CONTOURLET TRANSFORM BASED COMPLEX DIFFUSION FOR ULTRASONIC SPECKLE REDUCTION

Jinhua YU, Yuanyuan WANG
Department of Electronic Engineering, Fudan University
220 Handan Road, 200433 Shanghai, China
{jhyu, yywang}@fudan.edu.cn

ABSTRACT
A novel multiscale anisotropic diffusion method is proposed for ultrasonic speckle reduction. This method, which extends the nonlinear complex diffusion to a nonsubsampled contourlet domain, is called a contourlet transform based complex diffusion (CTCD). The nonsubsampled contourlet transform provides a flexible, multiscale, multidirection, and shift-invariant image decomposition with a good contour capturing and representation. Then, the nonlinear complex diffusion removes noise from the contourlet coefficients with the imaginary values serving as a robust edge-detector. The directionality and multiresolution features of the nonsubsampled contourlet transform and the robust anisotropic smoothing feature of the complex diffusion make the hybrid filter produce satisfactory despeckling results. Both synthetic and real ultrasound images are used to evaluate the performance of the filter, and denoising results are also compared with other three competitive speckle reduction algorithms. It is shown that the proposed method is superior to other methods in both noise reduction and structure preservation.

KEY WORDS
Ultrasonic speckle reduction, contourlet transform based complex diffusion, nonsubsampled contourlet transform, nonlinear complex diffusion.

1. Introduction
Ultrasound imaging is one of the most prevalent diagnostic techniques in a range of clinic applications, due to its noninvasiveness, portability and relatively low cost. However, automatic interpretation of ultrasound images is extremely difficult because of its low signal to noise ratio (SNR). One of the main reasons for this low SNR is the presence of speckle noise. Speckle noise is a phenomenon that inherents in any coherent imaging process in which the superposition of acoustical echoes produce an intricate interference pattern. As the most prominent artifact, speckle noise tends to obscure diagnostically important details in images. Therefore, in the past few decades extensive researches have been made in the field of speckle noise reduction.

Loupas et al. [1] proposed a speckle reduction filter that was called the adaptive weighted median filter (AWMF). AWMF suppressed the speckle noise by utilizing weighted median filter, in which weight coefficients and smoothing characteristics were adjusted according to the local statistics around each pixel of images. Inspired by Loupas et al’s work, Karaman et al. [2] proposed an adaptive speckle suppression filter (ASSF) which was also based on the local statistics of images. The filter adaptation was achieved by using appropriately shaped and sized local filtering kernels. Czerwinski et al. [3] proposed a ‘stick’ method to detect tissue boundary and suppress speckle noise based on an image enhancement technique. However, the denoising performances of above filters are highly sensitive to some empirically determined parameters, such as the size and shape of the filter window. In addition, these methods usually do not respect edges, which make the speckle remaining in the neighborhood of an edge.

The partial differential equation (PDE) - based nonlinear diffusion processes have been broadly used in noise smoothing over the past decade, especially in speckle noise reduction for synthetic aperture radar (SAR) images, medical ultrasound images, and optical coherence tomography images. Unlike conventional spatial filtering techniques that do not respect region boundaries or small structures, anisotropic diffusion techniques can simultaneously eliminate noise and preserve or even enhance edges. Yu and Acton [4] proposed a speckle reducing anisotropic diffusion (SRAD), which integrated the spatial adaptive filters into the diffusion technique, and thus provided a dramatic improvement in speckle suppression compared with other traditional anisotropic diffusion methods, such as Lee filter [5], Perona-Malik filter [6], and Catté filter [7]. Adb-Elmoniem et al. [8] proposed a nonlinear coherent diffusion models for speckle reduction and coherence enhancement. This method combined isotropic diffusion, anisotropic coherent diffusion and mean curvature motion, and pursued maximally low-pass filtering for parts of the image correspond to fully developed speckle while preserving information associated with object structures. Salinas and Fernández [9] introduced the nonlinear complex diffusion (NCD) proposed by Gilboa et al. [10] into the speckle reduction for optical coherence
tomography images. The nonlinear complex diffusion combined properties of both forward and inverse diffusion, and achieved performance improvement over the traditional Perona-Malik filter [6] in term of noise suppression, image structural preservation and visual quality [9], [10].

For speckle reduction in ultrasound images, several methods also have been proposed based on multiscale analysis. One group of these methods was based on the soft-thresholding in wavelet domain, which was first presented by Donoho [11]. Due to the complexity of speckle statistics, the main difficulty in applying these thresholding methods in wavelet domain is how to balance between speckle suppression and feature preservation. Yue et al. [12] presented a novel ultrasound speckle suppression method by employing the iterative multiscale diffusion on the wavelet domain. Yue et al.’s work indicated that by combining the sparsity and multiresolution properties of wavelets, the nonlinear diffusion in multiscale can provide improved speckle suppression performance. Recently, Zhang et al. [13] presented a Laplacian pyramid-based nonlinear diffusion (LPND) method. Different from Yue et al.'s work, Laplacian pyramid was utilized for multiscale nonlinear diffusion instead of wavelet transform. The main idea behind the Yue et al. and Zhang et al.'s work was to achieve a more accurate separation between speckle noise and useful signal components based on the multiscale analysis, and thus a better denoising result.

Contourlet transform (CT) is a newly developed multi-scale and multi-direction framework of discrete image [14]. Compared with other multiscale analysis tools such as wavelet and Laplacian pyramid, contourlet transform not only holds their main features in multiscale and time-frequency localization, but also possesses several advantages in multiresolution image representation. The most favorable advantage of CT in denoising filter design is its ability in capturing intrinsic geometrical structures. Nonsubsampled contourlet transform (NSCT) [15] is an improved version of CT. The shift-invariant feature of NSCT makes it possible to design filters with better frequency selectivity and thereby better subband decomposition than CT. In this paper, nonsubsampled contourlet transform is integrated with the nonlinear complex diffusion process (NCD). When the directionality and contour detection ability of NSCT and the robust anisotropic denoising ability of NCD can be well utilized, it is expected that NSCT based complex diffusion can produce better despeckling effect for ultrasound images. We refer to the integration of nonsubsampled contourlet transform and nonlinear complex diffusion as the contourlet transform based complex diffusion (CTCD).

The remainder of this paper is organized as follows. Section 2 introduces the background of the proposed method. Section 3 describes the proposed contourlet transform based complex diffusion (CTCD) method. The numerical implementation of CTCD is also discussed in this section. Section 4 compares CTCD with other three competitive despeckling algorithms. Section 5 concludes this paper.

2. Background

2.1 Nonsubsampled contourlet transform (NSCT)

Nonsubsampled contourlet transform (NSCT) is constructed by combining two distinct and successive decomposition stages: a multiscale decomposition followed by a multidirectional decomposition. The nonsubsampled Laplacian pyramid (NSLP) is used to decompose an image into a number of NSLP subbands, and the nonsubsampled directional filter bank (NSDFB) decomposes each NSLP detail subband into a number of directional subbands. Fig. 1(a) shows the filter bank structure of NSCT and Fig. 1(b) shows the idealized frequency partitioning obtained with NSCT. The overall result of NSCT is an image expansion using basic elements like contour segments.

The NSCT offers a flexible multiresolution and directional decomposition for images, since it allows for a different number of directions at each scale. Compared with NSCT, two dimensional (2D) wavelets only provide three directional components, namely horizontal, vertical, and diagonal. Thus, 2D wavelets will only be good at isolating the discontinuities at edge points, but can not capture the geometrical smoothness of contours. Laplacian pyramid provides a multiscale representation of images; however the direction of contours also can not be captured. Because NSCT offers a much richer set of directions and shapes, they are more effective in capturing smooth contours and geometric structures in images [15].

2.2 Anisotropic Diffusion and Nonlinear Complex Diffusion

Perona and Malik [6] first proposed a nonlinear diffusion based on a partial differential equation (PDE) to smooth an image in a continuous domain:

![Fig. 1. Nonsubsampled contourlet transform, (a) the filter bank structure of NSCT and (b) idealized frequency partitioning.](image-url)
where $\nabla$ is the gradient operator, $\|\|$ denotes the magnitude, $c(\|\nabla I\|)$ is the diffusivity function or the “edge-stopping” function, $\text{div}$ is the divergence operator, and $I_0$ is the original image. The anisotropic diffusion produces an image with less noise from a set of increasingly smooth images $\{I_p\}$, indexed by a simulated time parameter $t$. The diffusion is discouraged across boundaries with large gradient magnitudes and encouraged within homogeneous regions with small gradient magnitudes.

For images corrupted by the heavy noise, the gradient operator cannot achieve an effective separation of edges and noise. Thus, the original Perona and Malik model is highly sensitive to noise and can not produce satisfactory despeckling effect for ultrasound images. Gilboa et al. [10] generalized the linear scale spaces in the complex domain by combining the diffusion equation with the free Schrödinger equation. Compared with the Perona-Malik model, the nonlinear complex diffusion (NCD) approach does remove noise from edges and reduce noise more effectively. The PDE equation for NCD can be represented by

$$\frac{\partial I}{\partial t} = \text{div}[c(\text{Im}(I)) \cdot \nabla I]$$  \hspace{1cm} (2)

where $\text{Im}(\cdot)$ is the imaginary value and the diffusion function is defined as

$$c(\text{Im}(I)) = \frac{\exp(i\theta)}{1 + \left(\frac{\text{Im}(I)}{\eta_k}\right)^2}$$  \hspace{1cm} (3)

where $k$ is a threshold parameter and $\theta \in (-\pi/2, \pi/2)$ is the phase angle. Gilboa et al. proved that when $\theta \to 0$, the imaginary part can be considered as a smoothed second derivative of the initial signal; factored by $\theta$ and the time $t$. So, the imaginary part serves as a robust edge-detector, which can handle noise well and provide better control for nonlinear process.

### 3. Contourlet Transform Based Complex Diffusion (CTCD)

We refer to the integration of nonsubsampled contourlet transform (NSCT) and nonlinear complex diffusion (NCD) as the contourlet transform based complex diffusion (CTCD). The principle for speckle noise suppression in CTCD is to use nonlinear complex diffusion filter applied on the contourlet transform coefficients. CTCD consists of three stages. The first stage is to transform an image into its corresponding nonsubsampled contourlet domain. The second stage is to manipulate the contourlet transform coefficients by using the nonlinear complex diffusion filter. The final stage is the reconstruction of the diffused contourlet coefficients. The second stage is the crucial step for speckle reduction. In the frequency domain, both noise and weak edges produce low-magnitude coefficients. Since weak edges are geometric structures and noise is not, a more efficient discrimination between weak edges and noise can be achieved by NSCT rather than by wavelet or Laplacian pyramid. In addition, as high frequency component, speckle noise mainly exists in fine scales of decomposition coefficients. With the increasing of the decomposition layer, the scale turns to be coarser and the high frequency component attenuates, which means the speckle noise component decreases in coarser scale. Therefore, performing different complex diffusion in each subband image can effectively suppress noise without blurring weak edges. An important parameter controlling the performance of the complex diffusion is to select an appropriate diffusion threshold. According to the above analysis, a relatively large diffusion threshold should be applied in the lower decomposition layers since the speckle noise component is dominant, and a smaller one should be used in higher layers since the speckle noise component decreases. Suppose the pyramid layer is represented by $l$, and the initial diffusion threshold is represented by $k$, then the diffusion threshold for each layer can be calculated by [13]

$$k(l) = k \cdot 2^{l+1}$$  \hspace{1cm} (4)

The initial diffusion threshold is estimated using the robust median absolute deviation (MAD) estimator [16]

$$k = \frac{\text{MAD}(\frac{I_\lambda}{\lambda})}{0.6745}$$  \hspace{1cm} (5)

The constant is based on the fact that $MAD$ of a zero-mean normal distribution with unit variance is 0.6745.

The differential equation (2) can be solved numerically using iterative approach. A discrete form of the nonlinear complex diffusion in each subband image can be expressed as:

$$I_p^{t+1} = I_p^t + \frac{\lambda}{|\eta_p|} \sum_{q \in \eta_p} c(\text{Im}(I_p^t, q)) \nabla I_p^t$$  \hspace{1cm} (6)

where $I_p^t$ is the discretely sampled image, $p$ denotes the pixel position in a discrete two-dimensional grid, $t$ denotes the discrete time step, $\eta_p$ represents the spatial neighborhood of pixel $p$, and $|\eta_p|$ is the number of pixels in the window (usually four, except at image boundaries). The gradient value is linearly approximated as:

$$\nabla I_p^t = I_p^t - I_q^t, \quad \forall q \in \eta_p$$  \hspace{1cm} (7)

and

$$c(\text{Im}(I_p^t, q)) = \frac{\cos \theta + i \sin \theta}{1 + \left(\frac{\text{Im}(I_p^t, q)}{\eta_q}\right)^2}$$  \hspace{1cm} (8)

The implementation of the proposed CTCD can be summarized in the following steps:

1. **Step 1:** Transform an image into its corresponding nonsubsampled contourlet domain.
2. **Step 2:** Perform the nonlinear complex diffusion in each subband.
   a. Estimate the diffusion threshold using (4).
   b. Calculate the gradient value within neighborhood $\eta_p$ using (7).
   c. Compute the diffusion coefficients using (8).
   d. Perform the diffusion using (6).
e) Perform above procedures in Step 2) until the diffusion convergence.

Step 3) Reconstruct the diffused contourlet coefficients.

4. Experimental Results and Discussion

To evaluate the performance of CTCD, experiments are done with synthetic and real clinical ultrasound images. For all test images, the performance of CTCD is compared with that of nonlinear complex diffusion filter (NCD) [10], speckle reducing anisotropic diffusion (SRAD) [4], and the Laplacian pyramid-based nonlinear diffusion (LPND) [13].

To quantitatively evaluate the performance of the proposed method, we first experiment with synthetic images. We quantify the algorithm performance in terms of the structure similarity (SSIM) [17] and Pratt’s figure of merit (FOM) [4], [12]. SSIM is based on the adaptation of the human visual system to the structural information in a scene, and it takes into account three different similarity measures, namely, luminance, contrast, and structure

\[ SSIM(I,J) = \frac{1}{l(I,J)^2 \times c(I,J)^2 \times s(I,J)^2} \]  

where \( I \) and \( J \) are the images to be compared, and \( l(\cdot), c(\cdot), \) and \( s(\cdot) \) are the luminance, contrast, and structure comparison function, respectively. All involved parameters are set as suggested in [17]. SSIM ranges between 0 and 1, with unity for the ideal structure preservation.

To compare edge preservation performance of different methods, we use the FOM defined as:

\[ FOM = \frac{1}{\max\{N_{\text{real}}, N_{\text{ideal}}\}} \sum_{i=1}^{N_{\text{ideal}}} 1 + \frac{d_i^2}{e} \]  

where \( N_{\text{real}} \) and \( N_{\text{ideal}} \) are the number of detected and ideal edge pixels, respectively, \( d_i \) is the Euclidean distance between the \( i \)th detected edge pixel and the nearest ideal edge pixel, and \( e \) is a constant typically set to 1/9. FOM ranges between 0 and 1, with unity for the ideal edge detection.

We first apply four algorithms to a synthetic ultrasound image. The backscatter cross section distribution of the synthetic image is illustrated in Fig. 2(a), which contains three regions of interest (ROIs), with several small structures in each ROI. The spatial correlated speckle noise is generated by low-pass filtering a complex Gaussian random field and taking the magnitude of the output. The synthetic image is shown in Fig. 2(b). Fig. 2(c)-(f) show the filtered results of NCD, SRAD, LPND and CTCD, respectively. Table 1 summarizes the SSIM and FOM performance of four filtering methods.

In order to further test the performances of four methods in different noisy conditions, we change speckle noise degrees by using complex Gaussian random field with different variations. A set of synthetic images with the mean square error (MSE) 1483.2 ± 216.26 (mean ± deviation) are processed and compared. Table 2 summarizes the comparison results of SSIM and FOM in terms of (mean ± deviation).

From the experimental comparisons, we can see that all of four algorithms eliminate the speckle in the image. However, NCD produces slight “blocky” effect around object boundary. LPND and CTCD smooth background better than NCD and SRAD. CTCD produces the best noise removal and detail preservation.

Four despeckling filters are also applied to real clinical ultrasound images. Fig. 3(a) shows a clinical ultrasound image of an embryo sac. Fig. 3(b) to (e) shows results of NCD, SRAD, LPND and CTCD, respectively. For clear illustration, the profiles, along the highlight line in the
original image, are also compared. Compared to other methods, CTCD shows the most contour smoothness and continuity while producing satisfactory speckle suppression in homogenous region. NCD enhances edges but produces unpleasant “fluctuant” effect. SRAD dilates bright regions such as area around fetal eye. Literature [13] also pointed out that, with the SRAD, the boundaries of bright regions will be broadened and those of dark regions are shrunk. Due to the contour capturing ability of NSCT, CTCD preserve the contour coherence and suppress more speckle noise than LPND.

5. Conclusion

This paper proposed a novel despeckling method for ultrasound images. Speckle noise is a multiplicative noise with a quite complex statistical distribution. The proposed CTCD method incorporated the nonsubsampled contourlet transform into the nonlinear complex diffusion to improve the performance of the anisotropic diffusion in denoising ultrasound images. Thanks to the multiscale and contour capturing ability of NSCT and the robust anisotropic smoothing ability of complex diffusion, the proposed CTCD method outperforms the nonlinear complex diffusion, speckle reducing anisotropic diffusion, and the Laplacian pyramid-based nonlinear diffusion by effectively suppressing noise, while accurately preserving image structure.

Acknowledgements

This work was supported by the National Basic Research Program of China (No. 2006CB705707), Natural Science Foundation of China (No.30570488), Shanghai Leading Academic Discipline Project (No.B112) and Postgraduate Innovation Fund of Fudan University (No. EYH1220001).

References