MODELLING AND SIMULATION OF AN USER-WHEELCHAIR-ENVIRONMENT SYSTEM

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ABSTRACT
Research on wheelchairs is on its growing phase and assistive aids brought to wheelchair users in terms of adaptability to electric wheelchair have not yet been perfected. This paper presents the development of a User-Wheelchair-Environment (UWE) system in order to ease driving evaluation of wheelchair user. The system is modelled using Matlab/Simulink and it provides useful information such as linear and angular velocity and displacement in a 3D-virtual environment. The model consists of three main components (user interface, wheelchair model and virtual environment) which are individually modelled and later integrated altogether to form the UWE system. Simulation results are also presented.

KEY WORDS
Wheelchair, driver, simulation, virtual reality, modelling

1. Introduction
With the increase in the number of both disabled and senior citizens, there is a great need of assistive systems for mobility enhancement of wheelchair users. Driving a wheelchair today imposes burdens on their users both physically and emotional [1][2]. Some improvements were done by developing dynamic models which can be controllable [3][4][5]. Most of these dynamic models were restricted to movements on flat surfaces ignoring the effects of gravitational forces experienced by wheelchair users in real life. In [6] a dynamic model considering the movement on an incline surface, which can improve safety of the user, was developed. Other methods using virtual reality were also proposed [7][8][9]. However, these methods were still based on a 2D motion. Therefore the development of a system which integrates both a 3D dynamic model and a 3D virtual model is presented. This system can be used as an evaluation tool to improve manoeuvring skills of wheelchair users. Critical conditions can be simulated without endangering the life of the subject. The UWE system consists of three main components. See Figure 1 for block diagram. In section 2, the modelling of the UWE system is presented. Simulation of the UWE system is discussed in section 3 and simulation results are also included.

A brief summary and further work are mentioned in the concluding section.

2. User-Wheelchair-Environment Modelling
The UWE system comprises a user controlling the wheelchair via a joystick interface in a virtual environment. In this section, we elaborate on the user interface model, dynamic wheelchair model and a virtual environment model also referred to as a 3D world.

2.1 User interface
This part of the overall model allows the user to control the wheelchair using the joystick. It also acts as a data acquisition block for our UWE system.

Figure 1. Block diagram of the UWE system

Figure 2. User interface Simulink model

The joystick input in Figure 2 gets a command from the external joystick connected to the computer via an USB port. That command obtained for linear and angular velocity is then amplified and fed to the Dead Zone Simulink symbol. The Dead Zones in Figure 2 filter out...
all slow movements in any direction due to the high sensibility of the joystick and only the clear intention of user is then used to control the wheelchair.

2.2 Wheelchair dynamic model

Most electric wheelchairs are conceived as nonholonomic mechanical systems. The rear wheels are independently driven by a motor. The two front wheels are castor wheels. The dynamic models used in this paper is comparable to the one proposed in [6]. This model considers a third dimension which is the wheelchair on an inclined surface. We will briefly discuss the mathematical model representing the wheelchair in an incline as illustrated in Figure 4.

2.2.1 Mathematical model of the wheelchair

The mathematical model of the wheelchair is based on its geometry and kinematic equations. The geometry of the wheelchair is dependent on its physical construction in the plane (see Figure 3) while the kinematic equations characterize its motion in the space.

Considering the position of the wheelchair in the 3D space \((xyz)-coordinate system) indicated by \(q = (x_g, y_g, z_g, \theta)^T\) with \((x_g, y_g, z_g)\) being the Cartesian coordinate the centre of gravity of the wheelchair, and \(\theta\) the angle of rotation of the wheelchair the Lagrange function is expressed on the following form:

\[
M(q)\ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = E(q)\tau + A^T(q)\lambda \tag{2}
\]

where

\[
M(q) = \begin{bmatrix}
M & 0 & 0 & IM \cos \phi \sin \theta \\
0 & M & 0 & -IM \cos \phi \cos \theta \\
0 & 0 & M & 0 \\
IM \cos \phi \sin \theta & -IM \cos \phi \cos \theta & 0 & I_z
\end{bmatrix}
\]

\[
C(q, \dot{q}) = \begin{bmatrix}
0 & 0 & 0 & \dot{\theta}IM \cos \phi \cos \theta \\
0 & 0 & 0 & \dot{\theta}IM \cos \phi \sin \theta \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
G(q) = \begin{bmatrix}
Mg \sin \phi \cos \theta \\
Mg \sin \phi \sin \theta \\
0 \\
Mg \sin \phi (y_g \cos \theta - x_g \sin \theta)
\end{bmatrix}
\]

\[
E(q)_{6x2} = \begin{bmatrix}
\cos \theta & \cos \theta \\
\sin \theta & r \\
0 & \sin \theta \\
0 & r \\
b & -b \\
r & r
\end{bmatrix}
\]

\[
A(q)_{2x4} = \begin{bmatrix}
-\cos \phi \sin \theta & \cos \phi \cos \theta & \sin \phi & -l \\
\sin \phi \sin \theta & -\sin \phi \cos \theta & \cos \phi & 0
\end{bmatrix}
\]

and \(\lambda\) the vector of Lagrange multipliers. The left and right torque is designated by \(\tau_L\) and \(\tau_R\) respectively. When both are equal, a rectilinear motion occurs and rotational motion in the contrary case.

Assuming that the wheels of wheelchair roll without slipping, lateral movements are not feasible, therefore the matrix associated to the nonholonomic constraints is perpendicular to the axis of the driving wheels are written in equation form as \(A(q) \dot{q} = 0\). Considering

\[
\mathcal{S} = \frac{1}{2}M \left(\dot{x}_g^2 + \dot{y}_g^2 + \dot{z}_g^2\right) + \frac{1}{2}I \dot{\theta}^2 + IM \theta \cos \phi (\dot{x}_g \sin \theta - \dot{y}_g \cos \theta) - Mg \sin \phi (\dot{x}_g \cos \theta + \dot{y}_g \sin \theta)
\]

(1)
$S(q) \in \mathbb{R}^{n \times m}$, a set of smooth and linearly independent vector fields spanning the null space of $A(q)$ such that $S^\top(q)A^\top(q) = 0$, we derive the kinetic equation as follows:

$$\dot{q} = S(q)\eta \quad (3)$$

With $S(q) = \begin{pmatrix} \cos \theta & -l \cos \phi \sin \theta \\ \sin \theta & l \cos \phi \cos \theta \\ 0 & 0 \end{pmatrix}$ and $\eta = \begin{pmatrix} \nu \\ \omega \end{pmatrix}$. The velocities $\nu$ and $\omega$ are respectively the linear and angular velocities of the distinguished point $G$, the centre of gravity of the wheelchair and user combined.

The derivative of Equation 2 with respect to time gives

$$\ddot{q} = \dot{S}(q)\eta + S(q)\ddot{\eta} \quad (4)$$

With the matrix $\dot{S}(q) = \begin{pmatrix} -\dot{\theta} \sin \theta & -\dot{l} \cos \phi \cos \theta \\ -\dot{\theta} \cos \theta & -\dot{l} \cos \phi \sin \theta \\ 0 & 0 \end{pmatrix}$.

Taking Equation 3 and 4 into Equation 2 and after simplifications, we obtain a more appropriate control system representation for the dynamic model of the wheelchair as follow

$$\begin{pmatrix} \dot{\nu} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ M \phi' \cos (2\phi - l_z) & M \phi' b \cos (2\phi - l_z) \end{pmatrix} \begin{pmatrix} \tau_u \\ \tau_v \end{pmatrix} \quad (5)$$

The model as expressed above, show that the wheelchair in an inclined flat surface is a nonlinear system. Results found in [6] can also confirm this nonlinearity, thus the need of a controller.

### 2.2.2 Wheelchair controller

Many techniques have been used to control nonholonomic systems. Dabo [10] applied two approaches to control the wheelchair and positive results were obtained for both methods. They are NCGPC (Nonlinear Continuous-time Generalized Predictive Control) and feedback linearization. The latter, also implemented in [6], is used in this paper. The goal being, for a given command, the motion along the trajectory should correspond.

The wheelchair dynamic model can be rewritten as a square-MIMO nonlinear system on the following form

$$\begin{cases} \dot{x} = f(x) + \sum_{j=1}^{2} g_j(x)u_j \\ y = \begin{bmatrix} h_1(x), h_2(x) \end{bmatrix}^T \end{cases}$$

where $x \in \mathbb{R}^2$, $u \in \mathbb{R}^2$ and $y \in \mathbb{R}^2$ are state, control input, and output vectors respectively.

As the goal is to track the trajectory from a given pair of reference linear velocity $\nu_r$ and angular positioning of the wheelchair $\theta_r$, the tracking error is then expressed as

$$\begin{cases} \epsilon^1 = \nu - \nu_r \\ \epsilon^2 = \theta - \theta_r \end{cases}$$

and should be forced to zero. This is achieved by input-output linearization. A system is input-output linearizable in a field $E \in \mathbb{R}^2$ if it has a relative degree vector $\rho = (\rho_1, \rho_2)$ obtained from successive derivation of Equation 6 with respect to time until the control input $(u_1, u_2)^T$ appears. Hence, after applied Lie derivative to Equation 6, we have

$$\begin{cases} \epsilon^1 = \dot{\nu} - \dot{\nu}_r = L_\rho \epsilon^1 + \left(L_\rho \epsilon^2 \right) u_1 \\ \dot{\epsilon}^2 = \dot{\theta} - \dot{\theta}_r = \omega - \omega_r = \epsilon^2 \\ \dot{\epsilon}^2 = \dot{\omega} - \dot{\omega}_r = L_\rho^2 \epsilon^2 + \left(L_\rho L_\rho \epsilon^2 \right) u_1 \\ \end{cases}$$

where

$$L_\rho \epsilon^1 = -g \sin \phi$$

$$L_\rho^2 \epsilon^2 = \frac{Mg \sin \phi \left( y_\phi \cos \theta - x_\phi \sin \theta \right) \cos 2\phi}{Ml^2 \cos 2\phi - l_z} - \dot{\omega}_r$$

with a relative degree vector equal to $\rho = (1, 2)$ and the decoupling matrix

$$D(x) = \begin{pmatrix} L_\rho \epsilon^1 & L_\rho \epsilon^2 \\ L_\rho L_\rho \epsilon^2 & L_\rho L_\rho \epsilon^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{M} \frac{1}{b} \\ \frac{1}{M} \frac{1}{b} \frac{1}{r(l_z - Ml^2 \cos 2\phi)} \end{pmatrix} \quad (7)$$

Being that the sum of the relative degrees $\left(\rho_1 + \rho_2\right)$ equal to 3 greater than the wheelchair system described in
Equation 5, it is then convenient to re-write Equation 3 as follow

\[
\begin{pmatrix}
\dot{\nu} \\
\dot{\omega}
\end{pmatrix} =
\begin{pmatrix}
-g \sin \theta \\
Mg \sin \phi \left( y_g \cos \theta - x_g \sin \theta \right)
\end{pmatrix} +
\begin{pmatrix}
\frac{1}{Mr} \\
\frac{1}{Mr} \\
br \left( I_z - Mr \cos 2\phi \right)
\end{pmatrix}
\begin{pmatrix}
\tau_x \\
\tau_l
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{Mr} \\
\frac{1}{Mr} \\
br \left( I_z - Mr \cos 2\phi \right)
\end{pmatrix}
\begin{pmatrix}
\tau_x \\
\tau_l
\end{pmatrix}
\]

where \( f(x) = \frac{-g \sin \theta}{Ml^2 \cos 2\phi - I_z} \),

\[
g_1(x) = \frac{1}{Mr}{\begin{pmatrix}
\frac{1}{Mr} \\
b \\
r \left( I_z - Mr \cos 2\phi \right)
\end{pmatrix}},
\]

\[
g_2(x) = \frac{1}{Mr}{\begin{pmatrix}
\frac{1}{Mr} \\
b \\
r \left( I_z - Mr \cos 2\phi \right)
\end{pmatrix}}, \text{ with } x = (\nu, \omega, \theta)^T.
\]

2.3 Environment modelling

In order to simulate the wheelchair behaviour one requires a 3D model for a wheelchair and a 3D model for a preferred environment. In this work, we used Virtual Reality Modelling Language to create the 3D model of the wheelchair and also two virtual environments; one representing a Turn-Right path and the other one a Ramp path of slope 5 degrees. All the 3D models were created according to the ADA standards. Figures 5-7 illustrate our 3D models. These 3D models were integrated with the abovementioned models (user interface and the wheelchair dynamic model) to form the UWE system. The Simulink model of UWE system is presented in Figure 8.

Figure 5. 3D model of the electric wheelchair

Figure 6. 3D-model of a Turn-Right Path

Figure 7. 3D-model of a Ramp Path
3. Simulation Results of the UWE System

In this section, both simulation setup and results are presented. Parameters such as the maximum velocities, the combined mass of the wheelchair and the user, the calculated gains for stability control and the slope of each surface plane in the virtual environment are displayed in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>User interface</td>
<td>Maximum velocity $V_{max} = 12$Km/h, $\omega_{max} = 0.4$rad/s</td>
</tr>
<tr>
<td>Wheelchair</td>
<td>Kinematics $b=0.35$m, $l=0.25$m, $r=0.2$m</td>
</tr>
<tr>
<td></td>
<td>Dynamics $Mass=80$kg, $I_z=39.733$kgm$^2$</td>
</tr>
<tr>
<td></td>
<td>Control gains $K1=7$, $K22=10$, $K21=200$</td>
</tr>
<tr>
<td>Environment</td>
<td>Flat surface $\phi=0^\circ$</td>
</tr>
<tr>
<td></td>
<td>Inclined surface $\phi=5^\circ$</td>
</tr>
</tbody>
</table>

3.1 User interface results

This part of the overall model allows the user to control the wheelchair using the joystick. The joystick used here is a Logitech ATTACK 3; it also includes various buttons which can be used to customize other applications. The gains (Gain1 and Gain2) appearing in Figure 3 correspond respectively to the maximum angular and linear velocities shown in Table 1. Resulting graphs of velocities are illustrated in Figure 9 and Figure 11. These graphs are represented with respect to the trajectories created by the user.

Figure 9. Velocities corresponding to the travelled trajectory on Turn-Right path presented in Figure 10.

Figure 10. 3D representation of the travelled trajectory by an user on a flat surface (Turn-Right path, $\phi=0^\circ$)
3.2 Dynamic model results

Results proving the stability, controllability and manoeuvrability of the whole system are represented in this subsection. The velocities, both linear and angular, obtained from the user interface correspond to the travelled trajectory. Graphs shown in Figure 11 and Figure 12 describe respectively the 3D representation of the trajectories in Figure 9 and Figure 10.

Figure 11. Velocities corresponding to the travelled trajectory on Ramp path presented in Figure 12.

Figure 12. 3D representation of the travelled trajectory by an user on an inclined surface (Ramp path $\phi_i=5^\circ$).

The driving activities of an ordinary user are clearly mapped by the resulting velocities and also by the trajectories travelled. The straight trajectory is a result of a very small or null angular velocity. The turning trajectory as shown in Figure 9 and Figure 11, is a result of a significant angular quantity.

4. Conclusion

The UWE system was modelled and simulated in this paper. The UWE system comprises three key components: User interface, wheelchair and Environment. Each component interacts with one another. This interaction is beneficial as relevant information such as velocities (linear and angular), accelerations (linear and angular), and displacement of user-wheelchair in the environment can be measured from the UWE system. Besides the fact that the UWE system is a good measuring tool and can be used to ease driving evaluation of the wheelchair user, it can also be used to develop a wheelchair user model. This development of wheelchair user model and its integration in the UWE system will be explored in future work.

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References


