DETECTION OF SACCADIC EYE MOVEMENTS USING THE ORDER STATISTIC CONSTANT FALSE ALARM RATE TECHNIQUE

Pekka-Henrik Niemenlehto
Department of Computer Sciences
University of Tampere
Kanslerinrinne 1, FIN-33014 Tampere
Finland
email: phn@cs.uta.fi

Martti Juhola
Department of Computer Sciences
University of Tampere
Kanslerinrinne 1, FIN-33014 Tampere
Finland
email: martti.juhola@cs.uta.fi

ABSTRACT
A saccade detection method built upon the order statistic constant false alarm rate technique was applied to the detection of saccadic eye movements from electro-oculographic signals. The technique is based on the adaptive thresholding approach, in which the sensitivity of the detector is adjusted according to the observed signal in order to achieve a constant false alarm rate. The investigated method is capable of sequential detection of saccades, can operate in the presence of noise, can be implemented in a computationally efficient manner, and can operate autonomously without user assistance. Therefore, it finds potential use in applications that comprise analysis of electro-oculographic signals.

KEY WORDS
Electro-oculography, Saccade detection, Constant false alarm rate.

1. Introduction
Measurement and analysis of electro-oculographic (EOG) signals has been extensively used in clinical work and research [1, 2, 3, 4, 5] and also in research on human-machine interfaces [6, 7, 8]. Depending on the goal of the particular application, the detection of rapid eye movements known as saccades and the extraction of related saccade parameters, such as maximum angular velocity, amplitude, and duration, are usually performed during the analysis of EOG signals. Although other approaches to saccade detection exist, the traditional approach is to compare the velocity or the acceleration waveform of the EOG signal to a predetermined threshold [5, 9, 10]. In our previous investigation on saccade detection [11], we considered the application of the cell averaging (CA) constant false alarm rate (CFAR) technique. The basic principle behind CA CFAR is the use of an adaptive threshold instead of a fixed one. The threshold value is continuously computed by estimating the average noise level in the observed signal. The current investigation considers the application of a derivative of the CA CFAR called the order statistic (OS) CFAR. As the name would suggest, the technique uses an order statistic in the computation of the threshold value. Previously, radar receivers have been the main application area for different types of CFAR techniques [12, 13].

2. Method
The considered saccade detection method consists of several sub-units that operate in cascade. The overall architecture is portrayed in the block diagram of Figure 1. The measured EOG signal is first pre-processed. The pre-processed signal is input to the differentiator that extracts the velocity waveform. The velocity waveform is then full-wave rectified and input to the OS CFAR processor. The threshold value is computed and, finally, the decision logic determines whether a saccade is present or not. Furthermore, if a saccade is detected, its onset and termination locations are determined. The CFAR processor and the decision logic are different from those used in the method presented in [11].
2.1 Pre-processor

The role of pre-processing is both the attenuation of noise and the suppression of artefacts. Depending on whether an alternating current (AC) or a direct current (DC) powered amplifier is used, the measured EOG signal is preprocessed with a bandpass or a lowpass filter, respectively. Regardless of the filter type, the lowpass cutoff frequency is usually chosen to lie below 50 Hz because this set-up attenuates high frequency noise as well as power-line noise. Eye blink rejection should be also performed if vertical eye movements are to be observed since strong eye blink artefacts are a potential source of false alarms. Also, before further processing, the raw voltage values are typically transformed so that they represent the angle of eye rotation in degrees. In this investigation we applied a procedure in which the signal samples are multiplied with a predetermined constant. The constant was determined according to the average saccade duration because it ensures that there are fewer reference samples from the duration of the saccade when the test sample coincides with the saccade midpoint. The number of reference samples should be large enough so that necessary noise distribution parameters can be reliably estimated. If there are $q$ reference and $g$ guard samples on both sides of the test sample, the $n$th test sample can be expressed as

$$t(n) = |v(n - q - g)|,$$

(2)

where $q$ and $g$ are integers. The OS CFAR processor sorts the $N = 2q$ reference samples

$$R_1(n), \ldots, R_q(n), R_{q+1}(n), \ldots, R_N(n),$$

(3)

where

$$R_k(n) = \begin{cases} |v(n - k + 1)| & \text{if } 1 \leq k \leq q \\ |v(n - k - 2g)| & \text{if } q + 1 \leq k \leq N \\ \end{cases},$$

(4)

in ascending order so that

$$R_{(1)}(n) \leq R_{(2)}(n) \leq \cdots \leq R_{(N)}(n)$$

(5)

and returns the $k$th smallest sample

$$r(n) = R_{(k)}(n),$$

(6)

also known as the $k$th order statistic ($k \in [1, N]$). When multiplied with a constant $s$, the order statistic provides an adaptive threshold

$$\gamma(n) = s \cdot r(n)$$

(7)

to maintain a constant false alarm rate. The constant $s$ can be estimated if the noise in the velocity waveform can be assumed to follow some known distribution.

Assuming that the noise at the differentiator output follows the normal distribution with zero mean and standard deviation $\sigma$, the noise at the output of the full-wave rectifier follows the half-normal distribution. The related probability density function is

$$f(w) = \frac{2}{\sqrt{2\pi}\sigma} \exp\left(-\frac{w^2}{2\sigma^2}\right)$$

(8)

for $w \geq 0$ and 0 otherwise. Percentage point $\pi_z$, where $z \in [0, 1]$, is a statistic that gives the value of a random variable for which 100$z$ percent of the random variable’s values are smaller than $\pi_z$. For example, $\pi_{0.5}$ is known as the median. A half-normal distribution’s percentage point $\pi_z$ can be solved from the equation

$$z = \int_0^{\pi_z} f(w) \, dw$$

The test sample is the centermost sample in the processor’s memory. A number of samples chosen from both sides of the test sample are used as reference samples. In order to decrease information overlap between the test and the reference samples, some guard samples are placed between them. The number of guard samples should be chosen according to the average saccade duration because it ensures that there are fewer reference samples from the duration of the saccade when the test sample coincides with the saccade midpoint. The number of reference samples should be large enough so that necessary noise distribution parameters can be reliably estimated. If there are $q$ reference and $g$ guard samples on both sides of the test sample, the $n$th test sample can be expressed as

$$t(n) = |v(n - q - g)|,$$

(2)

where $q$ and $g$ are integers. The OS CFAR processor sorts the $N = 2q$ reference samples

$$R_1(n), \ldots, R_q(n), R_{q+1}(n), \ldots, R_N(n),$$

(3)

where

$$R_k(n) = \begin{cases} |v(n - k + 1)| & \text{if } 1 \leq k \leq q \\ |v(n - k - 2g)| & \text{if } q + 1 \leq k \leq N \\ \end{cases},$$

(4)

in ascending order so that

$$R_{(1)}(n) \leq R_{(2)}(n) \leq \cdots \leq R_{(N)}(n)$$

(5)

and returns the $k$th smallest sample

$$r(n) = R_{(k)}(n),$$

(6)

also known as the $k$th order statistic ($k \in [1, N]$). When multiplied with a constant $s$, the order statistic provides an adaptive threshold

$$\gamma(n) = s \cdot r(n)$$

(7)

to maintain a constant false alarm rate. The constant $s$ can be estimated if the noise in the velocity waveform can be assumed to follow some known distribution.

Assuming that the noise at the differentiator output follows the normal distribution with zero mean and standard deviation $\sigma$, the noise at the output of the full-wave rectifier follows the half-normal distribution. The related probability density function is

$$f(w) = \frac{2}{\sqrt{2\pi}\sigma} \exp\left(-\frac{w^2}{2\sigma^2}\right)$$

(8)

for $w \geq 0$ and 0 otherwise. Percentage point $\pi_z$, where $z \in [0, 1]$, is a statistic that gives the value of a random variable for which 100$z$ percent of the random variable’s values are smaller than $\pi_z$. For example, $\pi_{0.5}$ is known as the median. A half-normal distribution’s percentage point $\pi_z$ can be solved from the equation

$$z = \int_0^{\pi_z} f(w) \, dw$$

The test sample is the centermost sample in the processor’s memory. A number of samples chosen from both sides of the test sample are used as reference samples. In order to decrease information overlap between the test and the reference samples, some guard samples are placed between them. The number of guard samples should be chosen according to the average saccade duration because it ensures that there are fewer reference samples from the duration of the saccade when the test sample coincides with the saccade midpoint. The number of reference samples should be large enough so that necessary noise distribution parameters can be reliably estimated. If there are $q$ reference and $g$ guard samples on both sides of the test sample, the $n$th test sample can be expressed as

$$t(n) = |v(n - q - g)|,$$

(2)

where $q$ and $g$ are integers. The OS CFAR processor sorts the $N = 2q$ reference samples

$$R_1(n), \ldots, R_q(n), R_{q+1}(n), \ldots, R_N(n),$$

(3)

where

$$R_k(n) = \begin{cases} |v(n - k + 1)| & \text{if } 1 \leq k \leq q \\ |v(n - k - 2g)| & \text{if } q + 1 \leq k \leq N \\ \end{cases},$$

(4)

in ascending order so that

$$R_{(1)}(n) \leq R_{(2)}(n) \leq \cdots \leq R_{(N)}(n)$$

(5)

and returns the $k$th smallest sample

$$r(n) = R_{(k)}(n),$$

(6)

also known as the $k$th order statistic ($k \in [1, N]$). When multiplied with a constant $s$, the order statistic provides an adaptive threshold

$$\gamma(n) = s \cdot r(n)$$

(7)

to maintain a constant false alarm rate. The constant $s$ can be estimated if the noise in the velocity waveform can be assumed to follow some known distribution.

Assuming that the noise at the differentiator output follows the normal distribution with zero mean and standard deviation $\sigma$, the noise at the output of the full-wave rectifier follows the half-normal distribution. The related probability density function is

$$f(w) = \frac{2}{\sqrt{2\pi}\sigma} \exp\left(-\frac{w^2}{2\sigma^2}\right)$$

(8)

for $w \geq 0$ and 0 otherwise. Percentage point $\pi_z$, where $z \in [0, 1]$, is a statistic that gives the value of a random variable for which 100$z$ percent of the random variable’s values are smaller than $\pi_z$. For example, $\pi_{0.5}$ is known as the median. A half-normal distribution’s percentage point $\pi_z$ can be solved from the equation

$$z = \int_0^{\pi_z} f(w) \, dw$$
\[ z = \text{erf}\left(\frac{\pi z}{\sigma\sqrt{2}}\right) \]

\[ \text{erf}^{-1}(z) = \frac{\pi z}{\sigma\sqrt{2}} \]

\[ \sigma\sqrt{2}\text{erf}^{-1}(z) = \pi z. \]  

where \( \text{erf}(\cdot) \) is the (Gauss) error function. Hence,

\[ \gamma = \sigma\sqrt{2}\text{erf}^{-1}(1 - P_{\text{fa}}). \]  

If the noise characteristics did not vary, the threshold value corresponding to a certain false alarm probability \( P_{\text{fa}} \) could be obtained with the inverse half-normal cumulative distribution function

\[ \gamma = \sigma\sqrt{2}\text{erf}^{-1}(1 - P_{\text{fa}}). \]  

Order statistics can be computed with the above equation by expressing them as percentage points. Now that \( N \) is even, the \( k \)th order statistic is given by

\[ \beta_k = \pi(k-0.5)/N = \sigma\sqrt{2}\text{erf}^{-1}\left(\frac{k-0.5}{N}\right). \]  

If the noise characteristics did not vary, the threshold value corresponding to a certain false alarm probability \( P_{\text{fa}} \) could be obtained with the inverse half-normal cumulative distribution function

\[ \gamma = \sigma\sqrt{2}\text{erf}^{-1}(1 - P_{\text{fa}}). \]  

Noting that \( r(n) \) gives a running estimate of \( \beta_k \) and \( \gamma(n) \) is an adaptive form of \( \gamma \), the constant \( s \) can be estimated by substituting \( \beta_k \) and \( \gamma \) into Equation (7):

\[ s \approx \gamma/\beta_k \]

\[ = \frac{\sigma\sqrt{2}\text{erf}^{-1}(1 - P_{\text{fa}})}{\sigma\sqrt{2}\text{erf}^{-1}\left(\frac{k-0.5}{N}\right)} \]

\[ = \frac{\text{erf}^{-1}(1 - P_{\text{fa}})}{\text{erf}^{-1}\left(\frac{k-0.5}{N}\right)}. \]  

The estimate is a function of \( P_{\text{fa}}, k, \) and \( N \). Consequently, the probability of false alarm can be constrained for a combination of parameters \( k \) and \( N \) simply by choosing the constant \( s \) appropriately.

2.5 Decision Logic

Detection decisions are made by comparing the test sample to the derived threshold value. A decision that a saccade candidate is present is made if

\[ t(n) > \gamma(n). \]  

Further detections are suppressed until

\[ t(n) < \gamma(n) \quad \text{and} \quad t(n) < \gamma_r(n) = r(n), \]  

where \( \gamma_r(n) \) is called refraction threshold. As a result, new detections can not be made until the overall detector leaves the immediate range of influence of the previously detected saccade. After a saccade candidate has been detected, a total of \( d_{\text{max}} \) previous values of the test sample are searched backwards until

\[ t(n) < \gamma_r(n). \]  

If such refraction does not take place during the search period, then the local minimum is chosen to represent the refraction location. The saccade onset location can be estimated with

\[ n_{\text{test}} = n_{\text{ref}} - (p + q + g), \]  

where \( n_{\text{ref}} \) is the refraction location and the sum in the parenthesis is the group delay of the overall system, excluding the group delay of the pre-processor. The saccade termination location can be estimated in a similar manner, but there is no need to perform the search because the refraction location is available as soon as \( t(n) \) falls below both thresholds. Additional control over the decision logic is administered with the suppression threshold \( \gamma_s \). A saccade candidate is rejected if

\[ |\text{avg}_\text{ons} - \text{avg}_\text{trim}| < \gamma_s. \]  

3. Computational Requirements

Because of its simple design, the two-point central difference can be realized with only one multiplier and one adder. The full-wave rectifier, on the other hand, is essentially a single absolute value operator. Therefore, both the differentiator and the rectifier operate in constant time. The computation of the \( k \)th order statistic is a more time consuming operation. Particularly, it requires the sorting of the reference samples, but at the same time their temporal ordering has to be preserved. A feasible approach is to use two data structures: an order statistic tree and a circular buffer. The first maintains spatial and the second maintains temporal ordering of the reference samples. An order statistic tree is simply a red-black tree with additional information stored in each node that enables retrieval of order statistics [15]. This self-balancing binary search tree is computationally efficient since insertion, removal, and retrieval operations require \( O(\log N) \) time. First, a new reference sample is inserted into the tree. A pointer to the new node is then inserted into the circular buffer. Finally, the oldest pointer that was replaced is used to remove the corresponding node from the tree and the specified order statistic is retrieved. The estimation of the saccade onset and termination locations take \( O(d_{\text{max}}) \) and constant time, respectively. Hence, the overall time complexity for a single time step is the maximum of \( O(\log N) \), \( O(d_{\text{max}}) \), and the time complexity of the pre-processor.

31
4. Application Examples

Figure 2 shows the investigated method in operation on a horizontal EOG signal that was measured with bipolar surface electrodes and a DC powered amplifier. The sampling frequency was 400 Hz with a passband of 0–100 Hz and the amplitude resolution was 13 bits (that is to say, 12 bits and the sign bit). The following parameters were used in the saccade detection: \( pT = 6/400 \) s, \( q = 100, g = 20 \), median as the order statistic, whereupon \( k = 100.5, P_{th} = 10^{-3} \), which yields \( s \approx 4.879, \gamma_s = 1.5^\circ, d_{\text{max}} = 50, \) and \( M = 50 \). Because of the high quality of the signal, even the small corrective saccades that are of a larger amplitude than the specified suppression threshold were detected.

Figure 3 shows the method in operation on a horizontal EOG signal that was measured with bipolar surface electrodes and an AC powered amplifier. The sampling frequency was 200 Hz with a passband of 0.05–40 Hz and the amplitude resolution was 24 bits. Because of the type of the amplifier, there is noticeable drift during fixations. The following parameters were used in the saccade detection: \( pT = 3/200 \) s, \( q = 50, g = 10 \), median as the order statistic, whereupon \( k = 50.5, P_{th} = 10^{-3} \), which yields \( s \approx 4.879, \gamma_s = 2.5^\circ, d_{\text{max}} = 25, \) and \( M = 25 \). The signal is evidently of a lower quality than the one used in the previous example. Therefore, only saccades of a relatively large amplitude can be distinguished. A larger suppression threshold had to be used in order to suppress false alarms caused by impulse noise artefacts, such as the one shown in the figure. Indeed, the presented decision logic did not function here as well as it did with the DC measured signal. This is mainly because of the drift that is present in AC measured signals. The drift distorts the averages \( \text{avg}_{\text{ons}} \) and \( \text{avg}_{\text{off}} \), whereupon it affects the functioning of the decision logic.

5. Discussion

The current investigation considered the application of the OS CFAR technique to saccade detection in electro-oculography. The presented method is capable of sequential detection of saccades, can operate in the presence of noise, and can be implemented in a computationally efficient manner. In addition, excluding the choice of certain parameters, the method can operate autonomously without user intervention. Hence, the method can be used in a multitude of real-time and non-real-time applications that comprise analysis of EOG signals. The capability of operating in the presence of noise is due to the CFAR property. However, as the detection threshold is adjusted to accommodate for noise, the probability of detection is decreased, which means that smaller saccades could be missed. Also, the CFAR property holds only if the true noise distribution resembles the assumed noise distribution. The decrease in detection probability has to be tolerated when using a detector that has the CFAR property and, particularly, if the desired false alarm probability is low. On the other hand, the desired false alarm probability could be higher if a more robust decision logic was employed. The decision logic has to be improved so that it can cope with the drift that is present in AC measured signals. In addition, more detailed experiments have to be conducted in order to assess the performance of the method.

Acknowledgements

The first author is grateful to Tampere Graduate School in Information Sciences and Engineering (TISE) for financial support.

References


Figure 3. Example of application of the investigated saccade detection method on an AC measured horizontal EOG signal. Detected saccade onsets and terminations have been marked with circles (○) and squares (□), respectively. An impulse noise artefact that initially caused a false alarm has been marked with an asterisk (*).


