ABSTRACT

Forest and bush fires are a continued hazard to lives and livestock but remain difficult to model and accurately predict. The Drossel-Schwabl Forest-fire model is investigated with various localised combustion neighbourhoods. The transient behaviour as model systems approach equilibrium is studied using bulk system metrics as well as a study of individual like-like cellular components. Periodic dynamics of fire-re-growth behaviour is identified and a link is made between the parametric ratio of tree re-growth to fire strikes and the observed periodic time-constant of the model. A firebreak cell type that blocks fire spread is introduced into the model and the effect of a fractal watercourse introduced into the system is investigated. The presence of the water-break is shown to significantly reduce initial transients and lowers the overall fire damage to the model system.

KEY WORDS
fire-model; simulation; dynamics; fire-break; transient; cycles.

1 Introduction

Understanding the manner in which forest and bush wild fires [15] propagate [5] is an important problem with implications for the safety of both lives and property in many countries [20, 23–25, 33].

Scenarios where there are multiple interacting fires are particularly difficult to predict and model. A large scale system of combustible vegetative matter such as bush or trees will typically behave as a complex system [2] with some statistical and long term scaling properties that are known from models. The shorter term or transient behaviours are often equally of not more important to understand however, as they will affect how a particular fire or combination of fires interact and can be modelled [8] or combusted.

A useful starting point is the Drossel-Schwabl model of a forest fire [9, 10]. This model is described in detail in Section 2 but in summary allows study of quite large scale simulated systems. Modelling a combination of randomly located fires amongst slowly re-growing vegetation or trees is possible in this model. An earlier model by Bak et al [1] did not allow this feature although it displayed some similar scaling behaviours.

Figure 1 shows a typical configuration of the Drossel-Schwabl model on a 256² cell square grid 69 steps after initialisation with an empty system. Cells catch fire if one or more of he cells in their immediate neighbourhood are on fire. The Drossel-Schwabl model and its variant models [13] are normally studied as an example of critical systems [6, 19] with size scaling phenomena and phase transitional behaviour [7] in the parameters and densities.

There is still disagreement in the reported literature on the large system sized equilibrium scaling behaviour [21, 29, 30] of the Drossel-Schwabl model [17, 26, 34]. Even for simplified variants of the model [3] it is unclear whether there are power laws [22, 27] or universal behaviour [11] exhibited. The model may also have different theoretical limiting behaviours in different dimensions [31].

In this present paper however, we restrict ourselves to two-dimensional systems and study the transient effects of changing the neighbourhood size of the cells so that fires can spread more or less easily and can therefore propagate...
more or less rapidly across the whole model system. This is shown to give rise to interesting variations in the dynamics of the system as a whole, and we find there are cycles of oscillatory fire and re-growth in the model that are affected by a single parameter as well as by the chosen geometry and the neighbourhood size.

We also introduce a firebreak capability in the model and we experiment with the blocking effect of fractal patterns as might arise from realistic natural water-courses through an environment. We show that such water-courses or other patterns can trap fires within gully-like regions, and even when they do not completely block fires spreading across the whole system, they do reduce the effect of a single fire.

As we discuss further below, this model and work provides a foundation for more systemic investigation of realistic transient effects in forest or bush fire prone environments [4, 18].

Our article is structured as follows: In Section 2 we summarise the Drossel-Schwabl forest fire model and describe how we have simulated it and extended it for different combustion neighbourhoods and other conditions. In Section 3 we present some selected snap shots of typical model configurations as well as some plotted bulk properties of the model system. We give a discussion of the implications of the firebreak and varied neighbourhoods in Section 4 and offer some concluding remarks and scope for further studies in Section 5.

2 Simulation Model

The Drossel and Schwabl model is formulated in terms of a square lattice of individual cells which represent: trees or patches of vegetation which are already burning (B); or which are empty (E), having been burned out; or which have re-grown and contain combustible vegetation or trees (T).

Algorithm 1 Drossel and Schwabl Forest Fire model with added “water” or firebreak cells W

\begin{verbatim}
for all runs do
   for all steps do
      initialise sites randomly
      for all sites \( i \in E, T, B \) do
         update according to:
         \( B \rightarrow E \), unconditionally
         \( E \rightarrow T \), with re-growth probability \( p \)
         \( T \rightarrow B \) if \( R = 1 \) or more neighbours are burning
         \( T \rightarrow W \) if lightning strikes with probability \( f \)
         \( W \rightarrow W \), unchanged
      end for
      record measurements
   end for
   normalise averaged measurements
\end{verbatim}

Algorithm 1 shows the Drossel and Schwabl forest fire model algorithm as it applies to a cellular system of \( N = L^2 \) cells on a square lattice, each with a definite neighbourhood.

The site states are cyclic amongst Burning, Empty, and Tree with \( B \rightarrow E \rightarrow T \rightarrow B \) with conditional probabilities depending upon environment and the parameters. As Grassberger and others have noted [14], for large scale systems approaching equilibrium over long time scales, the model scales so that the two parameters re-growth probability \( p \) and lightning strike probability \( f \) effectively combine into a single scaled parameter \( \theta = p/f \). We have made use of this in our model and by choosing fixed values of \( p \) we have experimented with variations in \( f \) and hence \( \theta \) as discussed in Section 3. We have also experimented with differing neighbourhoods, so cells can catch fire from any of differing numbers of connected cells nearby.

We have introduced another effect into the model to investigate possible firebreak models. We incorporate an additional cell type \( W \), which is immovable and which is unaffected by the dynamics of the system. Water course-way or firebreak cells \( W \) therefore only affect the ability of fires to spread across the other model cells, by obstructing neighbourhood chains.

Figure 2 gives a simple illustration of \( W \) cells blocking fire spread in a simulated system. Normally the red fire-front could spread across the model space unimpeded, but the pattern of blocking \( W \) cells disrupts the fire, and can trap it in gullies or similar spatial patterns.

3 Time Series Results

A number of useful bulk properties can be measured from the simulated system. The fraction \( f_B \) of burning cell sites; the fraction \( f_T \) of unburned or growing tree cell sites; and the fraction \( f_E \) of empty cell sites can be defined so that these sum to unity. When waterway sites are introduced, it is convenient to redefine these fractions so that they only include the active or combustible cell sites. In the work reported here, we plot these for a model system that is (unless stated otherwise) of size 512\(^2\) cells in size, and run for 1,000 time steps, after being initialised with all empty cells. We use vegetative growth probability \( p = 5 \times 10^{-5} \) and a lightning strike probability \( f \) of \( \{5 \times 10^{-2}, 5 \times 10^{-3}, 5 \times 10^{-4}\} \) giving corresponding values of \( \theta = p/f \) of: \( \{10, 100, 1, 000\} \).

Figure 3 shows the initial transient and subsequent damped periodic behaviour of the model system. The system is initialised with empty cells, and tree/vegetative matter gradually grows to a point where a fire creating lightning strike hits a site that is no longer isolated, and consequently the fire can spread across the system. The initial transient is a steady growth in tree cover until a crash occurs as fires spread throughout the system wiping out the vegetative cover. The fraction of trees \( f_T(t) \) is therefore seen to be exactly out of phase with the fraction of empty cells \( f_E(t) \).

Figure 4 shows that the behaviour is simply that of a damped oscillatory motion and is on a time-scale ten times
Figure 2. Drossel-Schwabl Forest fire model, on 128x128 mesh, with burning distance of 3, with and without a fractal waterway.

Figure 3. Cyclic variations in the fraction of empty cells and cells with unburned (grown) vegetation. The oscillatory motion is complicated by the spatial patterns of fires interfering with or joining other fires. There are consequently several periodic frequencies present in the system over a longer period of time. Investigating many different starting conditions (different random number generator seed values) reveals that the same general behaviours result but that there is a random phase difference for each system. The observed exponential envelope shows a decay in the amplitudes of the oscillations, and when results are averaged over many systems the effect is to integrate out any phase dependence and the mean behaviour is indeed confirmed to be a slowly converging equilibrium value for a given parameter $\theta = p/f$.

Figure 4. Bulk measurements on the simulated fire model recorded over a longer time scale, showing the transient and oscillations as the system approaches a dynamic equilibrium. Note the relatively small number of instantaneously burning sites.

Figure 5. Bulk measurements on the simulated fire model recorded over a longer time scale, showing the transient and oscillations as the system approaches a dynamic equilibrium. Note the relatively small number of instantaneously burning sites.

Another useful bulk metric is the fraction $f_B$ of cells that are burning at a given time step. Figure 5 shows $f_B(t)$ for various neighbourhood conditions. The Drossel-Schwabl model is normally run with periodic boundary conditions as this implies a useful symmetry of neighbourhood connectivity for all cells. The resulting torus-mapping is useful for investigating theoretical scaling properties of the model, but for our work we are interested in how the boundary conditions and the applied connectivity.
neighbourhoods affect the behaviour of the system and the spread of fires.

In each of the four plots shown in Figure 5 we show the measured fraction of instantaneously burning tree cells as a function of time for the three values of parameter \( \theta = p/f \). The nearest neighbour system as used in most published work on the Drossel-Schwabl model is the baseline for comparison. We see that lowering \( \theta \) retards the initial transient and assists in dampening the oscillatory behaviour. There is typically less combustible material available compared to the incidence of fire strikes and the system reaches a dynamic equilibrium mean value sooner. This behaviour is retained even when we adjust boundary conditions and combustibility neighbourhoods.

As we see, applying fixed boundaries instead of periodic boundaries does not have a marked effect on the system. The model we have used is \( 512^2 \) cells in size and it is likely that this is large enough that bulk and not boundary behaviour dominates this measurement. However we do see interesting effects from increasing the combustibility neighbourhood. In the case of N1, each cell can only catch fire from one of 4 neighbouring cells on the square lattice. For a Moore neighbourhood [12] consisting of nearest and next-nearest this is increased to 8 cells. We also experimented with a radial proximity distance of 3, which gives a neighbourhood of 28 cells that could set the central cell alight. The effect of increasing the combustion neighbourhood is to rapidly increase the time-scale of the oscillations of the system. Fires can spread further and faster with increased local neighbourhood.

After studying the effect of different neighbourhoods, we can experiment with a specific pattern of firebreak - in the form of a simulated water course. The assumption is that a realistic watercourse might take a fractal form as shown. It is convenient to have a mechanism to generate multiple such patterns rather than study just a single pattern from a real watercourse. We have generated these patterns using a diffusion-limited aggregation (DLA) model [32] with the starting seed at the centre of the model system, and have allowed the fractal to grow until it touches the model edges. This results in a semi-realistic watercourse pattern against which we can study fire-breaking properties.

Figure 6 shows the effect on the fraction of burning cells, when a fractal watercourse is incorporated into the model. The fractal [28] fills a lot of the space but does not actually occupy a large number of cells. The \( W \) water-
firebreak cells are typically quite small fraction of the system as a whole, but they have an interesting effect on the model. The effect of the W cells in the simulated watercourse has removed the initial large rapid growth transient. The system now grows steadily with smaller spatial pockets of cells able to sustain fires. The number of burnt/re-growth cycles occurring within the simulated 1,000 steps is not significantly different however, although the amplitude of the oscillations has dropped. There are some distortions to the wave trains that likely accrue from the particular fractal aggregate pattern used, but which might be expected to integrate out, when an average is taken over many such particular patterns.

We can investigate visually what is occurring around the fractal watercourse pattern of W cells as different neighbourhoods are applied. Figure 7 shows the pattern from a nearest neighbourhood fire combustion connectivity and for a Moore neighbourhood connectivity. Close examination reveals that the fractal watercourse pattern, which was grown with nearest-neighbour connectivity, will completely block a fire that is simulated also on nearest-neighbour connectivity. However, when a Moore combustion neighbourhood is used, a propagating fire has is only partially blocked by the watercourse. This is analogous to fire sparks being spread over short distances. The behaviour is thus one of partially reduced fire spreading, but not complete blocking of fires that could spread across the whole system.

4 Discussion

Additional analysis of the fire model properties is possible using more sophisticated metrics. Component clusters can be automatically measured using a number of different algorithms. For the purposes of the work presented here, we have simply counted the number of individually isolatable components - which could be clusters of B, T or E cells.

Figure 8 shows the number of isolatable connected components as a function of time in a typical model run. As can be seen there are a number of manifest behaviours depending upon the value of parameter $\theta = p/f$, but we have also included cases for the same value of $\theta$ obtained by lowering or raising the vegetative re-growth parameter $p$ rather than clamping it, and adjusting lightening strike probability $f$ as was done with the remaining experiments.
The plotted curves emphasise earlier points. If tree/vegetative re-growth is too low, then the time-scale of the model is stretched out. It takes a lot longer for sufficient tree mass to accumulate to be dense enough to allow large scale fires to spread. Consequently the fire-re-growth envelope is stretched out, although with similar amplitudes to before. If the re-growth parameter is too high compared to the lightening probability, then the result is very rapid and high level of burning. The system has a lot more turbulence in it and many small fires spread and interfere with one another, so that the temporal fluctuations are small, but the average burn rate is higher.

Realistically the density of trees and vegetation is unlikely to be uniform. Gradients of density or fuel load are likely [16] and this could be added into the model by having individual cells burn for longer than a single time step. Similarly some cells could be more resistant to catching fire, and this could be modelled by using non-unit values for parameter $R$ in algorithm 1.

Trees of vegetation requires a more intense local fire for combustion effects to be triggered and thus the system becomes more resistant to fire spreading. There is scope to add these effects to the model from suitably scaled fire resistance data [5].

5 Conclusion

In summary, we have shown how the Drossel-Schwabl Forest Fire simulation model has a number of interesting transient behaviours that have not been previously well studied. We have shown that the fire combustion neighbourhood does significantly affect the model time-scale, We also introduced a simple firebreak model based upon a passive cell type $W$ that could represent some immovable non-combustible material such as water or bare rock.

We experimented with a simple watercourse pattern using a semi-realistic fractal pattern generated using a diffusion-limited aggregation (DLA) model. We found that some limitation to fire spreading arises when the connectivity pattern of the DLA cluster is lower than that of the fire combustion neighbourhood. In such cases the firebreak limits but does not totally contain the spread of a fire throughout the whole system.

This class of model has potential for studying more realistic patterns of vegetative cover and other combustible material for assessing the spread patterns and risk management probabilities for real forest or bush fires. The model has scope for additional parameters to be added, and an obvious one would be a probability range parameter for combustion.

Instead of a fixed neighbourhood of definite fire propagation, a probabilistic propagation based on declining radial likelihoods would be interesting to explore. Such a model could incorporate wind direction in the form or a preferred propagation direction. Other possibilities include changes to cell combustion duration based on fuel type or
height, if realistic terrain height data could be incorporated.

The Drossel-Schwabl model is usually studied from a perspective of equilibrium behaviour and critical properties, but its transient and dynamic properties have also been shown to be complex and interesting. There is scope for further detailed and systemic study of the oscillatory amplitudes and periods as parameterised by the growth and fire-strike parameter pairs.

References