ABSTRACT

Electrical power distribution networks are becoming more complex with increased variability in the type and properties of power station generators. National grid scale distribution networks are governed by many physical and geographic constraints and it is difficult to identify points of vulnerability. We employ some static graph analysis metrics to study how an approximation of the UK National Grid islands first into regions then small clusters of generators and substations. We use a progressive node culling procedure based upon the between-ness centrality metric to investigate numerically the properties of the overall network and individual islands as nodes and hence distribution lines fail. We find that the UK Grid system breaks first into regional islands, but is remarkably stable against node failures, breaking further into islands that are potentially viable at some service provision level. The between-ness metric identifies the most critical nodes in the network.

KEY WORDS

Islanding; network fragmentation; electrical power distribution; complex networks; risk analysis.

1 Introduction

Complex networks [26,33] such as power distribution systems have typically evolved over decades with many physical and geographical constraints. In the UK system the nuclear power stations are constrained to be on the coastline. Wind farms are obviously constrained to be in hilly or mountainous regions where there is sufficient wind. It is therefore useful to study such legacy systems to analyse critical points of failure and to hypothesise how additional resource could be added to ameliorate failure modes.

There is ongoing interest and reported research on complex network properties including hierarchical networks [7]; failure analysis [35] and distribution systems [23]. We focus on electrical power distribution networks [1,8] and associated network analyses [24,28] although the techniques described can also be applied to other infrastructural networks such as water [37] or transport systems [17].

Electrical distribution networks have particular spatial structural properties and constraints [21,30]. These can cause potential failure points [4,5] which can lead to power outages and other public consequences [3,22,32].

Network analyses can make use of agent based flow techniques [29] but a range of graph analysis metrics can also yield interesting insights. In this present article we use the between-ness centrality to characterise network nodes and study the consequences of their failure. Network centrality [13] is not new and various centrality measures [14]...
have been developed in recent years. The between-ness centrality metric [25] is discussed in Section 2 below and it identifies the most central or crucial node in a given network - and therefore the node whose failure or removal will have the greatest impact on the whole network.

Recent use of this metric [20] is reported for communications networks [34] applications and social [10] networks, as well as for electrical power distribution networks [19]. Figure 1 shows the test data set we analyse in this paper. It is an approximation of the UK Electricity National Grid/Super-Grid based upon 2006 data from [27].

An area of recent interest in the power generation and distribution communities [11] concerns accidental or intentional islanding [38]. As the name suggests, islanding involves the isolation or break up of a power distribution network into a number of separate component clusters [2, 36]. In principle these islanded regions could function independently of one another and intentional islanding might be used as a procedure to protect the whole network against cascade failures originating in particular nodes or regions. Accidental islanding as a consequence of node failure is also of interest. We study the islanding consequences of a progressive failure of the most critical – and therefore most likely heavily loaded – individual nodes in an approximation of the UK National Grid. Although islanding has been simulated in detail for micro grid systems [6], we are able to study realistically large scale network sizes such as the ≈ 500 nodes in the UK power-station/substation grid structure using computationally optimised graph analysis software.

Our article is structured as follows: In Section 2 we describe the between-ness metric and how we apply it to a network and progressively remove the highest ranked nodes to develop a static islanding procedure. We present a range of numerical results from the study of the UK National grid network in Section 3. We discuss the implications of the progressive islanding in Section 4 and offer some conclusions and areas for further work in Section 5.

2 Static Islanding Experiments

We first establish some terminology to discuss a power distribution network as a Graph $G$ with a set $V$ vertices or nodes. We thus have $N_V$ nodes (power stations or substations) and a set of $E$ edges containing the $N_E$ power lines. In this present article we do not differentiate between different voltages or carrying capacities of the individual lines for our model.

Centrality metrics such as the between-ness rank the nodes in an order denoting which is the most connected or crucial to the network. A simple centrality measure is simply the in or out degree of a node but this is not particularly insightful for the UK Grid data network we study here. The between-ness centrality metric ranks the node or vertex through which the maximal number of paths connecting any two other nodes pass. Computing the between-ness requires computation of the shortest distance between each pair of nodes $(s, t); s \in V, t \in V$ in the network. The fraction of the shortest paths that pass through each vertex $v$ is computed and summed over all possible pairs of vertices $(s, t)$. This is written as:

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}}$$

where $\sigma_{s,t}$ is the number of shortest paths from vertex $s$ to vertex $t$ and $\sigma_{s,t}(v)$ is the number that pass through vertex $v$. We normalise by dividing by the number of node pairs that do not include vertex $v$, which thus scales the computed between-ness values by a factor of $(n-1)(n-2)$.

We develop an experimental procedure to investigate a network by culling nodes progressively in their between-ness ranked order.

Algorithm 1 Computational Procedure for Systemic Static Islanding of Power Grid Network

1. set up network of generating and substation nodes
2. repeat
3.  compute between-ness of all nodes
4.  identify highest ranked node
5.  remove or cull highest-ranked node
6.  count remaining islanded component clusters
7.  record largest component clusters
8.  for all nodes in network do
9.    compute Dijkstra distance
10.   compute number of leaf nodes
11.  end for
12.  compute number of remaining power line edges
13.  record metrics
14. until no components with generation capacity left

Algorithm 1 shows our node-culling experimental procedure which gives us a systemic way to observe how a power distribution system will “island” as a consequence of a series of progressive failures, based upon the most critical – and presumably highest loaded component – failing at each stage. We also list the various properties we measure of the network and the islanded components at each stage. Plots of these are presented in Section 3 below.

A certain amount of book-keeping code was necessary to develop in order to manage the network component structure as nodes were progressively culled. A C++ code was developed that could manage an iterative series of in-memory graphs and their properties. Islanded components can be identified by various algorithms. In the case of this network of ≈ 500 nodes, a simple breadth-first colouring algorithm was sufficient [18] and could relabel the components clusters in near interactive time on a desktop computer.

Computing the Dijkstra shortest paths for a network is a well-known problem and there are several algorithms available [9, 16, 31]. Practically, for networks that are not too large the choice is dominated by the how easy it is to manage the memory structures in tandem with those for the other calculations such as component labelling and the
Floyd-Warshall algorithm [12] was sufficient for the work reported here.

The leaf nodes are identified using a simple one-pass through the network looking for nodes with unit connectivity. The edges are also simple to identify and cull when either of their end point nodes is culled.

Leigh’s map data [27] records generator type and gives some generator capacities. In this present work we do not use the specific 2006 capacity values but just the distinction between generators and substations to identify when an islanded cluster no longer has a connected generation capability.

3 Experimental Results

The network we study is an approximation of the UK National grid/super-grid based upon 2006 data from [27]. It contains 524 generator and substation nodes connected by 1,164 power line edges, with an average connectivity degree of 2.22, but with nodes varying in connectivity from 1 (leaf nodes) up to 6 (hub nodes). The map in Figure 1 shows both the high voltage grid and super-grid transmission lines and for the purposes of this present paper we treat them as one uniform network spanning Scotland, Wales and England.

We first measure the betweenness metric for the complete UK Grid (> 220kV) and Super-Grid (> 380kV) network data set. Figure 2 shows a plot of the natural logarithm of the betweenness for each node in the network, ranked in descending order. Not surprisingly the most crucial node as ranked by the betweenness centrality metric is at the Scotland/North-England border and is the key link line for the western connection between these two regions. The fitted slope for the straight line region of this plot is -0.10 and the intercept of the fitted line is -4.86. The quality of the fit suggests there is a fairly smooth relationship of the form \( C_B \approx A e^{-0.01r} \) for index rank \( r \). Note however the higher values for the most crucial nodes and the rapid fall off at low rank values indicated the presence of a small number < 10 of very highly used nodes on the main super-grid (> 380kV).

We apply the node culling procedure described in Section 2. Figure 3 shows the nearly linear decline in the number of operating power transmission lines in the network as individual nodes are progressively removed or culled in order of highest betweenness. The culling process effectively breaks the network and the loss of lines at 7.72 per node is disproportionate to the mean 2.22 node connectivity. Although this plot is relatively linear, the impact of the forced islanding can be seen in Figure 4 which shows how the number of islanded clusters grows as nodes are progressively culled and lines are effectively disconnected.

This plot is relatively linear with a slope of \( \approx 1.57 \) new islands per culled node initially. Once the top ten or so most crucial network nodes are gone however, the network begins to degrade at a different rate with a steeper slope.
This effect is also seen in Figure 5 which shows the size of the largest islanded cluster of nodes as nodes are progressively culled. There is rapid fall-off up to approximately ten culled nodes, thereafter the overall network declines more steadily. The lower plot in Figure 5 shows a log-log analysis of the same data. The linear region does not include the loss of the first five or so nodes, but thereafter appears to follow a power-law so that the largest island size $M \approx n^{-1.35}$ where $n$ is the number of culled network nodes.
Figure 5. Size of the biggest island cluster as nodes are progressively culled.

Figure 6 shows the network map where nodes are coloured (arbitrarily) by component cluster. As can be seen the first few culled nodes starts to separate the grid into Scotland and England-Wales. Further culled nodes breaks the system up into a surprisingly good approximation of the regional operating companies and finally with more than ≈ 8% of the network nodes culled, the network starts to break up into dysfunctional islands that do not necessarily have a generation capability within them.

4 Discussion

Other analyses we can apply to the network are to study the Dijkstra shortest path metric for the whole system, applied within the largest remaining component cluster and also to consider the change in the number of leaf nodes in the network.

Figure 7 shows how the Dijkstra distance fluctuates as nodes are culled and the largest remaining cluster varies. Note that this distance rises with poorer reachability/connectivity as well as declines with decreasing cluster size. The plot shows that most of the dramatic variation takes place within the loss of the first ten or so network nodes before it levels out to a steady value of around 8.

Figure 7. Dijkstra (hop) distance averaged over the whole system, as nodes are progressively culled.

Figure 8 shows the decline in the number $L$ of still-connected leaf nodes in the network. These leaf nodes are either generators or substations that are assumed to have lower voltage distribution sub nets attached to them.

The plot shows the number of leaf or terminal nodes declines relatively steadily with the number of culled nodes and a log-log plot suggests a power law relationship of approximately $L \approx n^{-1.23}$.

We have used naive network “hops” in our definition of the Dijkstra distance and not a proper physical distance-weighted metric nor a carrying capacity weight that would be more appropriate for a network of power distribution lines. Nevertheless it is likely that the gross power law behaviours found are indicative of how the real UK grid might behave in the presence of several failed major nodes.

The real UK grid is operated with the assumption of only a small number of likely actual node failures before some sort of demand management strategy being introduced to maintain frequency and power delivery levels. Not all clusters have generating capacity at the extreme number of culled nodes we have studied. However for realistic numbers of node failures ($< 20$) they do, and therefore service disruption is perhaps manageable without recourse to demand-side management [15].

There is scope to analyse the individual clusters in terms of their exact generation and power consumption amounts. There is published data on the generating capacity of individual power stations but it is harder to make a realistic model for the power consumption levels.

We have culled network nodes, since this was easiest to implement using the available graph analysis software. However it would be interesting to also study line failures. To an extent this can be done using the adjoint graph of the power distribution network. We have no realistic probabilistic values of line failure to incorporate into such a model and so the line between-ness could be used for such a study in a similar manner to the node between-ness we have used.
5 Conclusion

We have applied some static graph analysis metrics to an approximation of the UK National Electricity distribution grid. We have incorporated power stations and major substations and have applied an artificial islanding procedure whereby the most crucial node was identified as the node with the one with the largest number of pathways passing through it. Culling or removing the highest ranked node progressively we have been able to study how the grid network might break up into isolated islands of varying size and containing a mix of generating and power consumption nodes.

We found there are a relatively small number ($< 5\%$) of the nodes that play a pivotal role in the UK network, but that even with their removal, the network breaks up into regional supplier islands that are individually viable at some level of service. We note that as nodes were culled, the first fragmentation was into Scotland / England & Wales. This will remain a point of concern with the known steady annual flow of generated power across this boundary remaining a significant percentage of UK national generating capacity. Nevertheless the UK Grid appears to have evolved over time into a structure that is quite stable against a manageable number of failed nodes.

The data set exhibited a number of simple relationships consisting of regions of linear degradation and power-law behaviours. It would be interesting to determine if these are also found in other national-scaled electrical distribution networks or if they are unique to the UK system.

There is scope to consider more accurate line-carrying capacities, proper distance-weighted metrics, and Joule-heating and resistive weighting limitations to power flow in computing the metrics discussed. It would also be interesting to compute accurate generation and load properties on the individual islanded regions if representative load data were available.

Although we have used a relatively simple graph-oriented set of analysis techniques, we have found the between-ness centrality to be a useful metric to study overall network islanding properties. Intentional islanding of power generation and transmission networks is presently a topic of interest to many real generation and distribution companies and agencies and therefore this sort of study may be worthwhile extending to use more accurate assumptions about capacities and loads.

References