e}_{ev}(t), \dot{e}_{em}(t), \dot{e}_{sub}(t) = 0 \quad (40)
\]

provided that the initial conditions are selected such that:

\[
\dot{e}_{ov}(0), \dot{e}_{om}(0) \neq 0 \quad (41)
\]

**Proof:** The following Lyapunov-like function is proposed to analyze the stability of the error system given in (39):

\[
V = \frac{1}{2} r^T M r + \frac{1}{2} \dot{\Phi}^T \Gamma^{-1} \dot{\Phi} + K_z^{-1} \dot{\varepsilon}
\]

\[
+ k_{v2}[(1 - \dot{e}_{ov})^2 + \dot{e}_{ev}^2]
\]

\[
+ k_{m2}[(1 - \dot{e}_{om})^2 + \dot{e}_{em}^2]
\]

\[
+ \frac{1}{2} k_{v1} \sum_{j=1}^{N_1} w_{vj} [\max(0, f_{vj}(\delta p_{vj}))]^2
\]

\[
+ \frac{1}{2} k_{m1} \sum_{i=1}^{N_2} w_{mi} [\max(0, f_{mi}(\delta p_{mi}))]^2 \quad (42)
\]

where the parameter estimation error \(\dot{\Phi}(t) = -\dot{\Phi}(t)\) is assumed (i.e., dynamic parameters are constant). Differentiating \(V\) with respect to time and taking into account (33), (36), and (39) yields:

\[
\dot{V} = \frac{1}{2} r^T M r + r^T Y (\cdot) \Phi - K_r r - K_0 K_p \begin{bmatrix} \dot{e}_{pu} \\ \dot{e}_{ev} \\ \dot{e}_{pm} \\ \dot{e}_{em} \end{bmatrix} - C r - D r - v_R
\]

\[
+ \frac{1}{2} \dot{\Phi}^T \Gamma^{-1} \dot{\Phi}
\]

\[
- \dot{\Phi}^T Y^T ||r|| - \dot{\Phi}^T Y^T (\cdot) Y (\cdot) \dot{\Phi} + K_z^{-1} \dot{\varepsilon}
\]

\[
+ \dot{e}_{pu}^T K_{v1} R(r_{vu} - \lambda_{u1} R^T \dot{e}_{pu})
\]

\[
+ k_{v2} \dot{e}_{ev}^T (r_{vu} - \lambda_{v2} \dot{e}_{ev}) + [\dot{e}_{pm}^T K_{m1} \dot{e}_{em}] J_m
\]

\[
+ \sum_{i=1}^{N_2} w_{mi} [\max(0, f_{mi}(\delta p_{mi}))]^2 \quad (43)
\]

After applying Property 2 and Property 3 and cancelling common terms in (43), (35) can be used to place an upper bound on \(V\) in the following form [18]:
\[ \dot{V} \leq -r^T K_r r - r^T D r - \dot{e}^T_{pp} K_{e1} \lambda_1 e_{pv} - \dot{e}^T_{ez} k_{e2} \lambda_2 e_{ez} \\
- \dot{e}^T_{pm} K_{m1} \lambda_m e_{pm} \\
- \dot{e}^T_{zm} k_{m2} \lambda_m e_{zm} - \frac{1}{2} \dot{\Phi}^T Y T(\cdot) Y(\cdot) \dot{\Phi} \\
+ K_{e}^{-1} \dot{e} - r^T v_R + Y \dot{\Phi} \|r\| \]  

(44)

Substituting (34) into (44), yields:

\[ \dot{V} \leq -r^T (K_r + D) r - \dot{e}^T_{pp} K_{e1} \lambda_1 e_{pv} - \dot{e}^T_{ez} k_{e2} \lambda_2 e_{ez} \\
- \dot{e}^T_{pm} K_{m1} \lambda_m e_{pm} \\
- \dot{e}^T_{zm} k_{m2} \lambda_m e_{zm} - \frac{1}{2} \dot{\Phi}^T Y T(\cdot) Y(\cdot) \dot{\Phi} - \epsilon \\
- \left[ \|r\|^2 (Y \dot{\Phi})^2 + Y \dot{\Phi} \|r\| \right] \]  

(45)

where the bracketed terms can be manipulated such that (45) can be rewritten as follows:

\[ \dot{V} \leq -r^T (K_r + D) r - \dot{e}^T_{pp} K_{e1} \lambda_1 e_{pv} - \dot{e}^T_{ez} k_{e2} \lambda_2 e_{ez} \\
- \dot{e}^T_{pm} K_{m1} \lambda_m e_{pm} \\
- \dot{e}^T_{zm} k_{m2} \lambda_m e_{zm} - \frac{1}{2} \dot{\Phi}^T Y T(\cdot) Y(\cdot) \dot{\Phi} - \epsilon \\
- \frac{\epsilon Y \Phi \|r\|}{Y \Phi \|r\|} + \epsilon \]  

(46)

Since the sum of the last two terms in (46) is always less than zero, thus the new upper bound on \( \dot{V} \) is defined as:

\[ \dot{V} \leq -\lambda_{\min} \{k_t\} \|\zeta\|^2 \]  

(47)

where \( \zeta(t) \triangleq [r^T \quad \dot{e}^T_{pp} \quad \dot{e}^T_{ez} \quad \dot{e}^T_{zm}] \) denotes the composite state vector and \( k_t \in \mathbb{R} \) is a scalar constant. Based on (47), the new upper bound on \( \dot{V} \) can be placed as:

\[ \dot{V} \leq -\lambda_{\min} \{k_t\} \|\zeta\|^2 \]  

(48)

which implies that:

\[ \int_0^\infty \dot{V}(\alpha) d\alpha \leq -\lambda_{\min} \{k_t\} \int_0^\infty \|\zeta\|^2 d\alpha \]  

(49)

Multiplying (49) by \(-1\) and integrating the left-hand side yields:

\[ V(0) - V(\infty) \geq \lambda_{\min} \{k_t\} \int_0^\infty \|\zeta\|^2 d\alpha \]  

(50)

Since \( \dot{V} \) is negative semidefinite as defined in (48), therefore \( V \) is a non-increasing function and is upper bounded by \( V(0) \). The inertia matrix \( M(q) \) is lower bounded as delineated by the Property 1, therefore \( V \) is lower bounded by zero. Since \( V \) is non-increasing, upper bounded by \( V(0) \), and lower bounded by zero, (50) can be rewritten as:

\[ \lambda_{\min} \{k_t\} \int_0^\infty \|\zeta\|^2 d\alpha < \infty \iff \int_0^\infty \|\zeta\|^2 d\alpha < \infty \]  

(51)

The bound delineated by (51) informs that the composite vector \( \zeta \) is bounded. Note that, the constraint in (1) and (41) can be used with (51) to prove that \( e(t) \) is non-zero for all time, thus (27) is invertible for all time. The following lemma [19] permits one to conclude the asymptotic stability.

**Lemma 1.** Let \( e = H(s) r \) with \( H(s) \) a nxm strictly proper, exponentially stable transfer function. Then, \( r \in L_2^r \) implies that \( e \in L_2^e \cap L_\infty^e \), \( \dot{e} \in L_2^e \), \( e \) is continuous, and \( e \to 0 \) as \( t \to \infty \).

For proof of Lemma 1, see [19].

The transfer function relationship between the position and orientation tracking error and the filtered tracking error \( r \) can be established using (28) and (29) as follows:

\[ e(s) = H(s) r(s) \]  

(52)

where \( s \) is the Laplace transform variable and \( H(s) \) is a strictly proper asymptotically stable transfer function. Thus, utilizing Lemma 1, the position and orientation tracking error are asymptotically stable which can be stated as \( r \in L_2^r \Rightarrow e \in L_2^e \cap L_\infty^e \), \( \dot{e} \in L_2^e \), \( e \) is continuous and \( e \to 0 \) when \( t \to \infty \). However, the velocity tracking error and the bounding estimate are only guaranteed to be bounded. In addition, when \( r(t) \) is regulated, then \( \dot{e}_{sub} \) from (31) is also regulated and the control sub-tasks can be achieved as \( t \to \infty \).

### 5.  Multiple Self-Motion Objective Functions

This section presents several sub-tasks that can be monitored during the motion of a UVMS, e.g., maintaining joint limits [2], manipulator manipulability [20], drag minimization [21], and congruent buoyancy and gravity loading optimization [22]. The weighted sum approach as in [23] is utilized in order to formulate the multiple optimization criteria for the function \( H(q) \) defined in (16).

#### 5.1 Manipulability or Singularity Avoidance

The first sub-task objective is based on singularity avoidance for the kinematically redundant of mounted manipulator, in addition to the main sub-region tracking objective. The manipulability measure is defined by [20]:

\[ H_1(q) = \sqrt{\text{det}(J^T M_J J_M)} \]  

(53)

where \( \text{det}(\cdot) \) represents the determinant of the matrix.

#### 5.2 Joint Limit Avoidance

Let consider the joint limit avoidance as a sub-task for a redundant manipulator. As in [24], the following performance function is selected:

\[ H(q)^{-1} = \sum_{i=1}^n \frac{1}{4} \frac{(q_{max} - q_{min})^2}{C_i (q_{max} - q_j)(q_j - q_{min})} \]  

(54)
where \( n \) is the number of robot joints and \( C_i \) is a positive number defining the degree of strictness of the constraint for the \( i \)th joint. Note that only the manipulator joints are weighted since there are no physical limits on the vehicle’s DOFs. This relevant objective function automatically gives higher weight to the joints approaching their limits and reaches infinity at the joint bounds. Accordingly, each term of the summation (54) takes the value one when the robot is at the furthest angle from the associated upper and lower joint limits and reaches infinity at the limits. Moreover, this function offers normalization on the variation in the motion ranges and it can be represented in a discrete-time model as discussed in [25].

### 6. Simulation Studies

In this section, simulation studies are presented to demonstrate the performance of the proposed adaptive robust controller. A fully actuated 6-DOFs underwater vehicle equipped with a 3-link revolute arm is chosen in this work. The dynamic parameters of an ellipsoidal-shape vehicle can be found in [26]. For simplicity, planar motion is considered for the manipulator working in the vertical plane; hence the number of redundant DOFs of the UVMS is one. The masses and length of the manipulator link are set as \( m_L = 3.38 \text{ kg} \) and \( L_i = 0.33 \text{ m} \) where \( i = 1, 2, \text{ and } 3 \), respectively. The links are cylindrical and the radius of each link is 0.05 m. In the simulations, arbitrary constant values are used for unknown parameters such as hydrodynamic damping matrices and added masses. To verify the robustness of the proposed controller, a class of unmodelled effects and a hydrodynamic forces term are added in (17) which are bounded by (35) [27].

Using the multiple sub-regions, the vehicle is required to track the intersection of two spherical sub-regions with radii \( r_{v1} \) and \( r_{v2} \) defined by:

\[
\begin{align*}
    f_{v1}(\delta p_{v1}) &= (x_{vd1} - x_v)^2 + (y_{vd1} - y_v)^2 \\
    &+ (z_{vd1} - z_v)^2 - r_{v1}^2 \leq 0 \\
    f_{v2}(\delta p_{v2}) &= (x_{vd2} - x_v)^2 + (y_{vd2} - y_v)^2 \\
    &+ (z_{vd2} - z_v)^2 - r_{v2}^2 \leq 0
\end{align*}
\]  

(55)

where in the Simulation 1, the centers of these two spherical regions expressed in the inertial-fixed frame are specified as:

\[
\begin{bmatrix}
    x_{vd1} \\
    y_{vd1} \\
    z_{vd1}
\end{bmatrix} = \begin{bmatrix}
    x_{vd2} \\
    y_{vd2} \\
    z_{vd2}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
    2.2 - 0.22(2\pi t/t_f - \sin 2\pi t/t_f)/2\pi \\
    0 \\
    -1.6 + 0.01(2\pi t/t_f - \sin 2\pi t/t_f)/2\pi
\end{bmatrix} \text{ [m]} \tag{57}
\]

where the vehicle is kept in stationary mode within the intersection of two spherical sub-regions and \( t_f = 50 \text{ s} \). Meanwhile, the end-effector is required to track the intersection of two circles sub-regions with radii of \( r_{m1} \) and \( r_{m2} \) defined by:

\[
\begin{align*}
    f_{m1}(\delta p_{m1}) &= (v_x v,md1 - v_x v,m)^2 \\
    &+ (v_z md1 - v_z v,m)^2 - r_{m1}^2 \leq 0 \\
    f_{m2}(\delta p_{m2}) &= (v_x v,md2 - v_x v,m)^2 \\
    &+ (v_z md2 - v_z v,m)^2 - r_{m2}^2 \leq 0
\end{align*}
\]  

(58)

The centers of these two circles, which move in a straight line, are defined as follows:

\[
\begin{bmatrix}
    v_x v,md1 \\
    v_z md1
\end{bmatrix} = \begin{bmatrix}
    v_x v,md2 \\
    v_z md2
\end{bmatrix}
\]

\[= \begin{bmatrix}
    0.35 + 0.08(2\pi t/t_f - \sin 2\pi t/t_f)/2\pi \\
    0.15 - 0.16(2\pi t/t_f - \sin 2\pi t/t_f)/2\pi
\end{bmatrix} \text{ [m]} \tag{59}
\]

where (59) is expressed in the body-fixed frame and \( t_f \) is defined in (57). The system was initialized to be at rest at the following pose: \( p_c = [220 \ -1.6]^T \text{ m} \), \( \epsilon_v = [0001]^T \), and \( q_i = [\pi/2.75 \ -\pi/1.75 \ \pi/18]^T \text{ rad} \). The desired orientation trajectory for the vehicle is constant during simulation. Two set of simulation with different sub-task objectives were carried out. In the first simulation \( H(q) \) was set to zero where there was no restriction on the self-motion such that only the sub-region tracking objective is enforced. In the second simulation, \( H(q) \) was selected as a combination of two different sub-task objectives as:

\[
H(q) = 0.5(H_1(q)) + 0.5(H_2(q)) \tag{60}
\]

where \( H_1(q) \) is defined in (53) and the second term is given by:

\[
H_2(q) = \frac{1}{4} \sum_{i=1}^{n} (q_{i_{\text{max}}} - q_{i_{\text{min}}})^2/(C_i(q_{i_{\text{max}}} - q_i)(q_i - q_{i_{\text{min}}})) \tag{61}
\]

where \( C_i = [0.25 \ 0.5 \ 0.25]^T \), \( q_{i_{\text{max}}} = 2.10 \text{ rad} \), and \( q_{i_{\text{min}}} = -2.10 \text{ rad} \); \( n = 3 \). For both simulations, the sub-regions are set to \( r_{v1} = 0.07 \text{ m} \), \( r_{v2} = 0.05 \text{ m} \), \( r_{m1} = 0.15 \text{ m} \), \( r_{m2} = 0.07 \text{ m} \).
Figure 3. Desired sub-region and actual tracking trajectories. (a) Simulation 1 with multiple sub-region and no sub-task tracking and (b) Simulation 2 with multiple sub-regions and sub-tasks tracking.

and \( r_{m2} = 0.09 \) m and the controller gains are tuned to the following:

\[
K_p = \text{diag}\{400, 400, 400, 30, 90, 25, 25\}; \\
K_r = \text{diag}\{2.08, 2.08, 2.08, 1.1, 1.1, 1.1, 0.5, 0.5\};
\]

The weighted gains \( w_{v1}, w_{v2}, w_{m1}, \) and \( w_{m2} \) are all set to 0.8. In Fig. 3, the initial pose of the UVMS is drawn by blue lines and the final pose by red lines. The black lines represent traces of the actual tracking trajectory for the end-effector and vehicle while the yellow lines are intermediate poses of the UVMS. The intersection local sub-regions are defined by the dash-dot lines. A close-up view of the end-effector trajectory tracking is shown in Fig. 4. As observed in Fig. 3(a), there is no sub-task with multiple local sub-regions applied to the UVMS. In Fig. 3(b), the multiple sub-regions and sub-tasks objectives are utilized for UVMS tracking control. In this configuration, the multiple sub-tasks objectives which are the manipulability measure and the joint limit avoidance are formulated using (61). Notice that the second joint reaches its mechanical joint limit at \( q_2 = \pm \pi/1.5 \) rad, which also requires vehicle movement. Hence, it is preferable for the manipulator not to move close to these critical situations when it can be achieved using the self-motion tracking. Meanwhile, the other links are kept in a dexterous configuration due to the manipulability measure. Meanwhile, Fig. 5 shows the required forces, moments and joint torques for a UVMS where it can be concluded that the system with self-motion criteria provides better energy saving than the one without any sub-task.
Figure 5. Required forces, moments, and torques in Simulation 1 (dash-dot lines) and Simulation 2 (solid lines).

7. Conclusion

A new adaptive robust controller is proposed in this paper for a UVMS that achieves sub-regions, attitude and sub-task tracking under the influence of modelling uncertainties and additive disturbances. Two individual sub-regions for tracking are specified for the UVMS, that is, the vehicle sub-region and the end-effector sub-region. Each individual sub-region is obtained using the intersection of the multiple local sub-regions. The applied control strategy uses the pseudo-inverse of the manipulator Jacobian and does not require the computation of the inverse kinematics. Since the proposed controller does not place any restriction on the self-motion of the manipulator, extra DoF are available for sub-tasks like maintaining manipulability, drag minimization, and obstacle avoidance. The performance optimization criteria are formulated using a weighted sum approach when the UVMS is subject to multiple sub-task tracking objectives. In addition, a least-squares estimation algorithm is used instead of gradient-type approach. Lyapunov-like functions have been utilized for stability analysis and simulation results are presented to illustrate the effectiveness of the developed controller.

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References


**Biographies**

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