A MATHEMATICAL MODEL FOR ASSESSMENT OF PHYSICAL PROPERTIES OF FLYASH-SAND-CEMENT BRICKS

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ABSTRACT
This paper highlights the development of a mathematical model to find the relationship of compressive strength of flyash cement bricks with other process parameters namely, percentage of flyash, sand and cement and water. Here, non linear one parameter polynomials are fitted using the existing experimental data. It is seen that the fitting with the single parameter in some cases does not give good results. As such, a multi dimensional relationship among the mentioned parameters have been proposed to get desired result and the corresponding coefficient of determination are also reported to show the efficacy of the relationship. By using this model the compressive strength data may be approximated for the use of these bricks in construction sectors. As the flyash and other ingredients have been collected from thermal power station in Botswana, the model may be a beneficial to the people in Botswana working with flyash bricks.

KEYWORDS
Mathematical modeling, model development, flyash, brick, compressive strength.

1. Introduction
Nowadays researchers all over the world are investigating effective ways of utilization of industrial wastes such as flyash, red mud in production of building components. Fly ash has pozzolanic effect, so it can be used as a partial substitute of cement. In Botswana there is a thermal power station located at Palapye town. At present, the plant produces approximately 300 tonnes of flyash per day. Out of this huge amount of fly ash, only a small quantity of fly ash has been used in the partial replacement of Portland cement by the local cement industry. The remaining huge quantity of fly ash is continuously dumped in the near-by area and has a tremendous negative impact on environment [1, 2]. Sahu and Swarnadhipathi [3] and Sahu [4] have studied to use of fly ash in road construction works. Like other countries, brick is the main building material used in construction industry in Botswana. It is therefore appropriate that the brick industry presents an opportunity to efficient use of the vast quantities of fly ash. In [5] the authors have discussed how to use fly ash in manufacturing of bricks. Using fly ash in brick making appears to improve the optimum strength of the brick. The optimum strength could be achieved by reducing the percentage of cement to some degree. Under the sponsorship of Office of Research and Development, University of Botswana a brick making machine has been developed by Panigrahi et. al.[6]. By making repeated experiments the mix design has been optimized to give required strength. By slightly change in the product parameters the strength will change accordingly.

To find out the desired compressive strength of bricks, it is necessary to repeat the experiments with different mix designs. It is observed that every time the investigation of corresponding strength of bricks becomes a time consuming and costly process due to these repeated experiments. The simulation in computers in terms of mathematical models for identification of brick quality may substantially minimize the need of repeated experiments over and again. In this regard prediction of compressive strength of mortars made with Portland cement-blast furnace slag - flyash blends has been done by [7] and prediction of compressive strength of mortars with ternary blends by [8]. The authors in [9, 10 and 11] have developed mathematical relations to optimise the process parameters to get the optimum strength of fly ash-sand-lime bricks. Also in [12] the authors tried to develop a model for flyash-sand-cement bricks using the flyash collected from thermal power stations in India. However, they have not discussed in detail on effect of each mix designs on compressive strength. In [13 and 14] the authors tried to get the process parameters using different approaches which require more computational efficiency.

In the present investigation mathematical model in MatLab environment has been developed by using numerical modelling to assess physical properties of flyash cement bricks using experimental data. First it analyses for one parameter modelling where it can give the effect of single parameter (process parameters) viz. flyash (%), Cement (%), sand (%), fineness, cement reactivity, unburnt carbon (%), water added (%) etc. on the product parameters such as compressive strength, water absorption and bulk density for flyash cement bricks. Then it analyses the modelling for multi parameter where it can give the approximate equation for product parameters.
using the required number of process parameters as mentioned earlier.

2. Raw Materials, Mix Designs and Production of Bricks

The raw materials which are used for production of bricks are cement (43 Gr. OPC), local river sand and fly ash. The fly ash percentages have been varied 35% to 65% with an increment of 5% in each batch of production of the bricks. Ordinary Portland cement has been used from 6% to 12% with an increment of 2% and the rest of sand is used in the mix for producing the bricks. The precise quantity of water was added as per the requirement of mix. The bricks were produced using vibro compaction technique in a brick making machine developed in Mechanical Engineering Department of University of Botswana. The bricks were tested for compressive strength parameter. In this paper only the Compressive strength parameter has been considered for modeling. The details of mix design are as demonstrated by Table 1.

<table>
<thead>
<tr>
<th>Mix No</th>
<th>Fly ash%</th>
<th>Sand%</th>
<th>Cement%</th>
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</tr>
<tr>
<td>A-2</td>
<td>45</td>
<td>49</td>
<td>6</td>
</tr>
<tr>
<td>A-3</td>
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<td>39</td>
<td>6</td>
</tr>
<tr>
<td>A-4</td>
<td>65</td>
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<td>6</td>
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<th>Sand%</th>
<th>Cement%</th>
</tr>
</thead>
<tbody>
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<td>57</td>
<td>8</td>
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<tr>
<td>B-2</td>
<td>45</td>
<td>47</td>
<td>8</td>
</tr>
<tr>
<td>B-3</td>
<td>55</td>
<td>37</td>
<td>8</td>
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<td>B-4</td>
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<th>Sand%</th>
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<tr>
<td>C-2</td>
<td>45</td>
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<td>10</td>
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<table>
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<th>Mix No</th>
<th>Fly ash%</th>
<th>Sand%</th>
<th>Cement%</th>
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<tbody>
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<td>53</td>
<td>12</td>
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<tr>
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<tr>
<td>D-4</td>
<td>65</td>
<td>23</td>
<td>12</td>
</tr>
</tbody>
</table>

3. Mathematical Descriptions

As mentioned earlier mathematical model [15 and 16] can handle both the single parameter and multi parameter model. In the present methodology, the data available under ORD project of University of Botswana has been used.

3.1 Single parameter Model

Regression analysis has been done to get polynomial equations with the hope to get the desired product parameter. To elaborate the idea, let us consider that z is the desired parameter which depends on another parameter and \( p(r) \) is a polynomial in r with degree k and this is written as,

\[
z = p(r) = \sum_{j=0}^{k} u_j r^j
\]

Where \( u_j \) are \( k+1 \) unknown constants to be determined by the analysis. Let us suppose that we have N number of data for various r to fit the above Eq. (1). Then the error term \( E_r \) can be written as,

\[
E_r = \sum_{i=1}^{N} \left[ z_i - \left( \sum_{j=0}^{k} u_j (r_i)^j \right) \right]^2
\]

As per the Least-square Approach, \( E \) is differentiated with respect to \( u_i \) and equated to zero to get the \( (k+1) \) equations in \( (k+1) \) unknowns \( u_j \). Following are the corresponding equations:

\[
\frac{\partial E}{\partial u_j} = \sum_{i=1}^{N} \left[ z_i - \left( \sum_{j=0}^{k} u_j (r_i)^j \right) \right] \cdot (r_i)^j = 0, \quad j = 0, 1, k
\]

Eq. (3) actually is a matrix equation which can be solved for the unknowns \( u_j \), and the approximate value of the parameter can be obtained if these be substituted in the polynomial Eq. (1).

Varying the value of k, the degree of the polynomial various equations are developed and these theoretical results are compared with the experimental results and found to be in agreement as per the obtained coefficient of determination value. The final equations/terminology that has been used in simple form in the mathematical model are as follow for all the models.

One Parameter Model:

\[
Z = C_0 + C_1 X + C_2 X^2
\]

Where

\( Z \) : Comp. Strength or Bulk Density or Water Absorption and

\( X^i \) may be

a. Flyash % or
b. Sand % or
c. Cement % or
d. Water added in % or
e. Un-burnt carbon % or
f. Vibration time etc.
3.2 Multi Parameter Models

\[ Z = C_0 + C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + \ldots (5) \]

\( Z \): Comp. Strength or Bulk Density or Water Absorption

\( X_1 \): Flyash \%,
\( X_2 \): Sand \%,
\( X_3 \): Cement
\( X_4 \): Water added \% nd
\( X_5 \): Fineness etc. as desired

The efficacy of the various relationships are computed as

\[ \hat{y}_1 = \text{Computed values} \]

co-efficient of determination given in the software from

Where, \( \hat{y}_1 \) = Experimental values and the accuracy if

\[ R^2 = 0 \Rightarrow \text{Lack of fit} \quad \text{and} \quad R^2 = 1 \Rightarrow \text{Perfect Fit} \]

4. Numerical Results

Exhaustive numerical computations have been carried out. However, a limited of them are reported here. The polynomial equation (1) is taken from \( k = 1 \) to 6 degree and the experimental data set for the product parameter i.e. compressive strength of flyash cement bricks are used. Then the corresponding results from the above equation (1) has been obtained after obtaining the values of \( u_i \) from equation (3).

Results were analyzed and graphs have been plotted between compressive strength versus % of fly ash or % of cement keeping one parameter constant (Figure 1 and 2). It is observed that the compressive strength reduces as the % of fly ash increases (Figure 1).

Similarly it is also observed that as the % of cement increases the strength of the brick increases (Figure 2).

![Graph showing relationship between compressive strength and % of fly ash for various fixed % of cement](image1)

Fig. 1: Relationship between Compressive strength with % of Fly ash for various fixed % of Cement

Figure 3 shows the relationship between compressive strength and % of fly ash with a fixed % of cement (12%). In this case the % of sand is also varied. As % of fly ash increases compressive strength decreases gradually. A linear regression analysis has been carried out with \( k = 1 \) and \( R^2 \) value has been calculated. Here, \( R^2 \) value tends to unity.

![Graph showing linear relationship between compressive strength with % of fly ash for 12% cement](image2)

Fig. 3: Linear relationship between compressive strength with % of fly ash for 12% cement

\[ y = -0.073x + 13.875 \quad R^2 = 0.9989 \]
Figures 4 to 6 shows the above relation for 10%, 8% and 6% cement respectively. Corresponding linear fits have been drawn and shown in the graph.

Figure 7 shows the relation between compressive strength with the % of cement with fixed 35% of fly ash. As % of cement increases, the compressive strength increases. A linear regression plot and $R^2$ value are shown here. It is also observed that $R^2$ value tends to unity.

Fig. 4: Linear relationship between compressive strength with % of fly ash for 10% cement

Fig. 5: Linear relationship between compressive strength with % of fly ash for 8% cement

Fig. 6: Linear relationship between compressive strength with % of fly ash for 6% cement
All the above relations are modeled considering $k = 1$. When we are considering $k = 3$, the relationship between compressive strength with % of fly ash for 10% cement and the relationship between compressive strength with % of cement for 45% fly ash are presented in Figures 11 and 12 respectively as special cases.

From the graphs (Fig. 11 and 12) it is observed that the $R^2$ value comes to unity which is a good indication.

It is seen that the fitting with the single parameter in some cases like the case of relationship between compressive strength with % of cement for 55% fly ash (Figure 9) is not good. As such, the multi parameter model is developed and all the mix designs have been considered as data points and corresponding compressive strength both experimental and theoretical are presented as bar chart in Figure 13.

From the Figure 13, it is observed that the compressive strength obtained theoretically is in consistence with that obtained experimentally.

5. Conclusion

This study is a part of a research project at University of Botswana. The present paper demonstrates the inter-relationships among the product parameters namely compressive strength of flyash sand cement bricks with process parameters like % of fly ash, sand, cement and water added etc. As mentioned earlier, different degrees of polynomials for one parameter model are fitted using the existing experimental data and the corresponding coefficient of determination are reported to show the relationships of the parameters. Again using the multi parameter model the compressive strength for mix designs have been obtained from the model. It may be seen from the results that the proposed theoretical ones well fit with the experimental data. These results show the efficacy,
reliability and accuracy of the model. Using this model user may obtain the corresponding strength of the flyash sand cement bricks without doing repeated experiments.

Acknowledgement

The authors are thankful to Dr. C. Ketlogetswe, Head, and Mechanical Engineering Department University of Botswana, Botswana for assisting the sponsorship from University of Botswana for presenting this paper.

References


Fig. 13. Comparison of Expt. & Theo. Values of Compressive Strength of Flyash Sand Cement Bricks


