ROBUST SERVOMECHANISM PROBLEM FOR NEUTRAL DISTRIBUTED TIME-DELAY SYSTEMS

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ABSTRACT
An important problem in control engineering is to design a controller which stabilizes a given system and achieves robust tracking of a reference signal despite certain disturbance signals. The present paper considers this problem, which is usually termed as the robust servomechanism problem, for linear time-invariant time-delay systems of neutral type with distributed time-delay. It is assumed that the reference and the disturbance signals satisfy a delay-differential equation, which is linear and has constant-coefficients and is of neutral type with distributed time-delay. Discrete time-delays, whether in the system dynamics or in the delay-differential equation satisfied by the reference and the disturbance can also be included. The present paper presents the necessary and sufficient conditions for the solvability of the robust servomechanism problem, as well as the structure of the controller when it exists, for linear time-invariant neutral time-delay systems with distributed time-delay.

KEY WORDS
Control theory; robust control; neutral time-delay systems; distributed time-delay; servomechanism problem.

1 Introduction

Time-delays naturally exist in many systems. A controller design, which ignores such a time-delay may be satisfactory if the time-delay is small compared to the time-constants of the rest of the system. However, when these delays are large, they must be considered during controller design to obtain a satisfactory controller. Systems which involve time-delays, however, are infinite-dimensional. Hence it is more difficult to design a controller for such a system. When the time-delay is distributed, rather than discrete, the controller design problem is even more difficult [1, 2, 3]. As it has been indicated in [3], a number of different controller design methods exists for systems with time-delays. By now famous Smith predictor has been introduced by Smith [4]. Meinsma and Zwart [5] utilized J-spectral factorizations for multi-input multi-output systems with a single delay. Meinsma and Mirkin [6] extended this approach to systems with multiple delays. State-space methods were used by Nagpal and Ravi [7] and Tadmor [8]. Lyapunov-Krasovskii functionals [9] have been used by many researchers to analyze stability and design stabilizing controllers for time-delay systems (e.g., see [10]–[13] for retarded and [14]–[17] for neutral systems with discrete time-delays; see [18]–[23] for systems with distributed time-delay).

An important problem in control engineering is the robust servomechanism problem, which is to design a controller which stabilizes a given system and achieves robust tracking of a reference signal despite certain disturbance signals. The robust servomechanism problem has been well studied for linear time-invariant (LTI) delay-free systems (e.g., see [24]–[30] for the case of centralized control and [31]–[35] for the case of decentralized control). Discrete-event systems [36] and nonlinear systems [37, 38] have also been treated for this problem. Time-delay systems, however, has been treated only recently. A number of researchers (e.g., [39]–[41]) have considered this problem for specific type of controllers. The most general case (without any restrictions on the particular controllers to be used) has been considered in [42] for retarded time-delay systems and in [43] for neutral time-delay systems. A retarded time-delay system does not involve any delays in the derivative of its state vector. When the derivative of the state vector is subject to time-delays, the system is called neutral. It is known that retarded time-delay systems can have only finitely many poles in the right-half complex plane. Neutral time-delay systems, on the other hand, may have infinitely many poles in the right-half complex plane [2]. Hence, it is, in general, more difficult to control neutral time-delay systems. In [42, 43] the necessary and sufficient conditions for the solvability of the robust servomechanism problem, for the respective type of systems, have been derived. The structure of the controller, when it exists, has also been presented in [42, 43]. The treatment in [42, 43], however, have been restricted to systems with discrete time-delays. Retarded systems with distributed time-delay have been considered in [3]. In the present paper we extend the results of [3] to neutral distributed time-delay systems. Specifically, we present the necessary and sufficient conditions for the solvability of the robust servomechanism problem for such systems, where it is assumed that the reference and the disturbance satisfy a constant-coefficient linear delay-differential equation of neutral type with distributed time-
(delay. Furthermore, we also show that the discrete time-delays, whether in the system dynamics or in the delay-differential equation satisfied by the reference and the disturbance, can also be treated, besides the distributed time-delay, in the present framework. We also present the structure of the controller which solves this problem, when a solution exists.

Throughout the paper, as in [3], for positive integers $k$ and $l$, $\mathbb{R}^k$ denotes the space of $k$ dimensional real vector functions of a real variable. $\mathbb{R}^{k \times l}$ denotes the space of $k \times l$ dimensional real matrices. $k$ and $0_{k \times l}$ respectively denote the $k \times k$ dimensional identity matrix and the $k \times l$ dimensional zero matrix. $I$ and $0$ are used to denote, respectively, the identity matrix and the zero matrix of appropriate dimensions whenever the dimensions are apparent. $\otimes$ denotes the Kronecker product. For a (vector) function $f$ of a real variable, $\dot{f}$, $\ddot{f}$, and $f^{(k)}$ respectively denote the first, the second, and the $k$th derivative of $f$. A matrix function $M(\cdot) : \mathcal{I} \rightarrow \mathbb{R}^{k \times l}$, where $\mathcal{I}$ is an interval of the real line, is said to have full column-rank if $M(\theta)\xi = 0$, $\forall \theta \in \mathcal{I}$, where $\xi$ is a constant vector, implies that $\xi = 0$. Similarly, $M(\cdot) : \mathcal{I} \rightarrow \mathbb{R}^{k \times l}$ is said to have full row-rank if $\xi^T M(\theta) = 0$, $\forall \theta \in \mathcal{I}$, where $^T$ denotes the transpose, implies that $\xi = 0$.

2 Problem Statement

In this work, we consider LTI neutral time-delay systems with distributed time-delay. The dynamics of the system are described by

$$
\dot{x}(t) + \int_{-\tau}^{0} F(\theta) \dot{x}(t + \theta)d\theta = \int_{-\tau}^{0} [A(\theta)x(t + \theta) + B(\theta)u(t + \theta) + G(\theta)w(t + \theta)]d\theta .
$$

(1)

Here, $x \in \mathbb{R}^{n_x}$ denotes the state and $u \in \mathbb{R}^{n_u}$ and $w \in \mathbb{R}^{n_w}$ respectively denote the control input (also called as input) and the disturbance input (also called as disturbance). Furthermore, $\tau > 0$ is the maximum delay in the system and $t$ is the time variable. Moreover, $F(\cdot)$, $A(\cdot)$, and $B(\cdot)$ are matrix functions, which are given, and $G(\cdot)$ is a matrix function which is arbitrary (not necessarily known).

The system has an output to be regulated (also called as output), denoted by $y \in \mathbb{R}^{n_y}$, and a measured output (also called as measurement), denoted by $y^m \in \mathbb{R}^{n_y'}$. The output and the measurement are respectively given by:

$$
y(t) = \int_{-\tau}^{0} [C(\theta)x(t + \theta) + E(\theta)u(t + \theta) + H(\theta)w(t + \theta)]d\theta .
$$

(2)

and

$$
y^m(t) = \int_{-\tau}^{0} [C^m(\theta)x(t + \theta) + E^m(\theta)u(t + \theta) + H^m(\theta)w(t + \theta)]d\theta .
$$

(3)

Here, $C(\cdot)$, $C^m(\cdot)$, $E(\cdot)$, and $E^m(\cdot)$ are given matrix functions and $H(\cdot)$ and $H^m(\cdot)$ are arbitrary matrix functions. All the matrix functions involved are assumed to be bounded, except that they may contain Dirac delta terms. It is also assumed that $F(0)$ is bounded, i.e., $F(\theta)$ does not contain a term $\delta(\theta)$, but may contain $\delta(\theta + h)$, for some $h > 0$. Here $\delta(\cdot)$ is the Dirac delta function, defined as

$$
\int_{-\tau}^{0} \delta(\theta + h)\xi(t + \theta)d\theta = \xi(t - h)
$$

for any $\tau \geq 0$, any $h$ satisfying $0 \leq h \leq \tau$, and any continuous function $\xi(\cdot)$. By allowing these matrices to contain Dirac delta terms, we in fact allow representation of discrete delays besides distributed ones. For example, a term like

$$
A_1x(t - h) + \int_{-\tau}^{0} A_0(\theta)x(t + \theta)d\theta ,
$$

for some discrete delay $h \geq 0$, where $A_1$ is a constant matrix and $A_0(\cdot)$ is a bounded matrix function, in the right-hand-side of the system’s dynamics equation, can be represented as in (1) with $A(\theta) = \delta(\theta + h)A_1 + A_0(\theta)$. Here, of course, we assume that $h \leq \tau$. If $h > \tau$, then $\tau$ in (1) must be replaced by $h$, and we must define $A_0(\theta) = 0$ for $-h \leq \theta < -\tau$.

The problem involves a reference input (also called as reference), $r \in \mathbb{R}^{n_y}$, which is assumed to be available on-line. The output $y(t)$ is required to asymptotically track this reference. The reference, just like the disturbance, is not known in advance, but, it is known that they satisfy a linear constant-coefficient delay differential equation of neutral type with distributed time-delay. More specifically, given a LTI delay-differential operator, $D$, of neutral type with distributed time-delay, which is defined as

$$
D\xi(t) := \frac{d^\mu}{dt^\mu}\xi(t) + \sum_{k=0}^{\mu} \frac{d^k}{dt^k} \int_{-\tau}^{0} a_k(\theta)\xi(t + \theta)d\theta ,
$$

(4)

the reference and the disturbance respectively satisfy

$$
Dr(t) = 0 \quad \text{and} \quad Dw(t) = 0 .
$$

(5)

In (4), $\mu$, which is a positive integer, is the differential degree of the operator $D$. Furthermore, $a_k(\cdot)$, $k = 0, \ldots, \mu$, are bounded scalar functions, except that they may contain Dirac delta terms. $a_0(\theta)$, however, is assumed to be bounded, i.e., $a_0(\theta)$ does not contain a term $\delta(\theta)$, but may contain $\delta(\theta + h)$, for $h > 0$.

Now, let us consider the dynamics

$$
\dot{z}(t) + \int_{-\tau}^{0} \mathcal{F}(\theta)\dot{z}(t + \theta)d\theta = \int_{-\tau}^{0} A(\theta)z(t + \theta)d\theta ,
$$

(6)

where $z \in \mathbb{R}^{n}$ is the state vector,

$$
\mathcal{F}(\theta) := \begin{bmatrix}
0_{n-1 \times \mu-1} & 0_{n-1 \times 1} & a_\mu(\theta)
\end{bmatrix},
$$
and
\[
A(\theta) := \begin{bmatrix}
0 & 0 & \cdots & 0 & -a_0(\theta) \\
\delta(\theta) & 0 & \cdots & 0 & -a_1(\theta) \\
0 & \delta(\theta) & \cdots & -a_2(\theta) \\
\vdots & \ddots & \ddots & \ddots \\
0 & \delta(\theta) & \cdots & -a_{\mu-1}(\theta)
\end{bmatrix}.
\]

Then the reference and the disturbance can be represented as
\[
r(t) = C^rz(t) \quad \text{and} \quad w(t) = C^wz(t),
\]
where \(C^r \in \mathbb{R}^{n_y \times \mu}\) and \(C^w \in \mathbb{R}^{n_w \times \mu}\) are arbitrary constant matrices.

**Remark:** In general, the disturbance and the reference may satisfy different delay-differential equations. That is, there may exist two different operators, \(D^w\) and \(D^r\), such that the state of the system (say \(\tau_1\)) and for \(D\) (say \(\tau_2\)) may also be different. In such a case, however, one may simply define \(\tau = \max(\tau_1, \tau_2)\) and let \(a_k(\theta) = 0\) for \(-\tau \leq \theta < -\tau_2\), \(\forall k = 1, \ldots, \mu\), if \(\tau_2 < \tau_1\); or let \(M(\theta) = 0\) for \(-\tau \leq \theta < -\tau_1\), if \(\tau_1 < \tau_2\), where \(M\) represents any one of the matrix functions appearing in (1)–(3).

The following assumptions are the usual assumptions (e.g., see [3]), as adopted by the present problem, for the robust servomechanism problem. They are needed for non-triviality:

**Assumption 1:** The state of the system described by (6) is observable through the output
\[
\begin{bmatrix}
r(t) \\
w(t)
\end{bmatrix} = \begin{bmatrix}
C^r \\
C^w
\end{bmatrix} z(t)
\]
and \(G(\cdot) := \begin{bmatrix} G(\cdot) \\ H(\cdot) \\ H^m(\cdot) \end{bmatrix}\) has full column-rank.

**Assumption 2:** Let \(z(t)\) be a solution to (6). Then, \(\lim_{t \to \infty} z(t) = 0\) only if \(z(\theta) = 0, \forall \theta \in [-\theta_{\text{max}}, 0]\).

Here \(\theta_{\text{max}} \geq 0\) is the smallest number such that, for all \(k = 0, \ldots, \mu, a_k(\theta) = 0, \forall \theta \in [-\tau, -\theta_{\text{max}}]\). If there is no such \(\theta_{\text{max}}\), then \(\theta_{\text{max}} = \tau\).

**Assumption 3:** \(B(\cdot)\) has full column-rank.

**Assumption 4:** \(C(\cdot)\) has full row-rank.

The above assumptions can be made without loss of generality (as in other cases, such as the case of discrete delays [43]). Assumption 1 states that our plant, (1)–(3), is actually affected by all of the system (6). If some part of this system does not affect our plant, then that part can be eliminated. Assumption 2 states that the system (6) do not contain any part which is asymptotically stable. Any such part can be eliminated, since our problem is concerned with asymptotic stability and asymptotic tracking.

Assumption 3 states that the plant has no redundant inputs. If it does, those inputs can be eliminated. Finally, Assumption 4 states that the plant has no redundant outputs. If it does, those outputs can be eliminated.

The formal statement of our problem is as follows.

**Robust Servomechanism Problem:** Find a controller, which uses feedback from \(y^m\) to \(u\), such that the closed-loop system formed by the plant (1)–(3) and this controller is globally asymptotically stable. Furthermore, for all references \(r(t)\) and for all disturbances \(w(t)\) satisfying (5), and under all initial conditions,
\[
\lim_{t \to \infty} e(t) = 0,
\]
where \(e \in \mathbb{R}^{n_y}\) denotes the tracking error and is given by
\[
e(t) = y(t) - r(t).
\]
Moreover, (8) continuous to hold under all nondestabilizing perturbations in the matrix functions appearing in (1)–(3).

**Remark:** As for the case of delay-free systems [27, 28], a weaker version of the above problem can also be defined. The definition of this problem, which is called the **Weak Robust Servomechanism Problem**, is exactly same as the robust servomechanism problem, except that perturbations in the matrix functions appearing in (2) and (3) are not allowed in this case.

### 3 Main Result

In this section necessary and sufficient conditions for the solvability of the robust servomechanism problem and the structure of the controller which solves this problem will be presented. For this, let us first define \(\tilde{F}(\cdot) := F(\cdot) \otimes I_{n_y}, \tilde{A}(\cdot) := A(\cdot) \otimes I_{n_y},\) and \(\tilde{B} := \begin{bmatrix} 1 \\ 0_{n_y \times 1} \end{bmatrix} \otimes I_{n_y}\). The necessary and sufficient conditions for the existence of a solution to our problem are given by the following theorem.

**Theorem 1:** There exists a solution to the robust servomechanism problem (as defined in Section 2) if and only if the following are satisfied:

1. The system (1) with \(w(t) \equiv 0\) is stabilizable through the input \(u\).
2. The system (1) with \(w(t) \equiv 0\) is detectable through the output \(y^m\) given by (3) with \(w(t) \equiv 0\).
3. The system
\[
\tilde{\eta}(t) + \int_{-\tau}^{0} \begin{bmatrix} F(\theta) & 0 \\ 0 & F(\theta) \end{bmatrix} \tilde{\eta}(t + \theta) d\theta \\
\int_{-\tau}^{0} \begin{bmatrix} A(\theta) & 0 \\ B(\theta) & \tilde{A}(\theta) \end{bmatrix} \tilde{\eta}(t + \theta) \\
+ \begin{bmatrix} B(\theta) & \tilde{B}(\theta) \end{bmatrix} \tilde{\eta}(t + \theta) d\theta,
\]
where \(\tilde{\eta} \in \mathbb{R}^{n_y+n_y\mu}\) is the state, is stabilizable through the input \(\tilde{u} \in \mathbb{R}^{n_w}\).
4. The output $y(t)$, given in (2), is contained in $y^m(t)$, given in (3); i.e., $y$ is actually measurable.

**Proof:** **Necessity:** Conditions 1 and 2 are required so that a stabilizing feedback controller can be designed.

To show the necessity of Condition 3, let

$$
\dot{e}(t) :=\begin{bmatrix} e_{\mu-1}(t) \\
\vdots \\
e_1(t) \\
e(t) \end{bmatrix}
$$

where

$$
e_{\mu-1}(t) := \dot{e}(t) + \int_{-\tau}^{0} a_\mu(\theta) \dot{e}(t+\theta) d\theta + \int_{-\tau}^{0} a_{\mu-1}(\theta)e(t+\theta) d\theta + \frac{d}{dt} \int_{-\tau}^{0} a_{\mu-1}(\theta)e(t+\theta) d\theta + \ldots + \int_{-\tau}^{0} a_1(\theta)e(t+\theta) d\theta.
$$

It is easy to show that, the above relations give

$$
\dot{e}(t) + \int_{-\tau}^{0} a_\mu(\theta) \dot{e}(t+\theta) d\theta = e_1(t) - \int_{-\tau}^{0} a_{\mu-1}(\theta)e(t+\theta) d\theta ,
$$

$$
\dot{e}_j(t) = e_{j+1}(t) - \int_{-\tau}^{0} a_{\mu-j-1}(\theta)e(t+\theta) d\theta ,
$$

for $j = 1, \ldots, \mu - 2$, and

$$
\dot{e}_{\mu-1}(t) = \mathcal{D} e(t) - \int_{-\tau}^{0} a_0(\theta)e(t+\theta) d\theta .
$$

Using these relations, together with (1), (9), (2), and (5), one obtains

$$
\dot{\eta}(t) + \int_{-\tau}^{0} \begin{bmatrix} F(\theta) & 0 \\
0 & \tilde{F}(\theta) \end{bmatrix} \eta(t+\theta) d\theta = \int_{-\tau}^{0} \begin{bmatrix} A(\theta) & 0 \\
\mathcal{B} C(\theta) & \mathcal{A}(\theta) \end{bmatrix} \eta(t+\theta) + \begin{bmatrix} \tilde{B}(\theta) \\
\mathcal{B} E(\theta) \end{bmatrix} \hat{u}(t+\theta) d\theta
$$

(11)

and

$$
e(t) = \begin{bmatrix} 0_{n_y} \times (n_x + (\mu-1)n_y) \\
I_{n_y} \end{bmatrix} \eta(t) ,
$$

(12)

where $\dot{x}(t) := \mathcal{D} x(t)$, $\hat{u}(t) := \mathcal{D} u(t)$, and $\tilde{e} := \begin{bmatrix} \dot{x} \\
\dot{e} \end{bmatrix}$.

Thus, under Assumptions 1–4, in order to achieve (8) under all initial conditions, the part of the system (11) which is observable through (12) must be stabilizable through $\hat{u}$. Note that the part of the system (11) corresponding to $\dot{e}$ is observable through (12) and the remaining part, by Condition 1, is already stabilizable through $\hat{u}$. Therefore, the system (11) must be stabilizable through $\hat{u}$. This, however, is equivalent to Condition 3.

Finally, Condition 4 is necessary since error feedback is necessary for robust tracking. The error, $e(t)$, can not be obtained for feedback unless this condition is satisfied.

**Sufficiency:** In this part we will present a constructive proof. Since, $y(t)$ is measurable (by Condition 4) and $r(t)$ is available on-line (by assumption), $e(t) = y(t) - \tau(r(t))$ can be obtained. Suppose that we build the following system, which is named as the servocompensator, where $s \in \mathbb{R}^{n_x}$ is its state vector.

$$
\dot{s}(t) + \int_{-\tau}^{0} \mathcal{F}(\theta) \dot{s}(t+\theta) d\theta = \int_{-\tau}^{0} \mathcal{A}(\theta)s(t+\theta) d\theta + \mathcal{B} e(t) .
$$

(13)

Let us now augment this system to the plant dynamics. Then, we obtain

$$
\dot{\eta}(t) + \int_{-\tau}^{0} \begin{bmatrix} F(\theta) & 0 \\
0 & \mathcal{F}(\theta) \end{bmatrix} \eta(t+\theta) = \int_{-\tau}^{0} \begin{bmatrix} A(\theta) & 0 \\
\mathcal{B} C(\theta) & \mathcal{A}(\theta) \end{bmatrix} \eta(t+\theta) + \begin{bmatrix} \mathcal{B}(\theta) \\
\mathcal{B} E(\theta) \end{bmatrix} u(t+\theta) + \begin{bmatrix} G(\theta) \\
\mathcal{B} H(\theta) \end{bmatrix} w(t+\theta) d\theta - \begin{bmatrix} 0 \\
\mathcal{B} \end{bmatrix} r(t)
$$

(14)

where $\eta := \begin{bmatrix} x \\
s \end{bmatrix}$ is the state of the augmented system.

From this augmented system, the available mesurements are

$$
\begin{bmatrix} y^m(t) \\
s(t) \end{bmatrix} = \int_{-\tau}^{0} \begin{bmatrix} C^m(\theta) & 0 \\
0 & \delta(\theta) I \end{bmatrix} \eta(t+\theta) + \begin{bmatrix} E^m(\theta) \\
H^m(\theta) \end{bmatrix} u(t+\theta) + \begin{bmatrix} 0 \\
0 \end{bmatrix} w(t+\theta) d\theta
$$

(15)
Using Condition 2, it can be shown that the system (14) with \( w(t) \equiv 0 \) and \( r(t) \equiv 0 \) is detectable through the output (15) with \( w(t) \equiv 0 \). Moreover, using Condition 3, it can also be shown that the system (14) with \( w(t) \equiv 0 \) and \( r(t) \equiv 0 \) is stabilizable through the input \( u \). Therefore, a feedback controller from \( y^m \) and \( s \) to \( u \), named as the stabilizing compensator, which globally asymptotically stabilizes this system exists. Now, what remains to be shown is that such a controller also achieves (8) under all initial conditions, for all references \( r(t) \) and for all disturbances \( w(t) \) satisfying (5), and for all non-destabilizing perturbations in the matrix functions appearing in (1)–(3). To show this, let us define \( \dot{\eta}(t) := D\eta(t) \), \( \dot{\eta}^m(t) := D\eta^m(t) \), and
\[
\dot{\eta} := \begin{bmatrix} \dot{x} \\ \dot{s} \end{bmatrix},
\]
where \( \dot{x}(t) := DX(t) \) and \( \dot{s}(t) := DS(t) \).

Then, using (5), (14)–(15), we obtain:
\[
\dot{\eta}(t) + \int_{-\tau}^{0} \begin{bmatrix} \tilde{F}(\theta) & 0 \\ 0 & \tilde{F}(\theta) \end{bmatrix} \dot{\eta}(t + \theta) d\theta = \int_{-\tau}^{0} \begin{bmatrix} A(\theta) & 0 \\ BC(\theta) & \tilde{A}(\theta) \end{bmatrix} \dot{\eta}(t + \theta)
+ \begin{bmatrix} B(\theta) \\ E(\theta) \end{bmatrix} \begin{bmatrix} \dot{x}(t + \theta) \\ \dot{s}(t + \theta) \end{bmatrix} d\theta
\]
(16)

and
\[
\begin{bmatrix} \dot{\eta}^m(t) \\ \dot{s}(t) \end{bmatrix} = \int_{-\tau}^{0} \begin{bmatrix} C^m(\theta) & 0 \\ 0 & \delta(\theta)I \end{bmatrix} \dot{\eta}(t + \theta)
+ \begin{bmatrix} E^m(\theta) \\ 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t + \theta) \\ \dot{s}(t + \theta) \end{bmatrix} d\theta
\]
(17)

Now, note that the closed-loop dynamics of (14)–(15) and of (16)–(17) are equivalent. Thus, any controller which asymptotically stabilizes the former also asymptotically stabilizes the latter. Furthermore, since the latter system has no external inputs, its asymptotic stability implies \( \lim_{t \to \infty} \dot{\eta}(t) = 0 \). Comparing (16) and (11), it is seen that \( \dot{\eta}(t) = \dot{\eta}(t) \). Thus, \( \lim_{t \to \infty} \dot{\eta}(t) = 0 \). Desired result now follows from (12).

\[\triangleright\]

Remark: The structure of the desired controller, when it exists, is provided by the sufficiency part of the above proof. As in the case of delay-free systems [27, 28] or the case of discrete time-delays [43], this controller consists of two parts: a servocompensator and a stabilizing compensator. The servocompensator is given by (13); i.e., it is defined by the dynamics of the reference and the disturbance. The stabilizing compensator, on the other must be designed to asymptotically stabilize the augmented system described by (14)–(15). Any stabilizing controller design method which is valid for neutral time-delay systems with distributed time-delay can be used for this purpose.

Remark: By following steps as in the above proof, one can show that the necessary and sufficient conditions for the solvability of the Weak Robust Servomechanism Problem are also given by Theorem 1, except that Condition 4 must be replaced by the weaker condition that there must exist a matrix \( T \in \mathbb{R}^{n_y \times n_n} \) such that \( T C^m(\theta) = C(\theta) \) and \( TH^m(\theta) = H(\theta) \), all \( \theta \in [-\tau, 0] \). In this case, the tracking error can be obtained as \( e(t) = Ty^m(t) - \int_{-\tau}^{0} (TE^m(\theta) - E(\theta)) u(t) d\theta - r(t) \).

Remark: Note that, by defining \( F(\theta) = \sum_{i=1}^{\nu} \delta(\theta + h_i) F_i \), \( A(\theta) = \sum_{i=0}^{\nu} \delta(\theta + h_i) A_i \), \( B(\theta) = \sum_{i=0}^{\nu} \delta(\theta + h_i) B_i \), etc., and taking \( \tau \geq h_\nu \), the plant (1)–(3) reduces to the plant considered in [43]. Similarly, defining \( a_{\mu}(\theta) = \sum_{i=1}^{\nu} \delta(\theta + h_i) a_{\mu}^i \) and \( a_{\mu}(\theta) = \sum_{i=0}^{\nu} \delta(\theta + h_i) a_{\mu}^i \), \( k = 0, \ldots, \mu - 1 \), the delay-differential operator (4) reduces to the delay-differential operator considered in [43]. Therefore, the present case includes the case of discrete time-delays, which has been considered in [43], as a special case. An important difference between the present case and the case of discrete time-delays, however, is that Condition 3 of Theorem 1 in the present case can not be written as a condition on the transmission zeros of the system, as it was done for the corresponding condition in [43].

Remark: As a final remark, note that if \( F(\theta) = 0 \), \( \forall \theta \in [-\tau, 0] \), in (1) and \( a_{\mu}(\theta) = 0 \), \( \forall \theta \in [-\tau, 0] \), in (4), then the present problem reduces to the robust servomechanism problem for LTI retarded time-delay systems with distributed time-delay, where the disturbance and the reference satisfy a constant-coefficient linear delay-differential equation of retarded type with distributed time-delay [3]. Therefore, the present problem includes the retarded case, which was considered in [3], as a special case. Thus, both the necessary and sufficient conditions for the solvability of the problem and the structure of the controller in the retarded case can be deduced from the present results.

4 Conclusion

The robust servomechanism problem for LTI neutral time-delay systems with distributed time-delay has been considered. The reference and the disturbance are assumed to satisfy a constant-coefficient linear neutral delay-differential equation with distributed time-delay. It has been shown that, by using Dirac delta functions, discrete time-delays, whether in the system dynamics or in the delay-differential equation satisfied by the reference and the disturbance, can also be included besides the distributed-delay. Necessary and sufficient conditions for the solvability of this problem have been presented. Furthermore, in the case a solution exists, the structure of the controller which solves this problem has also been given. Similar to delay-free systems [27, 28] and systems with discrete time-delays [43], this controller consists of a servocompensator and a stabilizing compensator. Here the servocompensator (given by (13)) is defined by the dynamics of the reference and the disturbance. The stabilizing compensator, however, must be designed to stabilize a LTI neutral distributed time-delay system (described by (14)–(15)). Any stabilizing controller design method which has been developed for such systems (e.g., see [2], [44], [45], or references therein) can be used for this purpose.
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References


