BER ANALYSIS OF OFDM RECEIVER USING LS CHANNEL ESTIMATION METHOD WITH MODIFIED PTS FOR DIFFERENT SUB-BLOCKS

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ABSTRACT
Partial transmit sequence (PTS) is a distortion less method used in the Peak-to-Average Power Ratio (PAPR) reduction of Orthogonal Frequency Division Multiplexing (OFDM) signal. But PTS suffers from high computational complexity with increase in sub-blocks, because of exhaustive search of phase factors over all possible combinations. Modified PTS scheme has reduced this complexity by using neighbourhood search algorithm to find the optimum set of phase factors. In this paper, it is proposed to investigate the performance of OFDM receiver using Least Square (LS) channel estimation method. The Bit Error Rate (BER) analysis of the OFDM receiver using LS channel estimation method is made for PAPR reduced OFDM signal using PTS and Modified PTS. Simulation results show that BER increases with PAPR reduction.

KEY WORDS
Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), Partial Transmit Sequence (PTS), LS channel estimation, Bit error rate (BER)

1. Introduction
Orthogonal Frequency Division Multiplexing (OFDM) has become the technology of choice in broadband wireless communications because of its immunity against ISI and multipath fading. In OFDM system, the available spectrum is divided into a number of overlapping orthogonal narrowband channels. This method of dividing the spectrum converts the frequency selective channel into a non-frequency selective channel. ISI is overcome by using Cyclic prefix (CP) which is extension of the OFDM symbol with its tail portion. Because of these advantages, OFDM, a multi-carrier modulation technique is adopted in many high data rate communication systems, such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB) and in Wireless LAN standards [1], [2]. The main drawback of OFDM modulation is its high PAPR because the transmitted OFDM signal is the addition of many subcarrier components through IFFT operation. The high PAPR will degrade the efficiency of the high power amplifier at the transmitter by way of inter-modulation distortion. High PAPR of OFDM signal with non-linear power amplifier results in spectral emissions into adjacent channels with degradation in the BER. To prevent distortion, the high power amplifier should operate in the linear region. High PAPR dictates a linear power amplifier with large dynamic range. So, to combat the effects of high PAPR, the power amplifier should be linear in its operation with low efficiency and high power dissipation. Such linear power amplifiers cannot be incorporated in mobile units because of power constraints. Many methods of PAPR reduction in OFDM signals have been proposed. These include distortion techniques namely clipping [3], [4] and distortion less techniques such as coding [5], selective mapping [6] and partial transmit sequence (PTS) [7], [8]. In PTS, the OFDM data block after IFFT is divided into many sub-blocks and each sub-block is optimally shifted in phase, so that the probability of high PAPR in the combined signal is reduced. Here, PAPR reduction increases with increase in the number of sub-blocks. But, with increase in the number of sub-blocks, the computational complexity increases, due to many N-dimensional inverse FFT operations [9]. To reduce the computational complexity, Modified PTS technique [10] has been proposed. In Modified PTS, the optimal phase factor is selected by using neighbourhood search algorithm for an assumed threshold value of PAPR. Channel estimation is an integral part of wireless receiver designs. The transmitted signal travelling through the wireless channel are reflected and scattered and arrives at the receiver along multiple paths. The response of the wireless channel changes rapidly because of the mobility of transmitters, receivers and scattering objects. The arrival of signals along multiple propagation paths will cause it to spread in time, frequency and angle. Thus, the received signal is distorted by the detrimental effects of the wireless channel. The spreading of the signal limits the performance of the receiver. So, at the receiver, the effect of the channel on the transmitted signal is estimated to recover the transmitted information. In this paper, it is proposed to investigate the receiver performance using LS Channel estimation method for PAPR reduced OFDM signal using PTS and modified PTS.

2. PAPR Reduction with PTS
Consider an OFDM system with N subcarriers. Let the input data block $X_d = [X_d(0), X_d(1), ..., X_d(N-1)]$ represent the independently modulated data symbols $X_d(k)$. Then the transmitted signal in time domain is given by,
The PAPR of the OFDM signal is defined as the ratio of the maximum to the average power of the signal, as

$$PAPR(x) = 10 \log_{10} \left( \frac{\max |x(n)|}{E \left[ |x(n)|^2 \right]} \right)$$

where $E[.]$ is the expectation operator. To reduce PAPR using PTS approach, the input data block is partitioned into $V$ disjoint sub-blocks of equal size, represented by vectors $\{X_v, v = 1, 2, \ldots, V\}$. Each partitioned sub-block is multiplied by a corresponding phase factor $b_v = e^{j \phi_v}$ where $v = 1, 2, 3, \ldots, V$. After IFFT operation, the time domain signal is given by

$$x = IFFT \left\{ \sum_{v=1}^{V} b_v X_v \right\} = \sum_{v=1}^{V} b_v x_v$$

(3)

where $x_v$ is partial transmitted sequence. The phase factor vector is chosen from an allowed set $b_v = \{+1, -1, +j, -j\}$ such that the PAPR can be minimized [2]. Then the time domain signal with lowest PAPR is given by

$$x = \sum_{v=1}^{V} b_v x_v$$

(4)

where $b_v$ is optimum set of phase factor which will produce a low PAPR OFDM signal. The allowed set of phase factor $b_v = \{+1, -1, +j, -j\}$ should be searched to find the optimum set of phase vector. Because of this the computational complexity increases exponentially with increase in the number of disjoint sub-blocks. The PAPR performance is evaluated by the complementary cumulative distribution function (CCDF). The complementary cumulative distribution function (CCDF) is the probability that the PAPR of the OFDM signal exceeds a threshold value $PAPR_0$. 

$$CCDF(PAPR(x(n))) = \Pr(PAPR(x(n)) > PAPR_0)$$

(5)

Modified PTS is proposed to determine the optimum phase factor vector to reduce the computational complexity of PTS [10]. By using modified PTS algorithm, a better PAPR performance than PTS can be achieved but with reduced computational complexity.

3. Least Square (LS) Channel Estimation

In OFDM systems, Channel estimation techniques can be grouped into two categories: blind and non-blind. Blind channel estimation techniques use the statistical behaviour of the received signals and require large amount of data [12]. The non-blind channel estimation techniques use information about previous channel estimates or some portion of the transmitted signal is known to the receiver. Data aided channel estimation is a non-blind channel estimation technique wherein, a complete OFDM symbol or a portion of a symbol which is known by the receiver is transmitted so as to enable the receiver to estimate the wireless radio channel by the process of demodulation. The known information is sent over one or more OFDM symbols (Training symbols based channel estimation) or by inserting the known information into the data (Pilot aided channel estimation). Pilot aided channel estimation can have different structures based on the arrangement of pilot tones; block type, comb type and lattice type. In block type of pilot arrangement, pilot tones are inserted into all subcarriers of the OFDM symbols and are transmitted periodically to estimate the channel along the time axis. Thus, when the channel is varying across many OFDM symbols and in order to keep track of the channel variations, the pilots should be inserted at some ratio which is a function of coherence time. Now, the pilot spacing depends on the Doppler spread. The maximum spacing of pilot tones in time is given by

$$D = \frac{1}{t \cdot 2f_{dmax} T_f}$$

where $f_{dmax}$ is the maximum Doppler spread and $T_f$ is symbol duration of OFDM symbol. The block type of pilot arrangement is suitable for frequency selective channels. In comb type pilot arrangement, the pilot tones are used at periodical located subcarriers of every OFDM symbol. This type of pilot arrangement is used for estimation of the channel along the frequency axis. In pilot aided channel estimation, within an OFDM symbol the number of pilot tones is related to the channel delay spread. The spacing between pilots $D_p$ should be small enough to acquire all the variations of the channel in frequency and it is dependent on the coherence frequency of the radio channel. Thus

$$D_p \leq \frac{1}{\tau_{max} \Delta f}$$

where $\tau_{max}$ is the maximum delay of the channel and $\Delta f$ is the frequency separation between the subcarriers [13], [14]. Comb-type pilot arrangement is suitable for fast-fading channels. In lattice type pilot arrangement, the pilot tones are inserted along both the time and frequency axes with given periods. The pilot tones are scattered in both time and frequency axis to facilitate time/ frequency domain interpolations for channel estimation.

The OFDM signal with reduced PAPR when transmitted through the wireless channel undergoes distortion. At the receiver, the received signal is distorted by the characteristics of the channel. To recover the transmitted bits, the channel is estimated and
compensated at the receiver. The orthogonality of the subcarriers in OFDM allows that each subcarrier can be identified as an independent channel. So the received signal is the product of the transmitted signal and frequency response of the channel [11].

\[ Y(k) = X(k)H(k) + W(k) \]  \hspace{1cm} (6)

where \( Y(k) \) is the received signal, \( X(k) \) is the transmitted signal, \( H(k) \) is the frequency response of the channel and \( W(k) \) is the additive white Gaussian noise. The LS estimation of \( H(k) \) is given by

\[ H(k) = \frac{Y(k)}{X(k)} = \frac{H(k) + W(k)}{X(k)} \]  \hspace{1cm} (7)

To recover the transmitted signal the response of the channel is estimated at each subcarrier. Many channel estimation methods have been proposed by using pilot carrier or training symbols [9]. LS Channel estimation being the initial estimation technique for many channel estimation methods is considered here to analyse the BER performance of PAPR reduced OFDM signal, using comb-type pilot arrangement. In comb-type pilot arrangement, each OFDM symbol has pilot tones inserted periodically. The pilot tones are used for frequency domain interpolation to estimate the channel frequency response.

Let the locations of the pilot tones in the OFDM symbol of \( N \) subcarriers be represented by a set \( P = \{ 0,1,\ldots,N-1 \} \). Let the pilot tones in the OFDM symbol be represented by the set \( X_p = [X_p(0),X_p(1),\ldots,X_p(N-1)] \), where \( N \) is the number of pilot tones. Now, \( X_p(k) \) is the pilot tone when \( k \) is an element of \( P \) . Consider another set which is the complement of the set \( P \) , denoted by \( P' \) to represent the locations of the data symbols. The data symbols are represented in the set \( X_d = [X_d(0),X_d(1),\ldots,X_d(N-1)] \), where \( X_d(k) \) represents the corresponding data symbol for \( k \) an element in \( P' \). Thus, the OFDM symbol can be represented as,

\[ X(k) = X_d(k) + X_p(k) \]

\[ = \begin{cases} X_d(k), & k \in P' \\ X_p(k), & k \in P \end{cases} \]  \hspace{1cm} (8)

At the receiver, after FFT, the received signal is given by \( Y(k) = X(k)H(k) + W(k) \). Let \( H(k) \) be the channel estimate of \( H(k) \). Then,

\[ H(k) = H_d(k) + H_p(k) \]  \hspace{1cm} (9)

where \( H_d(k) \) is the channel frequency response at the location of the data and \( H_p(k) \) is the estimated channel frequency response at the location of pilot tones. Since the locations of the pilot are known at the receiver, the estimate of the channel frequency response at the location of pilot can be determined by using

\[ H_p(k) = \frac{Y(k)}{X(k)} \]  \hspace{1cm} (10)

The LS estimates at the pilot locations are then interpolated to estimate the channel \( H_d(k) \) at the locations of the data symbols. Then the estimated data symbol \( X_d(k) \) is given by

\[ X_d(k) = \frac{Y(k)}{H_d(k)} \]  \hspace{1cm} (11)

The performance measure employed in frequency domain channel estimates is MSE and it is defined by

\[ MSE = E \left[ \left( H(k) - H_d(k) \right)^2 \right] \]  \hspace{1cm} (12)

BER performance measure is used to evaluate the performance of the receiver using LS channel estimation method.

4. Results and Discussion

4.1 LS Channel Estimation with PTS

The BER analysis using LS channel estimation at the receiver for transmitted signal with PAPR reduction using PTS has been done using MATLAB simulation software. The simulation parameters considered for analysis are given in Table 1.

<table>
<thead>
<tr>
<th>Simulation parameter for PTS</th>
<th>Type value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarriers</td>
<td>256</td>
</tr>
<tr>
<td>Number of sub-blocks</td>
<td>2, 4, 8, 16</td>
</tr>
<tr>
<td>Guard band</td>
<td>8</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>16 QAM</td>
</tr>
<tr>
<td>Phase factors</td>
<td>+1, -1</td>
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</tbody>
</table>

In the system under consideration, PTS technique is applied to the OFDM signal and PAPR performance is evaluated using the complementary cumulative distribution function. Figure 1 shows the PAPR performance of OFDM signal without PTS and PTS for different sub-blocks of \( V = 2, 4, 8 \) and 16. The PAPR performance improves with increase in the number of sub-blocks. The PAPR of the OFDM signal is reduced with increase in the sub-blocks, since the phase of the OFDM symbols in each sub-block is rotated independently when multiplied by the corresponding phase factor, from the optimum phase factor vector.
Figure 1. PAPR performance for various sub-blocks $V = 2, 4, 8$ and $16$ with $N = 256$

Figure 2 shows the mean square error of the channel versus SNR for the LS channel estimator. LS channel estimator does not take into account the channel statistics. The MSE of OFDM without PAPR reduction is about 0.0043 at SNR = 20dB. Due to PAPR reduction with PTS, the MSE increases with increase in the number of sub-blocks. The mean square error at SNR = 20dB is 0.0086, 0.0165, 0.0279 and 0.0449 respectively for sub-blocks $V = 2, 4, 8$ and $16$.

Figure 2. Mean square error for LS estimator for different sub-blocks $V = 2, 4, 8$ and $16$ with $N = 256$

4.2 LS Channel Estimation with Modified PTS

The simulation parameters considered for BER analysis using LS channel estimation at the receiver for transmitted signal with PAPR reduction using modified PTS are given in Table 2.

Table 2 Simulation Parameters for modified PTS

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Type value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarriers</td>
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</tr>
<tr>
<td>Phase factors</td>
<td>$+1, -1, +j, -j$</td>
</tr>
</tbody>
</table>

The PAPR performance of OFDM signal using Modified PTS for different sub-blocks of $V = 2, 4, 8$ and $16$ is shown in Figure 4. The PAPR performance by using Modified PTS is better when compared to PTS scheme at reduced complexity with increase in the number of sub-blocks [10]. Figure 5 shows the MSE of the channel versus SNR for the LS channel estimator for modified PTS. Because of the greater reduction in the PAPR using modified PTS, the MSE increases with increase in the number of sub-blocks. The MSE at SNR = 20dB, is 0.0087, 0.0172, 0.0346 and 0.0687 respectively for sub-blocks $V = 2, 4, 8$ and $16$.

Figure 3 depicts the BER performance at the receiver using LS channel estimator. BER performance of the original OFDM signal without PAPR reduction is compared with PAPR reduced OFDM signal for different sub-blocks using the PTS scheme. Since PAPR reduces with increase in the number of sub-blocks, it is found that the BER rate increases with increase in the number of sub-blocks. The BER of OFDM signal without PAPR reduction is about 0.0068 at SNR = 20dB. The BER increases to 0.008, 0.010, 0.013 and 0.017 respectively for sub-blocks $V = 2, 4, 8$ and $16$ with increase in the reduction of PAPR.

Figure 3. BER for LS estimator for different sub-blocks $V = 2, 4, 8$ and $16$ with $N = 256$

Figure 4. PAPR performance using Modified PTS for different sub-blocks and $N = 256$
The bit error performance of the receiver using LS channel estimator of PAPR reduced transmitted signal using modified PTS is shown in Figure 6. It is observed that the BER at SNR = 20dB is 0.0084, 0.0118, 0.0184 and 0.0316 respectively for sub-blocks V = 2, 4, 8 and 16. The BER performance of OFDM signal without modified PTS is also shown in the figure. With increase in the number of sub-blocks for better PAPR reduction, the BER increases.

5. Conclusion

This paper investigates the BER performance of PAPR reduced OFDM signal using PTS and Modified PTS. LS channel estimator is used for the channel estimation at the receiver. It is observed there is a slight degradation in the BER performance of PAPR reduced OFDM signal at the receiver using Modified PTS as the PAPR reduction technique. BER performance can be improved by using MMSE channel estimator at the receiver.

References