REAL-TIME EXTRACTION OF VISUAL EVOKED POTENTIALS USING FOURTH-ORDER CUMULANT

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ABSTRACT
In order to detect and extract Visual Evoked Potentials (VEP), we use the fourth order cumulant of the observed noisy signal in an adaptive filter. The noisy signal is passed through a Finite Impulse Response (FIR) filter whose impulse response is estimated using the fourth order cumulant of the input signal. We also use a method to recursively utilize fourth-order statistics of the input signal for updating the coefficients of the adaptive FIR filter. This enables us to extract the VEP signal in real time. We show that the fourth order cumulant based method provides better results in comparison to the third order cumulant based method and both yield better results as compared to the widely used autocorrelation function.

KEY WORDS

1. Introduction
Visual evoked potentials (VEPs) represent the gross electrical activity of a specific region of the brain usually resulting from a visual stimulation. Like other types of evoked potentials, the raw VEP is corrupted by noise as a result of the on-going activity of the brain cells. Signal-to-Noise Ratios (SNRs) of the raw VEPs are often less than 0.0 dB [1]-[3]. Ensemble averaging and weighted ensemble averaging have been widely used to extract evoked potentials from a noisy background [3]-[5]. It has been shown that evoked potentials are non-stationary and therefore have characteristics that vary across stimuli [3],[5]. Thus, averaging methods fail to track dynamic changes that take place both in the latency and in the amplitude of the evoked potentials. Classical filtering with fixed bandwidths is not usable for SNR enhancement of VEPs, as the spectrums of VEPs and EEG overlap [3].

Adaptive filters have been extensively used for estimation of evoked potentials [1],[2],[6]. They can track dynamic variations of EPs and reduce the noise that is uncorrelated with the underlying signal. The performance of an adaptive filter greatly depends on its reference signal, several of which have been used to extract evoked potentials [1]-[2]. The objective is to construct a reference signal that resembles the signal of interest. Furthermore, such filters are effective when the noise is additive white Gaussian (AWGN). However, if the noise is colored, the adaptive filter’s impulse response is affected by the cross-correlation of the signal and colored noise, and its output contains the noise as well as the signal [6].

During the past decade various methods have been developed in which Higher-Order-Statistics (HOS or cumulants) are used for signal detection in Gaussian noise. The main advantage of using higher-order statistics is their insensitivity to the colored Gaussian noise. Such approaches have been developed for harmonic retrieval [7], spectral estimation [8], and line enhancement [9]. Gharieb and Cichocki in [10] estimated the impulse response of the FIR matched filter by using a selected slice of the third order correlation of the input noisy signal. In [11] we presented a framework for using higher-order statistics for detection of transient signals embedded in Gaussian noise. Here, we apply the same to extract VEPs to demonstrate the usefulness and applicability of the approach.

In this paper we use a method to recursively utilize higher-order statistics of the input signal for updating the coefficients of an adaptive FIR filter. This enables us to extract the signal in real time and improves the SNR at the output of the filter as compared to the use of third order correlation in [10]. We will show that the fourth order cumulant based method yields better results as compared to the second and the third order correlation based methods. The results are better than that of [10].
2. Problem Statement and Background Materials

A. Problem Statement

We wish to extract a VEP signal \( s(n) \) from a noisy observation \( x(n) \). The signal \( s(n) \) is modeled as sum of \( P \) exponentially damped sinusoids [10],[12], and is contaminated by additive colored Gaussian noise \( \nu(n) \) of zero mean and unknown covariance, i.e.,

\[
x(n) = s(n) + \nu(n) = \sum_{k=1}^{P} A_k e^{jw_k n} + \nu(n), \quad 0 \leq n \leq N-1
\]  

where \( N \) is the number of samples in \( s(n) \) and

\[
A_k = \alpha_k e^{jw_k}, \quad B_k = (\xi_k + j\omega_k)T
\]  

where \( \alpha_k \) is the amplitude, \( \xi_k \) is the damping coefficient, \( \omega_k \) is the frequency, and \( \phi_k \) is the phase of the \( k \)th sinusoid respectively, and \( T \) is the sampling time. We assume that \( P, \alpha_k, \xi_k, \omega_k \) and \( \phi_k \) are unknown constants and \( \nu(n) \) is a zero-mean additive noise statistically independent of \( s(n) \). We further assume that \( \nu(n) \) is the output of a stable, linear time-invariant (LTI) filter driven by independent and identically distributed (i.i.d.) random variable with Gaussian distribution and bounded HOS. Given a finite data length, the problem is to extract \( s(n) \) from \( x(n) \).

B. Fourth Order Cumulant

For ease of reference, we repeat some background material presented in [13] on the fourth order cumulant.

The fourth-order cumulant of a stationary zero-mean process \( x(t) \) is

\[
c_{4x}(\tau_1, \tau_2, \tau_3) = E[x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)] - \sum_{i,j} c_{2x}(\tau_i, \tau_j) c_{2x}(\tau_j, \tau_i) - c_{2x}(\tau_1, \tau_2) c_{2x}(\tau_2, \tau_3) - c_{2x}(\tau_1, \tau_3) c_{2x}(\tau_3, \tau_1) - c_{2x}(\tau_1, \tau_3) c_{2x}(\tau_2, \tau_3)
\]

where \( c_{2x}(\tau_1, \tau_{k-1}) \) is the cumulant of the \( k \)th order. For a finite length deterministic signal \( x(n), n=0,...,N-1 \), the \( k \)th order moment is

\[
m_k(\tau_1,...,\tau_{k-1}) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)(x(n+\tau_1)...x(n+\tau_{k-1}))
\]

To obtain a consistent sample estimate of cumulants, we assume \( \sum_{\tau_1,...,\tau_{k-1}} k_{kx}(\tau_1,...,\tau_{k-1}) < \infty \) for \( k = 1, ..., k_0 \), where \( k_0 \) is twice the value of the highest-order cumulant of interest. Given a sample sequence \( x(n), n=0,...,N-1 \) for \( k_0 = 8 \), and as stated in [14], the estimate of fourth-order cumulant is

\[
\hat{c}_{4x}(\tau_1, \tau_2, \tau_3) = \frac{1}{N} m_{4x}(\tau_1, \tau_2, \tau_3) - \frac{1}{N} [m_{2x}(\tau_1)m_{2x}(\tau_2 - \tau_3) - m_{2x}(\tau_2)m_{2x}(\tau_3 - \tau_1)]
\]

where \( m_{kx}(\cdot) \) is the deterministic \( k \)th order moment of \( x(n) \).

Now we consider several properties of cumulants. It is shown in [3] that for any Gaussian \( v(n) \) of unknown covariance, we have

\[
c_{kx}(\tau_1,...,\tau_{k-1}) = 0 \quad \text{for all } k > 2
\]  

Cumulants are additive, so the cumulant of the sum is equal to the sum of cumulants. This implies that if noise is added to the signal as in (1), we have

\[
c_{kx}(\tau_1,...,\tau_{k-1}) = c_{kx}(\tau_1,...,\tau_{k-1}) + c_{kx}(\tau_1,...,\tau_{k-1})
\]

Furthermore, we use (6) to write

\[
c_{kx}(\tau_1,...,\tau_{k-1}) = c_{kx}(\tau_1,...,\tau_{k-1})
\]

and use (8) to improve the SNR when noise is additive Gaussian.

3. Methods

As in [15], we utilize a matched filter whose impulse response is

\[
h(n) = s(N-1-n) \quad 0 \leq n \leq N-1
\]

where \( N \) is the length of \( s(n) \). Since \( s(n) \) is the unknown VEP, we obtain its estimate by utilizing the HOS of the noisy signal. We use a one-dimensional slice of the cumulants by setting \( \tau_1 = \tau \) and \( \tau_j = 0 \) for \( 1 < j \leq k-1 \), and write

\[
h(\tau) = \hat{c}_{kx}(\tau,0,...,0)
\]

For convenience, we use the simple notation \( \hat{c}_{kx}(\tau) \) to represent \( \hat{c}_{kx}(\tau,0,...,0) \).

Now, we compute the impulse response of the filter as

\[
h(\tau) = \hat{c}_{kx}(P-\tau), \quad \tau = 0,1,...,2P
\]

where \( P \) is the order of the matched filter. Eq. (13) implies that the length of the impulse response is \( P+1 \). The impulse response of a causal filter is symmetric in time.

For real time tracking of the signal, we use the following recursive algorithm to estimate the cumulant

\[
\hat{c}_{kx}(\tau | n) = \lambda \hat{c}_{kx}(\tau | n-1) + (1 - \lambda) |x(n)x(n+\tau)x(n+\tau)|
\]

where \( \tau = 0,1,...,K \), and \( 0 \leq \lambda < 1 \) is the so-called forgetting factor. Small values of \( \lambda \) yield fast tracking but poor smoothing, and large values result in slow convergence but better smoothing. The absolute sign in (12) is to avoid negative values in higher order cumulants.

4. Results

A. Artificial VEP

To demonstrate the effectiveness of our proposed method, we use an artificial VEP constructed from (1). Fig 1(a) shows \( s(n) \) vs. time, and Fig. 1(b) shows its power spectrum. We also generate \( v(n) \) in (1) as

\[
v(n) = z(n) + g(n)
\]

where \( z(n) \) is the white Gaussian noise and \( g(n) \) is the colored Gaussian noise generated by passing the white
Gaussian noise through a 6th order band pass Butterworth IIR filter with cutoff normalized frequencies of 0.04 and 0.08. Figs. 1(c) and 1(d) show $v(n)$ vs. time and its power spectrum respectively. Figs. 1(e) and 1(f) show the noisy signal with SNR = -10 dB, and its corresponding power spectrum respectively.

The results for non-adaptive method are shown in Fig. 2. We set the order of the matched-filter to $P=28$. Fig. 2(a) shows the enhanced signal vs. time using the conventional autocorrelation based method and Fig. 2(b) shows its corresponding power spectrum. In Figs. 2(c) and 2(d) the enhanced signal in the third-order cumulant based method and its corresponding power spectrum are shown. The output of the fourth order cumulant based filter and its corresponding power spectrum are shown in Figs. 2(e) and 2(f) respectively. We observe that the result in the case of fourth-order cumulant is better than those based on autocorrelation and third-order cumulant.

Now we apply the autocorrelation based, the third order cumulant based and the fourth order cumulant based adaptive approaches to a quasi-periodic version of the noisy VEP in Fig. 1(e). The SNR for quasi-periodic noisy VEP at the input is –10 dB. The results are shown in Fig. 3 for two matched filters with orders $P=32$ and 64. Note that the fourth order cumulant based (adaptive) (Fig. 3), and non-adaptive (Fig. 2) approaches yield better results as compared to the autocorrelation-based and to the third order cumulant based methods. Furthermore, the fourth order cumulant based adaptive method enables us to extract the VEP in real time. It is evident from Fig. 4 that increasing the order of the matched-filter from 32 to 64 yields a better SNR.

B. Real VEP Acquisition and Processing

Now we apply our proposed method to extract the VEP from actual recording of human subjects. The subjects are males, between 19 and 23 years old, and without any visual disorders. We apply a simple grating pattern which has 100% contrast and 2 cpd (cycle per degree) spatial frequency. Fig. 4(a) shows the recorded VEP and Fig. 4(b) shows its corresponding power spectrum. Each epoch has 512 samples with a duration of 250 msec and sampling frequency of $f_s = 2$ kHz.

The results for the non-adaptive matched-filter are shown in Fig. 5. It is evident that the SNR in the fourth order cumulant based approach is improved as compared to other methods. Now, we apply our adaptive filter with $P=40$ and $\lambda = 9995$. The results are shown in Fig. 6. It is evident that the results are improved as compared to non-adaptive approaches in Fig. 6. It can also be seen that the HOS based adaptive filtering achieves better results as compared to the autocorrelation based adaptive filtering. The results for the third order cumulant and the fourth order cumulant based methods are close to each other in this case.

5. Conclusion

We use an adaptive filter in which we use fourth-order statistics to detect VEPs in real time. Our proposed method is capable of extracting VEPs in real time and can
be used to detect VEP anomalies in each epoch, as compared to other methods that in effect average out the VEPs over the entire number of epochs. We have shown that the SNR is improved significantly as compared to other existing methods as well.

Figure 4- (a) Recorded VEP; (b) The power spectrum of (a).

Figure 5- The output of the non-adaptive filters for $P=40$ for real VEP data, (a) The output of the autocorrelation based filter; (b) The power spectrum of (a); (c) The output of the third order cumulant based filter; (d) The power spectrum of (c); (e) The output of the fourth order cumulant based filter; (f) The power spectrum of (e).

Figure 6- The output of the adaptive filter for $P=40$, (a) The output of the autocorrelation based adaptive filter; (b) The power spectrum of (a); (c) The output of the third order cumulant based adaptive filter; (d) The power spectrum of (c); (e) The output of the fourth order cumulant based adaptive filter; (f) The power spectrum of (e).

References


