Abstract

Design and implementation of a unilateral teleoperated pneumatic system is presented followed by the stability analysis. The system consists of a pneumatic actuator that is subject to an environmental force and is navigated by a human-operated hand controller. Unlike many unilateral systems, the external force is entirely handled on the slave side using an impedance control scheme which adopts a sliding mode control force controller. In contrary to bilateral teleoperation, force handling is independent to a human operator. The novelty of the work is design and implementation of a system with the benefits of unilateral teleoperation, i.e. simple design, more stability and less responsibility on the operator and also accurate force handling of bilateral systems. The concept of Lyapunov exponents is further applied to theoretically prove the stability of the entire control system. Parametric stability analysis is conducted to study the effect of changing parameters on stability. Finally, the performance of the control system is experimentally verified during force interaction between the teleoperated pneumatic actuator and the environment.

Key Words

Stability analysis, Lyapunov exponents, pneumatic actuator, impedance control

1. Introduction

Teleoperation refers to remote control of machines in applications whereby the operator cannot be present at the operation site or, the task is so precise that the operator’s movements need to be scaled down [1]. A teleoperated system is composed of a hand controller also known as master, a slave manipulator that follows the master and a central controller that coordinates the entire system through a communication channel [2]. If the slave manipulator reflects the interaction force with the environment (external force) back to the master, the teleoperation system is called bilateral; otherwise, it is called unilateral [3].

Using pneumatic actuators in the teleoperation is challenging. Although pneumatic actuators are inexpensive and easy to use, achieving precise control of pneumatic actuators can be difficult because of nonlinearity of air and friction [4]. Most of pneumatic teleoperation systems are bilateral [2], [5]. In [6], identical pneumatic slave and master actuators, driven by on/off valves, were employed in a bilateral manner. Stability analysis was conducted using a closed-loop model of the bilateral system [7] which models the combination of the slave and the master with a linear system [8]. Further stability analysis was done using the stability condition of the sliding mode controller (SMC) that assumes the external force to be model uncertainty [6]. Durbha et al. [9] navigated a pneumatic slave using an identical master in a bilateral way. To study the stability, a system theory was implemented, which guarantees the stability of the closed-loop force interaction of a passive and a strictly passive system. This theory presumes that the environment and the control system were passive [9]. A similar stability analysis approach was applied to a pneumatic rescue crawler which was paired with a phantom haptic device in a bilateral way [10]. As opposed to bilateral teleoperation, unilateral teleoperation has a simple structure. Unlike bilateral teleoperation, unilateral teleoperation is not depended on the operator’s dexterity. As a result, less training is required [2].

In this paper, a unilateral teleoperation pneumatic system is implemented. To handle the external force on the slave side, impedance control is applied which sends combination of external force and position feedback as desired force (called force-based) or position (position-based) to a control loop. The force-based and position-based impedance control methods are dual to each other, i.e. they have complementary characteristics [11]. Richardson [11] suggested the use of position-based impedance control for pneumatic systems. However, others preferred the
force-based method for pneumatic actuators [12]–[14]. This paper adopts the idea of impedance control, for the first time, on a teleoperation system. Force-based impedance control is preferred because the pneumatic actuators are naturally sources of force as they convert the force of air pressure to movement.

A force-based impedance control requires a force controller. Efforts have been done toward the development of an efficient force controller for pneumatic systems. The force was controlled through control of chamber pressures [14]. A linear proportional controller was used in some applications [15]. An adaptive controller was employed to pneumatic force tracking [16]. Richer and Hurmuzlu [17] developed a new techniques using an SMC for force control of a pneumatic muscle. In [17]–[19], the robust performance of SMC in force control of pneumatic systems was presented. Therefore, SMC is adopted to control the slave force in this paper.

Simple configuration of unilateral systems reduces the chance of instability [20]. However, it does not guarantee the stability due to the fact that the environment is not predictable which is a crucial factor in many applications [21], [22]. In this paper, the stability analysis of the system is conducted using the concept of Lyapunov exponents (LEs) [23]. Despite popularity of a Lyapunov direct method, lack of a method for finding the Lyapunov function has limited its application for many systems [24]. LEs characterize the stability of system through quantifying the average rates of exponential divergence/convergence of initially neighbouring trajectories in the state space. In addition, using the concept of LEs, one can study the stability of an already-designed system and also study the effect of changing parameters on the stability of the system. Using LEs showed promising results in stability analysis of a hydraulic manipulator [25] and a biped robot [24].

The resulted unilateral pneumatic system can be applied in tele-rehabilitation of patients with motor disabilities and also magnetic resonance imaging-compatible mechatronic devices of monitoring brain functions [11].

The rest of this paper is organized as follows. Section 2 describes the experimental set-up and modelling of the pneumatic system. Section 3 describes impedance control of a teleoperated pneumatic actuator using a force controller build upon SMC. In Section 4, the state-space model of the control system is established. Section 5 studies the stability of the entire control system using both simulation of the dynamic system and the concept of LEs. In Section 6, the under control system is experimentally evaluated.

2. Preliminaries

2.1 Force-based Impedance Control

Impedance control simultaneously maintains desired force and position in the same direction [26]. It can be position-based (also known as admittance control) or force-based depending on the lower-level controller. Richardson claims that position-based impedance control is the more suitable for pneumatic actuators [11]. In a recent work, however, the force-based impedance control is preferred over the position-based for a single pneumatic actuator [12]. In this paper, force-based impedance control is implemented for a teleoperated pneumatic system. Figure 1 shows a general block diagram of force-based impedance control.

In Fig. 1, \( e_x = x_d - x_s \), where \( x_d \) is the desired position generated by the hand controller. \( F_a \) is the actuating force of pneumatic actuator generated by the air pressure difference in the chambers. The slave actuator is modelled in Section 2. The environment can be anything depending on the application. The force controller and the correspondent control signal, \( u \), are detailed in Section 3.2. According to [11], the external force can be modelled as follows:

\[
-F_{ext} = M(\ddot{x}_d - \ddot{x}_s) + B(\dot{x}_d - \dot{x}_s) + K(x_d - x_s) \tag{1}
\]

where \( M \), \( B \) and \( K \) are inertia, stiffness and damping coefficients, correspondent to the impedance model. Considering \( e_x = x_d - x_s \), the formulation of the force-based impedance control is [4]:

\[
F_d = \frac{m}{M}(F_{ext} + M\ddot{x}_d + B\dot{x}_d + Ke_x) + (F_f + b\dot{x}_s) - F_{ext} \tag{2}
\]

where \( F_d \) is the desired force, \( m \) is the mass of pneumatic system and \( x_s \) is the position of the slave actuator. \( F_{ext} \) is the externally applied load, \( b \) is the viscous friction coefficient and \( F_f \) represents the dry friction. It is evident that the impedance model in (2) is a combination of external force, position information and position error.

2.2 Lyapunov Exponents

LEs are quantitative measures that demonstrate the asymptotic behaviour of a nonlinear system. For a nonlinear system with \( n \)-dimensional state space, a set of \( n \) LEs should be calculated. The stability property of system is determined by the signs of LEs. A positive LE corresponds to a chaotic system. All negative exponents show a stable...
2.2.1 Theory of Lyapunov Exponents

Consider a smooth dynamic system, \( \dot{x} = f(x) \), in an \( n \)-dimensional state space, where \( x \in \mathbb{R}^n \) is a state vector of the nonlinear system and \( f(x) \) is differentiable and continuous. To calculate LEs, the nonlinear equations of motion, \( \dot{x} = f(x) \), are solved, from initial conditions \( x(0) = x_0 \). The solution is called “fiducial” trajectory. In the next step, principal axes are defined orthogonally on the fiducial trajectory, the lengths of principal axes at each time instant show the behaviour of the nonlinear system. Each LE is defined as follows [28]:

\[
\lambda_i = \lim_{T \to \infty} \frac{1}{T} \ln \frac{\| \delta x_i(t) \|}{\| \delta x_i(t_0) \|}
\]

(3)

In (3), the length of a principal axis in iteration \( i \) is shown by \( \delta x_i \) and \( T \) is the duration of the most recent observation. To calculate \( \delta x_i \), linearized equations of motion are derived by calculating the Jacobian matrix, \( F(t) \) [28]:

\[
F(t) = \frac{\partial f}{\partial x} \bigg|_{x=x(t)}
\]

(4)

\[
\dot{\psi}_i = F(t)\psi_i
\]

(5)

With the initial condition equal to the unity matrix, (5) is integrated to find the principal axis \( \delta x_i(t) \). The nonlinear equations are also integrated simultaneously to provide the instantaneous state inputs for the Jacobian calculation [28]:

\[
\begin{pmatrix}
\dot{x} \\
\dot{\psi}_i
\end{pmatrix}
= \begin{pmatrix}
f(x) \\
F(t)\psi_i
\end{pmatrix}
\]

(6)

The principal axes tend to fall along the direction of the most rapid growth axis. When the orientations of axes converge beyond the computer limitation, they will not be distinguishable. To resolve the problem, the Gram–Schmidt scheme is used which normalizes the length of each vector and then orthogonalizes them in each iteration [28].

It is important to note that LEs are global characteristics of dynamical behaviour of a system, although only one set of data is used in their calculation. According to a theorem of Oseledec [29], LEs of a dynamical system with a hyperbolic equilibrium (all eigenvalues have non-zero real part), once calculated using any fiducial trajectory, guarantee stability of the dynamic system for any fiducial trajectory starting in the same stability region [29]. Presenting more details on this matter is avoided to maintain the focus of the work.

3. System Modelling

3.1 Experimental Set-Up

Figure 2(a) shows the configuration of the experimental set-up. It consists of a pneumatic actuator as the slave that interacts with a human subject, a PHANTOM OMNI joystick as the master manipulator and a computer that coordinates all of the connections via a data acquisition board. The network delay is small and negligible. The computer generates and forwards a control signal to a pneumatic actuator to move the slave according to the movement of master. The computer also received the measured data of a BOURNS incremental rotary encoder, a set of two Durham Industries pressure sensors and an ARTECH load cell. Data communication between computer and other parts is through a QUANSER data acquisition board which transmits data at a 500 Hz sampling rate. The sensors are assumed ideal, i.e. their dynamics are fast and do not have effect on the dynamics of the overall system. The pneumatic actuator set-up, shown in Fig. 2(b), is driven by a 5-port 3-way proportional directional flow control valve (FESTO MPYE). The actuator rod (FESTO DNC) is a 40-mm bore, with 500-mm stroke. To provide the force interaction point with environment, a shaft is attached to the pneumatic actuator. Considering Fig. 2, the equation of motion for the pneumatic slave manipulator is:

\[
m\ddot{x}_s = A(P_1 - P_2) + F_{ext} - (F_f + b\dot{x}_s)
\]

(7)

where \( A \) presents the area of the piston, and \( P_1 \) and \( P_2 \) are the air pressures at chambers. \( F_f \) denotes the dry friction using the LuGre friction model [30] excluding the viscous friction (viscous friction is presented by \( b\dot{x}_s \) in (7)):

\[
F_f = \sigma_0 z + \sigma_1 \dot{z}
\]

(8)

Figure 2. (a) Schematic diagram of unilateral pneumatic system and (b) experimental set-up and schematic diagram of pneumatic slave actuator.
where  is the equivalent spring constant of bristle, \( z \) is the average bristle deflection and  is the equivalent damping coefficient of bristles in the LuGre friction model. The average bristle deflection, \( z \), can be found by solving the following equation [30]:

\[
\dot{z} = \dot{x}_s - \frac{\sigma_0 |\dot{x}_s| z}{F_c + (F_s - F_c)e^{-(x_s/v_{sw})^2}}
\]  \( (9) \)

where \( v_{sw} \) is the Striebeck velocity and \( F_s \) and \( F_c \) denote the static and the Coulomb friction, respectively.

The motion of the actuator is generated by the air pressure difference in chambers. The pressures of chambers are related to the air mass flows, \( \dot{m}_1 \) and \( \dot{m}_2 \) [31]:

\[
\begin{align*}
\dot{P}_1 &= \gamma RT \frac{\dot{m}_1}{V_1} - \alpha \gamma A \dot{x}_s \frac{\dot{P}_1}{V_1} \\
\dot{P}_2 &= -\gamma RT \frac{\dot{m}_2}{V_2} + \alpha \gamma A \dot{x}_s \frac{\dot{P}_2}{V_2}
\end{align*}
\]  \( (10) \) \( (11) \)

In (10) and (11), \( \gamma \) is the ratio of specific heat, \( \alpha \) is the compressibility correction factor, \( R \) is the ideal gas constant and \( T \) is the air temperature [17]. \( V_1 \) and \( V_2 \) depict the volumes of chamber, as expressed in (12) and (13) where \( L \) is the actuator stroke and \( V_0 \) is the cylinder inactive volume:

\[
\begin{align*}
V_1 &= V_0 + A x_s \\
V_2 &= V_0 + A (L - x_s)
\end{align*}
\]  \( (12) \) \( (13) \)

Defining \( \tau = \sqrt{\frac{2}{\gamma}}(\gamma + 1)^{\gamma}(\gamma - 1)/R \), the mass flow rate of air is [32]:

\[
\begin{align*}
\dot{m}_1 &= w x_s \dot{\omega}_1 \\
\dot{m}_2 &= w x_s \dot{\omega}_2
\end{align*}
\]  \( (14) \)

where \( w \) and \( \dot{\omega}_i (i = 1, 2) \) are the orifice area gradient and mass flow per area unit, respectively. \( x_v \) is the displacement of the valve spool and \( C_d \) is the coefficient of discharge in the valve. \( P_s \) is the supply air pressure and \( P_a \) is the atmospheric pressure. The critical pressure ratio of the valve is shown by \( P_{cr} \). The valve spool displacement is related to the control signal by the following equation:

\[
\dot{x}_v = \frac{1}{\tau} (-x_v + K_v u)
\]  \( (17) \)

where \( K_v \) is the spool gain of the valve, \( \tau \) is the time constant and \( u \) is the control signal. The parameters of the test rig are achieved from the manufacturer datasheets and former research on the same test rig and shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder inactive volume</td>
<td>1.64 × 10⁻⁴ m³</td>
<td>( V_0 )</td>
</tr>
<tr>
<td>Actuator stroke</td>
<td>0.5 m</td>
<td>( L )</td>
</tr>
<tr>
<td>Piston annulus area</td>
<td>10.6 cm²</td>
<td>( A )</td>
</tr>
<tr>
<td>Valve coefficient of discharge</td>
<td>0.7</td>
<td>( C_d )</td>
</tr>
<tr>
<td>Total mass of piston, rod</td>
<td>1.91 kg</td>
<td>( m )</td>
</tr>
<tr>
<td>Compressibility correction factor</td>
<td>1.2</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>Valve critical pressure ratio</td>
<td>0.2</td>
<td>( P_{cr} )</td>
</tr>
<tr>
<td>Ideal gas constant</td>
<td>287 J/kg K</td>
<td>( R )</td>
</tr>
<tr>
<td>Ratio of specific heats</td>
<td>1.4</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>Valve spool position gain</td>
<td>0.25 mm/V</td>
<td>( K_v )</td>
</tr>
<tr>
<td>Temperature of air source</td>
<td>300 K</td>
<td>( T )</td>
</tr>
<tr>
<td>Atmospheric pressure</td>
<td>1 × 10⁵ Pa</td>
<td>( P_a )</td>
</tr>
<tr>
<td>Valve orifice area gradient</td>
<td>22.6 mm²/mm</td>
<td>( w )</td>
</tr>
<tr>
<td>Supply pressure</td>
<td>5 × 10⁵ Pa</td>
<td>( P_s )</td>
</tr>
<tr>
<td>Coulomb friction</td>
<td>32.9 N</td>
<td>( F_c )</td>
</tr>
<tr>
<td>Viscous damping coefficient</td>
<td>70 N s/m</td>
<td>( b )</td>
</tr>
<tr>
<td>Striebeck velocity</td>
<td>0.02 m/s</td>
<td>( v_{sw} )</td>
</tr>
<tr>
<td>Static friction</td>
<td>38.5 N</td>
<td>( F_s )</td>
</tr>
<tr>
<td>LuGre damping coefficient</td>
<td>93.13 N/m/s</td>
<td>( \sigma_1 )</td>
</tr>
<tr>
<td>LuGre spring constant</td>
<td>4500 N/m</td>
<td>( \sigma_0 )</td>
</tr>
</tbody>
</table>
3.2 Deriving the Sliding Mode Force Controller Formulation

SMC is a robust control approach and has been applied to force control of pneumatic systems [14], [17]. It generates the control signal based on the dynamic model of the system. For a pneumatic actuator, as shown in Fig. 2, the output is the actuating force, $F_a$, which is produced by pressure difference in chambers [17]:

$$F_a = A(P_1 - P_2) \quad (18)$$

The sliding mode force control scheme makes $F_a$ follow the desired force, $F_d$ [17]. The control signal, $u$, is composed of $Av_{eq}$, the equivalent component and $Av_{rb}$, the robust component [17]:

$$u = (Av_{eq} + Av_{rb})/(wK_v) \quad (19)$$

The sliding surface is selected as follows:

$$S = \left( \frac{d}{dt} + \delta \right) \int_0^t e \, dt \quad (20)$$

where $\delta$ is the control bandwidth. The force error is expressed as follows:

$$e = F_a - F_d \quad (21)$$

The dynamics on the sliding surface is expressed as follows:

$$\dot{S} = \dot{e} + \delta e = 0 \quad (22)$$

This leads to equivalent part of control signal:

$$Av_{eq} = \frac{\dot{F}_d - \delta e - \dot{F}_{ext} + (\dot{F}_f + b\ddot{x}_a) - F_x}{P_x} \quad (23)$$

where

$$F_x = -\alpha\gamma A^2 \left( \frac{P_1}{V_1} + \frac{P_2}{V_2} \right) \dot{x}_s \quad (24)$$

$$P_x = \gamma RT A \left( \frac{\dot{\theta}_1}{V_1} + \frac{\dot{\theta}_2}{V_2} \right) \quad (25)$$

The dynamics of the pneumatic system is faster than the rate of the change of the dry friction. Thus, $\dot{F}_f$ can be neglected in (23) [4], [35]. This simplification is compensated by the robust part of SMC which is formulated as follows:

$$Av_{rb} = \frac{K_{rb}}{P_x} \text{sign}(S) \quad (26)$$

In (26), $K_{rb}$ is the robustness gain. The discontinuity of the $\text{sign}$ function can affect implementation of $Av_{rb}$. Therefore, it is approximated by $\tanh(ax)$.

$$\text{sign}(S) = \tanh(ax), a \gg 0.5 \quad (27)$$

3.3 State Space Model of the Control System

The state space is formed by defining the following state vector:

$$\vec{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]^T$$

$$= [x_s \ v_s \ P_1 \ P_2 \ x_v \ \int_0^t e \, dt]^T \quad (28)$$

where $x_i (i = 1, \ldots, 7)$ is the state space and the right side variables are defined previously. Using (7)–(17), the state space model of unilateral pneumatic system is constructed as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m} \left[ A(x_3 - x_4) + F_{ext} - \left( \sigma_0 x_6 + \sigma_1 \left( x_2 - \frac{\sigma_0 |x_2| x_6}{F_c + (F_s - F_e) e^{-s(x_2/v_s)^2}} \right) + bx_2 \right) \right] \\
\dot{x}_3 &= +\gamma RT \frac{wx_s \dot{\theta}_1}{V_0 + Ax_1} - \alpha \gamma A \frac{x_2 x_3}{V_0 + Ax_1} \\
\dot{x}_4 &= -\gamma RT \frac{wx_s \dot{\theta}_2}{V_0 + A(L - x_1)} + \alpha \gamma A \frac{x_2 x_4}{V_0 + A(L - x_1)} \\
\dot{x}_5 &= \frac{1}{\tau} (-x_5 + K_v u) \\
\dot{x}_6 &= x_2 - \frac{\sigma_0 |x_2| x_6}{F_c + (F_s - F_e) e^{-s(x_2/v_s)^2}} \\
\dot{x}_7 &= e
\end{align*}
\]
The environment, which the slave actuator is interacting with, is assumed stiffness dominant. Thus, the external force is proportional to the slave position:

\[ F_{\text{ext}} = -K_{\text{ext}}x_1 \]  

(30)

where \( K_{\text{ext}} \) is a environment stiffness gain. The steady-state form definitions of variables in (29) are briefed in Appendix to avoid repetition. The initial condition is \( \bar{x}_0 = [0, 0, 3 \times 10^5, 3 \times 10^5, 0, 0, 0]^T \) and the equilibrium of (29) is \( \bar{x}_{\text{eq}} = [x_1^{ss} \ 0 \ x_3^{ss} \ x_4^{ss} \ 0 \ x_6^{ss} \ x_7^{ss}]^T \), where:

\[ A(x_3^{ss} - x_4^{ss}) - K_{\text{ext}}x_1^{ss} - \sigma_0x_6^{ss} = 0 \]  

(31)

\[ \dot{F}_d - K_{rb} \tanh(ax_7^{ss}) = 0 \]  

(32)

4. Stability Analysis

4.1 Calculation of LEs

Simulation studies are conducted to calculate LEs. The system model is stated in (29). Figure 3(a) shows the slave actuator position tracking results. The force tracking result is shown in Fig. 3(b). The desired force, \( F_d \), is obtained from the impedance model expressed by (2). The actuating force, \( F_a \), is formulated in (18). One can see the reasonable agreement between the motion of the master and slave and reaction of the slave to the external force. The control signal of the pneumatic actuator, shown in Fig. 3(c), is in the range \([0–10]\) V, i.e. a 5-V control signal corresponds to the closure of the valve. Figure 3(d) shows the external force, as defined in (30). The parameters of the controller are shown in Table 2.

To study the stability of the control system, the spectrum of the LEs is calculated from (3). To apply the LE to any system, one should make sure that the system satisfies some conditions such as existence of solution for the nonlinear system and uniqueness and continuity of the linearized equation. The authors used the Fillipov method [36], [37] to assure that the conditions satisfy for the under study system but the details are not presented to maintain the focus of the work. Table 3 shows the numerical values of LEs. It is evident that all LEs are zero or negative which justifies stability of the system despite nonlinearity of the pneumatic actuator and the external force.

When only the largest LE is zero, the magnitude of a principal axis has not changed as time evolves. In this case, the system has a one-dimensional equilibrium and so on [25]. To verify the values of LEs in Table 3, (31) and (32)
are considered again. Out of seven state space variables, \(x_3^a\) and \(x_5^a\) have numerical values at the steady state. The other five variables are formulated in (31) and (32). Having two equations and five unknown variables means the solution of (31) and (32) is three dimensions [25], [27]. Knowing that the number of zero LEs for any dynamic system represents the dimension of the equilibrium, one can see that the values in Table 3 are in line with the equilibrium of (29).

### 4.2 Parametric Stability Analysis

The concept of LEs can be used in the parametric stability analysis, which is the stability analysis of a dynamic system as its parameters change [27]. Parametric analysis assures the stability of the system when some of the physical parameters are not accurately known or measured. Likewise, it can be used to investigate the effect of changing controller gains on overall stability. Also, by varying the physical and controller parameters, one can find the stability regions of a dynamic system [27]. In this paper, typical parametric studies are presented for the unilateral pneumatic system described in Section 2. First environmental stiffness, \(K_{ext}\), was changed according to Table 4. Then, LEs were calculated for each value of \(K_{ext}\). It is observed that for all values of \(K_{ext}\), LEs are non-positive. Table 4 shows that \(K_{ext}\) does not have a major effect on the rate of convergence/divergence of trajectories in the state space. Therefore, we can conclude that the dynamic system is stable for the range of \(K_{ext}\) in Table 4.

In the next set of study, the value of SMC control bandwidth, \(\delta\), was changed. Similar to the last sample, all LEs are non-positive for all values of \(\delta\) presented in Table 5. Comparing to Table 4, the rate of changes of LEs in this sample is higher. This indicates that SMC control bandwidth can substantially influence the rate of convergence/divergence of state-space trajectories.

### 5. Experimental Results

Experiments were conducted on the test rig described in Section 3 to evaluate the performance of the proposed impedance control unilateral system. First, the effectiveness of the proposed SMC force controller in (19) was studied. Next, using the SMC and the impedance model in (2), unilateral teleoperation was tested.

#### 5.1 Force Controller

Figure 4 shows the performance of the SMC scheme in tracking the desired force. Force tracking of two sinusoidal waves with the magnitudes of 20 and 50 N, and the frequency of 0.3 rad/s is presented. To avoid the effect of motion on the actuator force, the piston rod was placed in the middle of the stroke by a set of bolts. As it is observed, the proposed SMC force controller is able to track the

<table>
<thead>
<tr>
<th>(K_{ext} = 10)</th>
<th>(\lambda_1)</th>
<th>(\lambda_2)</th>
<th>(\lambda_3)</th>
<th>(\lambda_4)</th>
<th>(\lambda_5)</th>
<th>(\lambda_6)</th>
<th>(\lambda_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
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<td>-22.45</td>
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<td>-64.134</td>
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<td>-64.189</td>
<td>-64.194</td>
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<tr>
<td>(K_{ext} = 100)</td>
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<td>0</td>
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<td>-64.161</td>
<td>-64.165</td>
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<tr>
<td>(K_{ext} = 125)</td>
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<td>0</td>
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<td>(K_{ext} = 200)</td>
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desired force successfully. The oscillation on the force error and the control signals is caused by the oscillatory chamber pressure signals. Comparing the relative error displayed in Fig. 4(a) (b), one can see that the performances of both force tracking experiments are similar. The control signals are always unsaturated and remain in the expected range. The above experiment shows that the proposed controller is capable of tracking the desired force with different amplitudes.

To further study the force-tracking performance, force tracking of 0.15, 0.3, 0.6 and 1.0 rad/s sinusoidal waves is experimented. The results in frequency domain are presented in Fig. 5. It is observed that the controller performs well for all tracking frequencies.

5.2 Unilateral Teleoperation System

The SMC force controller is applied to an impedance control loop as shown in Fig. 1 to track a desired position trajectory given by the master actuator. To observe the position tracking performance of the proposed controller, no external force is applied to the slave actuator firstly. The parameters of the impedance model defined in (2) were set as $M = 1\, \text{kg}$, $B = 20\, \text{Ns/m}$ and $K = 3,000\, \text{N/m}$. Figure 6 shows the results. From Fig. 6(a), it is obvious that impedance control method tracked the desired position successfully. The position error during the experiment, as shown in Fig. 6(b), is always less than 0.02 m, which is possibly caused by the notable dry friction of the pneumatic actuator. The control signal in Fig. 6(c) is reasonable and not saturated.

In the next experiment, a human subject applied an external force of 10 N magnitude to the slave actuator through an attached handle. A desired position trajectory, $x_d$, was given by the master actuator. The parameters of the impedance model are similar to the previous experiment. Fig. 7 shows the positions of the master and the slave. One can observe a reasonable position tracking in Fig. 7(a). Further, at the instants of force exertion, the position of the slave actuator is adjusted according to the force.

In the last experiment, the parameters of the impedance model are set to $M = 1\, \text{kg}$, $B = 20\, \text{Ns/m}$ and $K = 5,000\, \text{N/m}$. As shown in Fig. 8(a), the slave actuator was moved by 0.03 m reacting to a 50 N force, whereas in Fig. 7(a) the same displacement is observed for a force of 10 N. It shows that the impedance model provided higher stiffness as a result of increasing the value of $K$. The above experiments prove that the proposed impedance control can successfully control position given by a hand controller and at the same time incorporate the external force.
Figure 5. Frequency response of force tracking by SMC: (a) 20 N and (b) 50 N.

Figure 6. Experimental study of motion tracking in the absence of external force by impedance control: (a) position tracking; (b) position error; and (c) control signal.

Figure 7. Experimental study of low stiffness impedance model: (a) position tracking; (b) external force; and (c) control signal.
The control signal, \( u \), force error, \( e \), and mass flow rates \( \dot{O}_1 \) and \( \dot{O}_2 \) in (29) are defined in terms of state space variables here. The control signal is calculated by substituting (23) and (26) in (19):

\[
u = \frac{1}{w \cdot K_v} \left( \dot{F}_d - \delta c + K_{ext} x_2 + b \dot{x}_2 - Fx - K_{th} \tanh(aS) \right) \tag{A1}\]

where \( S \) is defined as follows:

\[
S = e + \delta x_7 \tag{A2}
\]

The force error is defined as:

\[
e = A(x_3 - x_4) - K_{ext} x_1
- \left( \sigma_0 x_6 + \sigma_1 \left( x_2 - \frac{\sigma_0 |x_2| x_6}{F_c + (F_s - F_c) e^{-(x_2/\nu)^2}} + b x_2 \right) - F_d \right) \tag{A3}
\]

In (33), \( F_x \) is defined as:

\[
F_x = -\alpha \gamma A^2 \left( \frac{x_3}{V_0 + A x_1} + \frac{x_4}{V_0 + A (L - x_1)} \right) x_2 \tag{A4}
\]

The desired force, \( F_d \), is achieved from the impedance model as follows:

\[
F_d = \frac{m}{M} (-K_{ext} x_1 + M \dot{x}_d + B(\dot{x}_d - x_2) + K(x_d - x_1))
+ (F_f + b \dot{x}_2) + K_{ext} x_1 \tag{A5}
\]

The derivation of \( F_d \) is:

\[
\dot{F}_d = \frac{m}{M} (-K_{ext} x_2 + B(\dot{x}_2 - \dot{x}_d) + K(x_2 - \dot{x}_d) + M \ddot{x}_d)
+ b \ddot{x}_2 + K_{ext} x_2 \tag{A6}
\]

The mass flow rates are calculated as follows:

\[
\dot{O}_1 = \begin{cases} 
\frac{C_{e_d} P_s}{\sqrt{\gamma}} \sqrt{1 - \left( \frac{x_2 (P_s - P_{cr})}{1 - \frac{P_0}{P_{cr}}} \right)^{\gamma - 1}} / x_2, & \frac{x_2}{P_s} \leq P_{cr}, \\
\frac{C_{e_d} P_s}{\sqrt{\gamma}} x_2, & \frac{x_2}{P_s} > P_{cr}, \quad x_2 \geq 0
\end{cases}
\]

\[
\dot{O}_2 = \begin{cases} 
\frac{C_{e_d} P_s}{\sqrt{\gamma}} \sqrt{1 - \left( \frac{x_2 (P_s - P_{cr})}{1 - \frac{P_0}{P_{cr}}} \right)^{\gamma - 1}} / x_2, & \frac{x_2}{P_s} \leq P_{cr}, \\
\frac{C_{e_d} P_s}{\sqrt{\gamma}} x_2, & \frac{x_2}{P_s} > P_{cr}, \quad x_2 \geq 0
\end{cases}
\]

\[
\dot{O}_3 = \begin{cases} 
\frac{C_{e_d} P_s}{\sqrt{\gamma}} \sqrt{1 - \left( \frac{x_2 (P_s - P_{cr})}{1 - \frac{P_0}{P_{cr}}} \right)^{\gamma - 1}} / x_2, & \frac{x_2}{P_s} \leq P_{cr}, \\
\frac{C_{e_d} P_s}{\sqrt{\gamma}} x_2, & \frac{x_2}{P_s} > P_{cr}, \quad x_2 \geq 0
\end{cases}
\]

\[
\dot{O}_4 = \begin{cases} 
\frac{C_{e_d} P_s}{\sqrt{\gamma}} \sqrt{1 - \left( \frac{x_2 (P_s - P_{cr})}{1 - \frac{P_0}{P_{cr}}} \right)^{\gamma - 1}} / x_2, & \frac{x_2}{P_s} \leq P_{cr}, \\
\frac{C_{e_d} P_s}{\sqrt{\gamma}} x_2, & \frac{x_2}{P_s} > P_{cr}, \quad x_2 \geq 0
\end{cases}
\]

\[
\dot{O}_5 = \begin{cases} 
\frac{C_{e_d} P_s}{\sqrt{\gamma}} \sqrt{1 - \left( \frac{x_2 (P_s - P_{cr})}{1 - \frac{P_0}{P_{cr}}} \right)^{\gamma - 1}} / x_2, & \frac{x_2}{P_s} \leq P_{cr}, \\
\frac{C_{e_d} P_s}{\sqrt{\gamma}} x_2, & \frac{x_2}{P_s} > P_{cr}, \quad x_2 \geq 0
\end{cases}
\]

\[
\dot{O}_6 = \begin{cases} 
\frac{C_{e_d} P_s}{\sqrt{\gamma}} \sqrt{1 - \left( \frac{x_2 (P_s - P_{cr})}{1 - \frac{P_0}{P_{cr}}} \right)^{\gamma - 1}} / x_2, & \frac{x_2}{P_s} \leq P_{cr}, \\
\frac{C_{e_d} P_s}{\sqrt{\gamma}} x_2, & \frac{x_2}{P_s} > P_{cr}, \quad x_2 \geq 0
\end{cases}
\]

\[
\dot{O}_7 = \begin{cases} 
\frac{C_{e_d} P_s}{\sqrt{\gamma}} \sqrt{1 - \left( \frac{x_2 (P_s - P_{cr})}{1 - \frac{P_0}{P_{cr}}} \right)^{\gamma - 1}} / x_2, & \frac{x_2}{P_s} \leq P_{cr}, \\
\frac{C_{e_d} P_s}{\sqrt{\gamma}} x_2, & \frac{x_2}{P_s} > P_{cr}, \quad x_2 \geq 0
\end{cases}
\]
References


Biographies


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