MODIFIED PTS BASED ON SUPERIMPOSED TRAINING SEQUENCE METHOD FOR PAPR REDUCTION OF OFDM SIGNALS WITH DIFFERENT SUBBLOCKS

Mukunthan Pandurangan¹ and Dananjayan Perumal²
¹Adhiparasakthi Engineering College, Melmaruvathur, Tamil Nadu, India
²Pondicherry Engineering College, Pondicherry, India
¹mukunthanec@pec.edu, and ²pdananjayan@pec.edu

ABSTRACT
Orthogonal Frequency Division Multiplexing (OFDM) is regarded as a promising technique for achieving high capacity in frequency selective fading channels. It is well known that OFDM signals have high peak-to-average power ratio (PAPR) and are thus not power efficient. High PAPR is caused by the large envelope fluctuations of the OFDM signal. High PAPR values lead to serious problems such as severe power penalty at the transmitter, which is not affordable in portable wireless systems where terminals are battery powered. Hence, Partial transmit sequence (PTS) scheme is an attractive solution to reduce PAPR without any distortion of transmitted signals. In the PTS scheme, one OFDM symbol is partitioned into disjoint subblocks, and each subblock is multiplied by a phase factor to generate OFDM signals with low PAPR. In the ordinary PTS scheme, the computational complexity increases extensively with the number of subblocks. In this paper, modified PTS based on superimposed training sequence method reduces the PAPR of OFDM signals, by different subblocks and it has low computational complexity has been presented. Simulation results demonstrate that all the training sequences in the proposed scheme have superior performance with different subblocks in the OFDM system.

KEY WORDS
OFDM, PAPR, PTS, Superimposed training sequence.

1. Introduction
The Orthogonal Frequency Division Multiplexing (OFDM) system has a serious drawback which is denoted as high Peak-to-Average Power ratio (PAPR) [1]. The high PAPR limits the average transmit power due to non-linearity of the transmit power amplifier. This high PAPR generates spectral spreading, in-band distortion and out of band noise which degrades the bit-error rate (BER). As a multicarrier modulation technique, OFDM has gained popularity in a many applications including digital audio broadcasting (DAB), terrestrial digital video broadcasting (DVB-T), as well as in asymmetric digital subscriber line (ADSL), the IEEE 802.11 a/g standard for wireless local area network (WLAN) and Broadband Wireless Access (BWA) applications [2] owing to its high bandwidth efficiency and robustness to multipath fading.

However, two important issues arise in OFDM systems such as reduced bandwidth efficiency due to the dedicated slots for training and generally low power efficiency due to the high PAPR of the OFDM signal [3, 4]. In addition, the high PAPR values of the OFDM signals make them sensitive to nonlinear effects in the high power amplifiers (HPAs). To minimize nonlinear distortions, either very linear HPAs have to be used, or the input signal has to be backed-off significantly. Both options are power inefficient.

The composite time signal in an OFDM system is formed from the linear addition of the independently modulated subcarriers. As a result, when the number of subcarriers is large, spurious high amplitude peaks appear in the OFDM signal which induces a large PAPR. Usually, the OFDM signal pass through a non-linear power amplifier before it is transmitted over the wireless channel. When the input signal exceeds a certain value, the output of the amplifier becomes saturated, causing distortion in the signal, and hence a deterioration in the BER performance.

Number of approaches have been proposed to deal with the PAPR problem. For instance, coding in the form of special forward error correcting codes (FEC) [5-7] excludes OFDM symbols with a large PAPR while probabilistic techniques like selective mapping (SLM) [8], partial transmit sequences (PTS) [9], and interleaving [10] reduce the probability of the occurrence of peak values. On the other hand, the amplitude clipping [11] technique involves signal distortion techniques which minimize the peak amplitudes by non-linearly distorting the OFDM signal at or around the peaks. These techniques achieve PAPR reduction at the expense of transmit signal power increase, BER increase, data rate loss, computational complexity increase, and so on.

Among the various PAPR reduction techniques, PTS is a distortionless phase optimization scheme that provides excellent PAPR reduction with a small amount of redundancy. It includes high computational complexity and reduced bandwidth efficiency due to the transmission of side information. Hence, Modified PTS scheme [12, 13] is proposed to lower the computational complexity while maintaining the similar PAPR reduction.
performance compared with the ordinary PTS scheme. To alleviate the problem of high complexity further an approach [14] has been proposed, in which real and imaginary parts are separately multiplied with phase factors, moreover PAPR is jointly optimized in real and imaginary parts.

One major advantage of the superimposed training method is that it incurs no loss of information rate. Since part of the transmitted power is diverted to the training signal, it is generally expected that for a fixed total average transmit power, superimposed training will perform much worse than traditional training in terms of BER, as a price paid for the higher information rate [15]. The training sequence has better correlation properties and lower PAPR due to its polyphase nature.

In this paper, performance of PAPR reduction has been analysed by modified partial transmit sequence (MPTS) based on training sequences composed of repeated pseudo-noise (PN) sequences. At transmit side, training sequence is placed at the front of OFDM symbol in time domain and transmitted with OFDM symbol. In the proposed scheme, the phase factors and subblocks are jointly estimated from the pilot subcarriers. The primary objective of this paper is to provide efficient PAPR reduction performance of OFDM signals. The amount of PAPR reduction depends on the number of subblocks and the number of allowed phase factors in OFDM system.

The rest of the paper is organized as follows. In the Section 2, the definition of PAPR and CCDF is given. Section 3 gives a description of PTS technique. In addition, modified PTS based on superimposed training sequence method to reduce PAPR is discussed in detail in Section 4. Section 5 presents the simulation results and the corresponding analysis, while conclusions are drawn in Section 6.

2. PAPR in OFDM System

In an OFDM system, the energy of the transmitted subcarrier-related information symbols fluctuates as a function of both the modulated sequence and that of the choice of the potentially on-constant-modulus modulation scheme itself. A block of subsequent OFDM symbols, \(X_k = (X_{k_0}, X_{k_1}, \ldots, X_{k_{N-1}})\), is formed with each symbol modulating the corresponding subcarrier from a set of subcarriers. \(N\) subcarriers chosen to transmit the OFDM signal are orthogonal to each other with pulse shape waveform of duration \(T\), and \(f_s = 1/T\) is the frequency spacing between adjacent subcarriers.

In OFDM system, the time-domain OFDM signal for \(N\) subcarriers can be written as

\[
x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi k t} , \quad 0 \leq t \leq T
\]  

(1)

where \(X_k\) denotes the modulated symbol in the \(k\)th subblock.

Replacing \(t = nT_b\), where \(T_b = T/N\), gives the time discrete signal with sampling factor \(L\) can be expressed as:

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi k n T} , \quad n=0,1,\ldots,LN-1
\]

(2)

The PAPR of the transmit OFDM signal, \(x(t)\), is then given as the ratio of the maximum instantaneous power and the average power, written as

\[
PAPR = \max_{0 \leq t \leq T} |x(t)|^2 / E[|x(t)|^2] 
\]

(3)

where \(E[ \cdot ]\) is the expected value operator. Usually, the continuous time PAPR of \(x(t)\) is approximated using the discrete time PAPR, which is obtained from the samples of the OFDM signals.

The complementary cumulative distribution function (CCDF) of the PAPR of the OFDM signal with superimposed training sequence method is used to measure performance for PAPR reduction techniques. It denotes the probability that the PAPR of OFDM symbol exceeds a certain threshold \(PAPR_0\) and can be expressed as [16]

\[
CCDF(PAPR(x(n))) = P_r(PAPR(x(n)) > PAPR_0)
\]

(4)

Due to the independence of the \(N\) samples, the CCDF of the PAPR of a data block with Nyquist rate sampling is given by

\[
P = P_r(PAPR(x(n)) > PAPR_0) = 1 - (1 - \frac{PAPR_0}{PAPR_N})^N
\]

(5)

This expression assumes that the \(N\) time domain signal samples are mutually independent and uncorrelated. This is not true, however, when oversampling is applied. Also, this expression is not accurate for a small number of subcarriers since a Gaussian assumption does not hold in this case. Therefore, the CCDF of PAPR computed of the \(L\)-time oversampled OFDM signal can be rewritten as

\[
P = P_r(PAPR(x(n)) > PAPR_0) = 1 - \left(1 - \frac{PAPR_0}{PAPR_N}\right)^{LN}
\]

(6)

The PAPR of the continuous-time OFDM signal cannot be precisely computed in the Nyquist sampling rate, which corresponds to \(N\) samples per OFDM symbol. When this is not the case, the achieved PAPR reduction might be lower and suboptimal [17]. To achieve effective PAPR reduction for this case, signal peaks may be skipped and PAPR estimates are not precise. Usually, the oversampling factor \(L\) larger than 4 is used for a PAPR reduction schemes to increase the resolution of discrete-time OFDM signals. However, such an oversampling process would significantly increase the computational complexity.

3. Partial Transmit Sequence OFDM

In partial transmit sequence [18] technique, the frequency domain random input signal is divided into a set of
disjoint subblocks and the OFDM subcarriers in each subblock are phase shifted separately. For further process the IFFT is computed. The objective is to find the weighting phase factors \{+1, -1, +j, -j\} with aim of minimizing PAPR. In general, the selection of the phase factors is limited to a set with finite number of elements to reduce the search complexity. The number of subblocks and the adjacent subblock partitioning scheme determine the PAPR reduction. However, the ordinary PTS scheme requires an exhaustive searching over all combinations of allowed phase factors. Consequently, the computational complexity increases exponentially with the number of the subblocks. Then for an OFDM system that has significantly large number of subcarriers, the required computational complexity can become prohibitively high.

Subblock partition for PTS OFDM is a method of division of the subcarriers into multiple disjoint subblocks. In general, it can be classified into 3 categories; interleaved partition, adjacent partition, and pseudo-random partition [19]. For the interleaved method, every subblock signal spaced apart is allocated at the same subblock [20]. In the adjacent scheme, successive subblocks are assigned into the same subblock sequentially. And each subblock signal is assigned into any one of the subblocks randomly in the pseudo-random scheme. It can be noted that the computational complexity of the interleaved subblock partitioning scheme is reduced extensively as compared to that of the adjacent and pseudo-random partition scheme. This arrangement reduces considerably the envelope fluctuations in the transmitted waveform.

In general, the partial transmit sequence \( S_m \) is partitioned by using an adjacent subblock partitioning method. Assume that the subblocks or clusters consist of a contiguous set of subcarriers and are of equal size [21]. Applying phase factors to subblocks allows optimization of combining partial transmit sequences is given by

\[
S' = \sum_{m=1}^{V} b_m S_m
\]  

where, \( \{ b_m, m=1, 2, ..., V \} \) are weighting factors and are assumed to be perfect rotations. In other words, the time domain is given by

\[
s = \sum_{m=1}^{V} b_m s_m
\]  

where, \( s_m \) consist of a set of subblocks with equal size and \( b_m \) is the phase factors, which are required to inform the receiver as the side information. In order to increase its performance, the size of side information is drastically decreased. The set of weighting factor for \( V \) clusters or subblocks are optimised in the time domain so as to achieve the better PAPR performance. PTS generates a signal with a low PAPR through the addition of an appropriate phase rotated signal parts. The code word to be transmitted are divided into several subblocks, \( V \), of length \( N/V \). Mathematically, expressed by

\[
A_k = \sum_{v=1}^{V} A_k^{(v)}, \quad v=1, 2, ..., V
\]  

All subcarriers positions in \( A_k^{(v)} \) which are occupied in another subblock are set to zero. Each of the blocks, \( v \), has an IFFT performed on it,

\[
a_k^{(v)} = IFFT\{ A_k^{(v)} \}
\]  

The output of each block except for first block which is kept constant, is phase rotated by the rotation factor as given by

\[
e^{j\theta(v)} \in [0, 2\pi]
\]  

The blocks are then added together to produce alternate transmit signals containing the same information as given by

Figure 1. A typical structure of transmitter for modified PTS based on superimposed training sequence method for PAPR reduction in OFDM system

Figure 1 depicts a simplified OFDM transmitter. At the transmitter, the serial data sequence in the frequency domain undergoes serial-to-parallel (S/P) conversion to be stacked into one OFDM symbol. The number of subcarriers is \( N \). After S/P conversion, \( N \)-points inverse fast Fourier transform (IFFT) follows to produce the \( N \)-dimensional signal in the time domain. Next, this \( N \)-dimensional signal is parallel-to-serial (P/S) converted, and a training sequence of length greater than or equal to the channel length is appended to introduce redundancy to mitigate interference between adjacent OFDM symbols and to preserve the orthogonality between the subcarriers. The resultant symbol is then applied to a power amplifier and subsequently sent over the wireless channel.
\[ \hat{a}_s = \sum_{n=1}^{V} a_n^{(v)} e^{j\phi_n(v)} \] (12)

Each alternate transmit signal is stored in memory and the process is repeated again with a different phase rotation value. After a set number of phase rotation values, \( W \), the OFDM symbol with the lowest PAPR is transmitted as given by

\[ \bar{\phi}_1, \bar{\phi}_2, \ldots, \bar{\phi}_W = \arg \min(\max |\hat{a}_n|) \] (13)

The weighting phase rotation factors are selected to minimize the PAPR. The computational complexity of PTS method depends on the different number of subblocks and phase rotation factors allowed. The phase rotation factors can be selected from an infinite number of phases \( \phi^{(v)} \in (0, 2\pi) \). But finding the best weighting factors is indeed a complex problem. To increase the potential capability of PAPR reduction performance for the PTS method, these phase factors combination correctly maintains the orthogonality between the different modulated carriers. However, the PTS PAPR reduction scheme’s performance improvement is achieved at the expense of high complexity and difficult parameter setting problems.

4. Superimposed Training Scheme for OFDM

In the superimposed training method for PAPR reduction, periodic pilots are superimposed on to the data at a low power, thus avoiding the need for additional time slots for training. For OFDM signal spreading, pseudo-random noise (PN) sequence with good cross correlation and autocorrelation properties are used. A PN sequence is made up of a number of chips for mixing the data with the sequence. Periodic pilots are added to data symbols in time domain before transmission, and first order statistics [22] are exploited to identify the larger peaks. As adding pilots can increase the PAPR, superimposed pilots must be carefully chosen to mitigate this problem.

In the OFDM transmission with superimposed training scheme is promoted for its high bandwidth efficiency, low computational complexity and possibly improved power amplifier efficiency at the transmitter. In superimposed training schemes, the interference is a composite of the unknown data and the additive white Gaussian noise. In accordance with the central limit theorem, if \( V \) is large, S can be regarded as a Gaussian-distributed random vector with zero mean. The deterministic mean of the data is subtracted from the data before transmission and iterative detectors are employed for symbol recovery. This scheme avoids transmitting data on those subcarriers which are influenced by the PN sequence. The number of such subcarriers in one OFDM symbol is equal to the period of the PN sequence. The IFFT based transformation from the subcarrier-related frequency domain to the time domain is invoked in order to exploit the frequency-domain correlation of the subcarrier-related coefficients as well as to reduce the complexity based on reducing the number of candidate signals. Therefore, the computational complexity is reduced clearly at the cost of performance loss for PAPR reduction. Unlike these methods, the proposed scheme reduces complexity by using the correlation among the adjacent candidates. Since the number of candidate signals is not reduced, it can achieve the same PAPR reduction as the modified PTS scheme.

The basic idea is that a periodic pilot sequence is added to the information sequence in every OFDM symbol. In this scheme, periodic pilots are superimposed on to the input data at a low power, thus avoiding the need for additional time slots for training. In superimposed training method, the sum powers of pilot and OFDM symbols are constrained by a fixed transmit-power budget [23- 25]. In the proposed scheme, if the number of pilots in a subblock is large by enlarging the modified PTS subblock size, the phase factor selection performance will be improved. The modified PTS based on superimposed training method reduces the PAPR more effectively. It has the advantages such as inhibiting PAPR of OFDM signals and reducing the efficiency of linear power amplifier, as well as further improving band-width utilization of the system.

5. Results and Discussion

The analysis of the modified PTS with superimposed training sequence method has been carried out using MATLAB software. The simulation parameters considered for this analysis is summarized in Table 1.

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Type/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM symbols</td>
<td>10000</td>
</tr>
<tr>
<td>Number of subcarriers (N)</td>
<td>64, 128, 256, 512, 1024</td>
</tr>
<tr>
<td>Number of subblocks (V)</td>
<td>2, 4, 8, 16</td>
</tr>
<tr>
<td>Oversampling factor(L)</td>
<td>4</td>
</tr>
<tr>
<td>Subblock partition scheme</td>
<td>Adjacent</td>
</tr>
<tr>
<td>Modulation Scheme</td>
<td>QPSK</td>
</tr>
<tr>
<td>Phase weighting factor</td>
<td>1, -1, j, -j</td>
</tr>
<tr>
<td>PN sequence</td>
<td>Superimposed training sequence</td>
</tr>
<tr>
<td>Cyclic prefix</td>
<td>16 samples</td>
</tr>
</tbody>
</table>

In the OFDM system under consideration, modified PTS technique is applied to the subblocks of uncoded information. This uncoded information of the cyclic mean
and data dependent pilot signal is modulated by QPSK modulation, and the phase rotation factors are transmitted directly to receiver through subblock. The PAPR reduction performances are empirically evaluated through computer simulations.

Figure 2 shows the variation in the PAPR value of the proposed scheme for different number of subblocks V= 2, 4, 8, and 16 for a random data of block size 10000 with N=64 subcarriers. From this figure it is observed that the values of PAPR for V= 2, 4, 8, and 16 become 6.8, 5.1, 3.8, and 3.5 dB respectively at complementary cumulative distribution function (CCDF) $10^{-2}$. The PAPR value decreases significantly as number of subblocks used in the OFDM transmission increase.

Figure 3 presents the PAPR reduction performance of the modified PTS based on superimposed training method for different number of subblocks V=2, 4, 8, and 16 for a random data of block size 10000 with N=128 subcarriers when oversampling factor L=4. It can be seen from the figure that as the subblock size is increased from 2 to 4, 8, and 16, the PAPR reduced to 6.9 dB, 5.6 dB, 4.3 dB and 3.7 dB respectively when CCDF $= 10^{-2}$, resulting in performance improvement as the number of subblocks increases.

Figure 4 illustrates the CCDF of the PAPR of the OFDM signal for the modified PTS based on superimposed training method with N=256 subcarriers and different number of subblocks V=2, 4, 8, 16. It is observed that modified PTS combined with superimposed training sequence method provide considerable gain in PAPR values by 7 dB, 6.1 dB, 4.6 dB and 4.1 dB for subblocks V=2, 4, 8, and 16 respectively at CCDF of $10^{-2}$.

Figure 5 depicts the CCDF of the PAPR of the OFDM signal for the modified PTS based on superimposed method with N=512 subcarriers and various values of V=2, 4, 8, 16 subblocks and oversampling factor (L) of 4. It can be seen from the figure that as the subblock size is increased from 2 to 4, 8, and 16, the values of PAPR is
reduced to 7.2 dB, 6.4 dB, 5.2 dB, 4.4 dB respectively at CCDF value is $10^{-2}$.

As shown in Figure 6 gives the performance of PAPR reduction for modified PTS based on superimposed training method when N=1024 subcarriers with oversampling factor L=4. From this Figure 6 it is observed that the values of PAPR for different subblocks V=2, 4, 8, and 16 become 7.3 dB, 6.8 dB, 5.6 dB, and 4.9 dB respectively at CCDF of $10^{-2}$. The PAPR values decreases significantly as number of subblocks used in OFDM transmission increase.

Table 2 gives the comparison of PAPR reduction performance with different subblocks in OFDM system. The results indicate that the modified PTS combined with superimposed training sequence method uses both the PN sequences and data subcarriers to reduce the PAPR. It is clear that when the number of subblocks are increased, the amount of reduction in PAPR is also increased.

### Table 2

<table>
<thead>
<tr>
<th>Subblocks(V)</th>
<th>Subcarriers(N)</th>
<th>Values of PAPR in dB at CCDF of $10^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>64</td>
<td>2 : 6.8, 4 : 5.1, 8 : 3.8, 16 : 3.5</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>2 : 6.9, 4 : 5.6, 8 : 4.3, 16 : 3.7</td>
</tr>
<tr>
<td></td>
<td>256</td>
<td>2 : 7.0, 4 : 6.1, 8 : 4.6, 16 : 4.1</td>
</tr>
<tr>
<td></td>
<td>512</td>
<td>2 : 7.2, 4 : 6.4, 8 : 5.2, 16 : 4.4</td>
</tr>
<tr>
<td></td>
<td>1024</td>
<td>2 : 7.3, 4 : 6.8, 8 : 5.6, 16 : 4.9</td>
</tr>
</tbody>
</table>

6. Conclusion

This paper presented a modified PTS based on superimposed training sequence method for reducing the PAPR of OFDM systems. By using modified PTS combined with superimposed training sequence, the computational complexity can be reduced dramatically and achieve the same PAPR reduction performance compared to the ordinary PTS scheme. This technique uses independent number of subcarriers and achievable PAPR reduction increases as the number of subblocks increases. The method avoids the use of any extra IFFTs and efficient use of the transmitter power amplifier. Simulation results show that the modified PTS based on superimposed training sequence method is an effective method to compromise a better trade-off between PAPR reduction and computation complexity. So this system will be suitable for the future wireless cellular mobile communication system which supports higher data rates.

### References


