MATHEMATICAL MODELING AND SIMULATION OF A STANDALONE SOLAR THERMAL ORGANIC RANKINE CYCLE WITH A THERMAL STORAGE


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ABSTRACT
Mathematical modeling followed by a dynamic simulation of an Organic Rankine Cycle (ORC) assisted by solar thermal energy and thermal energy storage is taken into consideration in this study for a standalone application. N-Pentane was taken as the working fluid of the cycle and three different Phase Change Materials (PCM) were taken as the thermal storage. Dynamic simulation is carried out taking hourly solar insolation at Hambanthota, a southeast location of Sri Lanka (06°07′ N 81°07′ E). Results from the analysis depicts that seasonal variation of solar energy potential is having strong impact when coming up with the optimum collector area and capacity of thermal storage.

KEY WORDS

1. Introduction
Rural electrification and development in energy infrastructure support economic growth. However, it is a challenging endeavour to extend the existing main grid into certain areas in order to meet the electricity demand when considering present Sri Lankan context. Off-grid energy systems become an attractive solution is such instances. Therefore several research groups have focussed on developing stand alone energy systems in order to meet-up this challenge considering the present Sri Lankan context [1–3]. This work evaluates the capability of using solar thermal Organic Rankine Cycle (ORC) with a thermal storage for such standalone applications.

Rankine cycle (RC) can be taken as the thermodynamic cycle used to generate majority of the present electricity demand. In most of the instances, water is used as the working fluid of RC with high steam temperatures. However, thermal efficiency of RCs drops significantly when the temperature of steam drops below 370°C [4]. Therefore, RC need higher initial capital investment with number of techno economical challenges when generating electricity by using steam. In such circumstances, Organic fluids can be used to replace water in RCs in order to harness thermal energy [5].

Mathematical modeling and experimental validation, simulation and optimization of solar thermal ORC has become a rich area of study [6-10]. Design of solar collector is important when considering the overall efficiency of the ORC since it becomes the thermal source. Flat plate, evacuated type and compound parabolic collectors are taken into consideration in most of the instances when it comes to low temperature ORC [11]. Use of low temperature non concentric solar collectors may extract both diffuse and beam radiation and reduce the cost of the system [5]. Heat losses of the solar collector field can be significantly reduced by subdividing solar field into different temperature stages that consists of both concentric and non concentric collectors [12]. Optimum balance between concentric and non concentric collectors has been taken into consideration in Ref [5,13]. Therefore, design of collector array and collector temperature need to be optimized when developing Solar assisted ORC. However, flat plate collectors are more suitable when considering Sri Lankan context compared to concentrated collectors due to low clearness index.

When it comes to stand alone applications, seasonal variation of solar irradiation plays a major role. Thermal storage plays a major role in such instances especially for standalone applications. Simulation of the ORC with a thermal storage by using a time series of solar irradiation data and Electricity Load Demand (ELD) becomes essential in order to assure higher power supply reliability which is not taken into consideration in recent literature which is taken into discussion in this work.

2. Mathematical Model for the Solar Assisted ORC with a Thermal Storage
This section illustrates the mathematical model developed to simulate the system considering hourly varying solar irradiation and Electricity Load Demand (ELD) which is later used to derive the area for solar collector and capacity of thermal storage. The system consists of evacuated type solar collectors, two thermal storage tanks and an ORC. The schematic diagram of the system is given in Fig. 1.
2.1 Mathematical Model for Solar Thermal Collector (STC)

Area of the STC is selected in order to provide the ELD throughout the year considering seasonal variation of solar irradiation and ELD since solar thermal energy becomes the energy source in this model. Both hourly solar irradiation on the tilted STC and its performance should have to be modeled in order to find the energy output from STCs. In order to achieve this, hourly global irradiation on a horizontal plane \( G \) is taken (Fig. 2) and it is used to calculate hourly diffuse fraction \( f \) using Climed-2 model [14], which was later used to calculate diffuse solar radiation \( G_d \) on horizontal plane according to Eq. (1).

\[
G_d = f \cdot G
\]  

(1)

Klucher model [15] was used to calculate diffuse solar radiation on tilted surface \( G_{d\beta} \). Finally, beam radiation \( G_{b\beta} \) and reflected solar radiation \( G_{r\beta} \) were calculated using Eq. (2) and (3). These values were used to find total solar radiation on tilted surface \( G_\beta \) using Eq. (4).

\[
G_{b\beta} = (G - G_d) \cdot \frac{\cos \theta}{\cos (\theta_z)}
\]  

(2)

\[
G_{r\beta} = \left(\frac{1}{2}\right) \rho \cdot G \cdot (1 - \cos(\beta))
\]  

(3)

\[
G_\beta = G_{d\beta} + G_{b\beta} + G_{r\beta}
\]  

(4)

where \( \theta \), \( \theta_z \), and \( \beta \) denote angle of incidence for an arbitrarily inclined surface oriented toward the equator, zenith angle and tilt angle of STC.

Evacuated type STC is taken into consideration in this work. The thermal efficiency of an evacuated type STC (\( \eta_c \)) can be modeled as a function of medium operating temperature of the STC (\( T_m \)), of the STC (\( T_m \)).
ambient temperature \((T_a)\) and solar irradiation on tilted STC \((G_p)\) according to Eq. 5 [16].

\[
\eta_c(t) = 0.84 - \frac{2.02 (T_m - T_a)}{G_p(t)} - 0.00461(t) \cdot \left[\frac{(T_m - T_a)}{G_p(t)}\right]^2
\]

(5)

![Fig. 2: Hourly variation of solar insolation](image)

Finally, hourly net thermal energy output from STCs is calculated using Eq. (6) where \(A_c\) denotes collector area of the STC.

\[
Q_{col}(t) = 3600 \cdot A_c \cdot I(t) \cdot \eta_c(t)
\]

(6)

The extracted thermal energy from the solar collectors is continuously supplied to evaporator and excess energy stored in a latent heat thermal storage tank by passing conduction oil through the solar collectors. Hourly mass flow of conduction oil through the solar collectors is calculated using Eq. (7).

\[
m_{\text{col}}(t) = \frac{Q_{\text{col}}(t)}{C_s \cdot (T_H - T_{L2})}
\]

(7)

where \(C_s\) denotes specific heat capacity of conduction oil and \(T_H\) denotes the temperature at the solar collector outlet.

### 2.2 Mathematical Model for Thermal Energy Storage

Main purpose of the thermal storage is to store excess thermal energy from STC in order to support continuous power generation under timely varying ELD and solar irradiation. Phase Change Material (PCM) is used as the thermal storage medium and energy is stored by melting the PCM and released by solidifying. It is essential to come up with a dispatch strategy for both store and release of energy. Dispatch strategy is based on the difference between solar energy potential and ELD which consists of three different states i.e Charging Cycle, Combined Discharge Cycle and Discharge Cycle.

#### 2.2.1 State 1 (Charging Cycle)

When thermal energy production is in higher than the requirement of ELD, the system starts following Charging Cycle. In this mode valves 1, 2, 4 and 5 are kept open in order to store excess thermal energy. Hourly mass flow of conduction oil through the evaporator in order to supply the ELD is calculated using Eq. (8).

\[
m_1(t) = \frac{Q_E(t)}{C_s \cdot (T_H - T_{L1})}
\]

(8)

where \(Q_E(t)\) denotes the evaporator heat requirement (discussed in detail in Section 2.3), and \(T_{L1}\) denotes the conduction oil temperature at evaporator outlet. Excess mass from the solar collectors \((m_2(t))\) is evaluated using Eq. (9).

\[
m_2(t) = m_{\text{col}}(t) - m_1(t)
\]

(9)

The amount of thermal energy stored \((Q_{\text{supply}}(t))\) in thermal storage can be determined using Eq. (9) and Eq. (10).

\[
Q_{\text{supply}}(t) = m_2(t) \cdot C_s \cdot (T_H - T_{L2})
\]

(10)

In this equation \(T_{L2}\) denotes the conduction oil temperature at thermal storage outlet.

Subsequently, available thermal energy capacity in the thermal storage available is calculated using Eq. (11) where \(Q_{\text{Losses}}(t)\) denotes thermal losses.

\[
Q_s(t) = Q_s(t-1) + Q_{\text{supply}}(t) - Q_{\text{Losses}}(t)
\]

(11)

Finally, melted PCM percentage of the thermal storage \((M_p)\) is computed using Eq. (12) where \(C_{LS}\) denotes latent heat of fusion of PCM and \(M_{PCM}\) denotes total mass of PCM.

\[
M_p = \frac{Q_s(t) \cdot 100}{C_{LS} \cdot M_{PCM}}
\]

(12)

#### 2.2.2 State 2 (Combined Discharge Cycle)

When solar thermal energy collected from the collector is not enough to drive the cycle to provide the ELD alone, system moves to Combined Discharge Cycle (State 2). In this mode, valves 2, 3, 5 and 6 are kept open and energy from both STC and thermal energy storage are used to supply the ELD. Thermal energy required from the thermal storage \((Q_{\text{require}}(t))\) in this state is calculated using Eq. (13).

\[
Q_{\text{require}}(t) = Q_E(t) - Q_{col}(t)
\]

(13)

Finally available thermal energy capacity is calculated using Eq. (14).

\[
Q_s(t) = Q_s(t-1) - Q_{\text{require}}(t) - Q_{\text{Losses}}(t)
\]

(14)
2.2.3 State 3 (Discharge Cycle)

Discharge cycle is used when solar irradiation is not available. Stored thermal energy in PCM is used in this state to provide the ELD. Valve 3 and valve 6 are kept open in this mode allowing conduction oil to flow through the thermal storage and evaporator. Thermal energy required from the thermal storage is calculated using Eq. 15 and available thermal energy capacity in thermal storage is calculated using Eq. 14.

\[ Q_{\text{require}}(t) = Q_E(t) \quad (15) \]

2.3 Mathematical Model for Organic Rankine Cycle (ORC)

This section provides a detailed explanation about the dynamic mathematical model formulated in order to simulate the ORC. Four basic components of the cycle i.e. pump, condenser, turbine and evaporator is modeled in order to come up with work and heat transfer from these devices. Fig. 3 provides the T/ S diagram of the state points of ORC.

When considering the work transfer of the cycle, work input at pump and work output from the turbine has being coupled together. Therefore, work input at the pump is initially calculated using Eq. 16. In this equation, \( \dot{m}_{\text{cycle}}(t) \) denotes the mass flow rate of working fluid, \( P_E \) and \( P_C \) denote evaporator and condenser pressures, \( \rho_{WF} \) denotes the density of working fluid and \( \eta_p(t) \) denotes the working efficiency of pump.

\[ \dot{w}_p(t) = \frac{\dot{m}_{\text{cycle}}(t)(P_E - P_C)}{\rho_{WF} \cdot \eta_p(t)} \quad (16) \]

The efficiency of pump is a function of the mass flow rate through the pump and calculated by using Eq. 17 [17]. In Eq. 17, \( \dot{m}_{\text{cycle}, \text{ref}} \) denotes the reference mass flow rate at peak demand and \( \eta_{p, \text{ref}} \) denotes the reference efficiency of the pump at peak demand.

\[ \eta_p(t) = 2 \cdot \eta_{p, \text{ref}} \cdot \left[ \frac{\dot{m}_{\text{cycle}}}{\dot{m}_{\text{cycle}, \text{ref}}} \right] - \eta_{p, \text{ref}} \cdot \left[ \frac{\dot{m}_{\text{cycle}}}{\dot{m}_{\text{cycle}, \text{ref}}} \right]^2 \quad (17) \]

Required turbine work is the addition of mechanical work required for electricity generation and pump work. This is calculated according to Eq. 18 assuming constant generator efficiency (\( \eta_{\text{gen}} \)).

\[ \dot{w}_T(t) = \frac{\dot{w}_{\text{load}}(t)}{\eta_{\text{gen}}} + \dot{w}_p(t) \quad (18) \]

The isentropic efficiency of the turbine is a function of turbine mass flow rate and \( \dot{m}_{\text{cycle}, \text{ref}} \) determined using Eq. 19 [17].

\[ \eta_t(\dot{m}_{\text{cycle}}) = -0.1423 \cdot \left[ \frac{\dot{m}_{\text{cycle}}}{\dot{m}_{\text{cycle}, \text{ref}}} \right]^2 + 0.2981 \cdot \left[ \frac{\dot{m}_{\text{cycle}}}{\dot{m}_{\text{cycle}, \text{ref}}} \right] + 0.6127 \quad (19) \]
Enthalpy of the working fluid at the turbine outlet \( (h_2) \) is calculated using Eq. 20 where \( h_1 \) denotes enthalpy of working fluid at the turbine inlet and \( h_{2S} \) denotes enthalpy under iso-entropic working conditions

\[
h_2 = h_1 - \eta_t (m_{cycle} \cdot (h_1 - h_{2S}))
\]  

Finally, hourly energy requirement from the evaporator \( (Q_E(t)) \) is calculated by using Eq. 21 where \( h_4 \) denotes the enthalpy of the working fluid at pump outlet

\[
Q_E(t) = m_{cycle}(t) \cdot (h_1 - h_4)
\]  

3. Simulation of the System

Simulation Program is developed using C++ language in Visual Studio® 2008 in Microsoft Windows® 07 environment. In this investigation, the system conditions are evaluated on hourly basis considering timely varying solar irradiation and required ELD. ELD is highly sensitive to the application. In this work, it was assumed that the ELD varies throughout the year according to summer - weekly load IEEE reliability test system [18], which is scaled to 7.5 kW (Fig. 4). Steady state conditions of system are assumed during each hour.

The simulation is conducted by evaluating thermal energy supply from solar collectors and thermal energy required for evaporator hourly. Subsequently, heat capacity of latent heat storage is calculated according to working mode of system as discussed in Section 2.2. The latent heat storage capacity is considered as the key condition for continuous operation of the system. The condition of the latent heat thermal storage depends on availability of the usable thermal energy inside the storage (availability of the molten PCM material). According to the availability of stored thermal storage energy, the collector area is increased until continuous power generation is feasible throughout the year. Using this procedure the collector areas are calculated for different thermal storage capacities. Finally, sensitivity of collector temperature and PCM are evaluated by varying the collector temperature and PCM. Selected thermodynamic properties of used PCM are given in Table 1[16].

<table>
<thead>
<tr>
<th>PCM</th>
<th>Melting temperature ( ^\circ\mathrm{C} )</th>
<th>Latent heat of fusion ( \text{kJ/kg} )</th>
<th>Density ( \text{kg/m}^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adipic acid</td>
<td>151-155</td>
<td>260</td>
<td>1360</td>
</tr>
<tr>
<td>Suberic acid</td>
<td>141-144</td>
<td>245</td>
<td>1020</td>
</tr>
<tr>
<td>Sebacic acid</td>
<td>130-134</td>
<td>228</td>
<td>1270</td>
</tr>
</tbody>
</table>

4. Results and Discussion

It is essential to determine the optimum storage capacity, temperature and area of the solar collector, evaporator pressure and temperature, and PCM in the final design. In order to analyze the relationship between solar collector area and thermal storage capacity, minimum collector area required to supply the ELD throughout the year for different solar collector temperatures are computed while varying the thermal storage capacity with Adipic acid...
Gradual reduction of STC area is expected with the increase of thermal storage capacity. However, complex variation takes place due to seasonal variation of solar irradiation. Variation of STC area with thermal storage is plotted for Suberic acid and Sebacic acid similar to Adipic acid (Fig. 7 and Fig. 8).

From simulation results it was validated that, the required solar collector area reduces with the increase of thermal storage capacity as expected. Further, it is observed that the system is getting optimized for the worst case operation where solar irradiation is at its minimum. Therefore, the system is over capacity for the rest of the period with a poor plant factor. However, it is impossible to find the optimum system design by only considering the thermal behaviour through a parameter such as area of STC which continuously decreases with the increase of thermal storage capacity.

5. Conclusion

As mentioned above, it can be seen that the proposed system runs below the designed capacity for most of the time. Therefore, the capital cost for the solar collectors and thermal storage increases unnecessarily. In order to utilize the excess energy while increasing power supply reliability and energy conversion efficiency addition other energy source such as biomass and wind energy is proposed in this work.

It is difficult to arrive at an optimal system design based on obtained results. Therefore, it is essential to combine a lifecycle cost model along with the dynamic simulation. In addition to that, multi objective optimization can be carried out considering both life cycle cost and exergy efficiency in order to come up with optimum organic fluids, solar collector types and thermal storage types with an additional energy source to fulfil the energy demand which will be taken into consideration in future publications.

Fig. 6 Variation of STC area with capacity of Thermal storage for Adipic acid

Fig. 7 Variation of STC area with capacity of Thermal storage for Suberic acid

Fig. 8 Variation of STC area with capacity of Thermal storage for Sebacic acid

Fig. 9 Impact of thermal storage medium on STC area at collector temperature of 160°C
References


