ABSTRACT
Multiple input multiple output (MIMO) systems have influenced nearly any mobile communication standard. However, in many cases the theoretically possible potential can’t be reached since of the proximity of transmitter and receiver side antenna arrays. The phenomena called correlation affects the channel capacity and the link BER (bit-error rate). Therefore in this paper receiver-side antennas correlation and its modelling are investigated. Together with the appropriate signal processing (e.g. singular values decomposition), the effect of receiver-side antennas correlation is studied. Our results show that under the effect of correlation not necessarily all layers might be used for the data transmission since the weighting of the stronger layer within the MIMO system becomes even more stronger. Simulation results are shown to underline these effects and the benefits of using appropriate power distribution strategies.

KEY WORDS
Multiple-Input Multiple-Output System, Antennas Correlation, Wireless Communication, Singular-Value Decomposition, Bit Allocation.

1 Introduction
During the last decades multiple input multiple output (MIMO) communication systems have attracted a lot of attention to the research community due to the possibility to increase the channel capacity and decrease the bit-error-rate (BER) by maintaining the available power at the transmitter-side. In order to obtain such advantages some signal processing techniques must be applied at the transmitted (pre-processing) and received (post-processing) signals based on the channel knowledge. Hence, in order to obtain the full promised advantages of the MIMO systems perfect channel state information is required at both the transmitter and receiver sides.

A MIMO system is characterized by the channel matrix which arranges the individual descriptions of the paths from each transmit antenna to every receive antenna. In wireless communications it is found that those paths are appropriately characterized as Rayleigh ones and the channel matrix is composed by an arrangement of paths descriptions (matrix elements) which are independent and identically distributed (i.i.d.) Rayleigh fading channels. A popular technique to remove the inter-antenna interferences produced by the use of multiple transmit and receive antennas is the singular values decomposition (SVD) which provides the pre- and post-processing matrices required to benefit the MIMO properties. By doing so the MIMO channel can be described as multiple independent Single-Input Single-Output (SISO) channels characterized by the gains given by the singular values. The ideal situation is that where the singular values are equal and for that a rich scattered environment is required, i.e., the off-diagonal elements of the MIMO channel matrix must largely differ. Due to the proximity of the multiple antennas available at the transmitter and receiver sides the antennas correlation effect appears. In consequence the transmit-to-receive antenna paths becomes similar affecting the channel behaviour by decreasing the channel capacity and increasing the BER with respect to their bounds. The antennas correlation due to its proximity has been studied since time ago and analysed for space diversity applications [1, 2, 3].

An indicator for the unequal weighting of the SISO channels is the ratio between the smallest and the largest singular value. A ratio close to 1 indicates that the MIMO-channel is well-suited for multi-layer transmission since all the layers offer nearly the same quality when transmitting bits over the different layers. However, the advantage of multi-layer transmission diminished as the ratio decreased further. Fig. 1 highlights the effect of antenna correlation within an exemplarily studied (4 × 4) MIMO system. The unequal weighting of the SISO channels requires intelligent solutions for bit- and power allocation. The novel contribution of this paper is that optimal and suboptimal power allocation solutions are investigated under the assumption of strongly unequal SISO channels. This paper analyses and characterizes the antennas correlation effects focussing on the receiver-side antennas correlation. The results allow a proper description of the MIMO link in the presence of receiver-side antennas correlation and enable a comparison with the non-correlated one. The main goal of this paper is analysing the MIMO systems performance affected by the presence of receiver-side antennas correlation. The main contribution of the paper is the characterization of the correlation coefficient between receiver-side antennas.
antennas as a function of the main parameters impacting that coefficient, showing that dependency and how it affects the singular values probability distribution function. The remaining part of this contribution is organized as follows: Section 2 describes the physical antennas adjustment as well as the corresponding variables that will impact the computation of the receive antennas correlation. The corresponding correlated MIMO system model is introduced in Section 3. Section 4 briefly reviews the underlying quality criteria and derived different power allocation solutions. The associated performance results are presented and interpreted in section 5. Finally, in section 6 the concluding remarks are discussed.

2 Receiver-side Antenna Correlation

Receive antennas correlation introduces undesirable effects which affect the MIMO system performance. The correlation coefficient attempts to measure the affinity of the various paths in a MIMO link. The receive antennas correlation coefficient determines the similarity between the path from a given transmit antenna to different receive antennas. The smaller the correlation coefficient the larger the possibility to benefit from the MIMO system properties.

Consider the physical antennas set-up described in Fig. 2, where the transmitter side antenna transmits a signal $s_1(t)$ that is received by the receive antennas #1 and #2 obtaining the signals $s_{11}(t)$ and $s_{21}(t)$ respectively. These signals can be described as a function of the channel coefficients describing the paths from each transmit to each receive antenna. Let’s $h_{11}$ be the channel coefficient describing the path from transmit antenna #1 to receive antenna #1, and let’s be $h_{21}$ be the channel coefficient describing the path from transmit antenna #1 to receive antenna #2. Under these conditions the received signals can be described by: $s_{11}(t) = h_{11} \cdot s_1(t)$ and $s_{21}(t) = h_{21} \cdot s_1(t)$, respectively. Given the transmit signal is the same for each receive antenna, the receive antennas correlation coefficient can be obtained by computing the correlation between the received signals. In consequence, the receive antennas correlation coefficient can be calculated as follows:

$$
\rho = \frac{E\left\{s_{11}(t) \cdot s_{21}^*(t)\right\}}{\sqrt{E\left\{|s_{11}(t)|^2\right\} \cdot \sqrt{E\left\{|s_{21}(t)|^2\right\}}} ,
$$

under the assumption that the transmitted signal $s_1(t)$ is zero mean and hence $s_{11}(t)$ and $s_{21}(t)$ are zero mean valued variables, too. The channel coefficients depend on several factors. In general terms they can be described by:

$$
h_{11} = G_{TX}(\theta_{TX}) \cdot G_{RX}(\theta_{RX}) \cdot A(d_{11}) \cdot e^{-j2\pi d_{1}/\lambda},
$$

where $G_{TX}(\theta_{TX})$ is the transmit antenna gain in the direction of the transmit departure angle $\theta_{TX}$ to the receive antenna, $G_{RX}(\theta_{RX})$ is the receive antenna gain in the direction of the arrival angle $\theta_{RX}$, $A(d_{11}) \leq 1$ describes the path attenuation for the path distance $d_{1}$ and $\lambda$ is the signal wavelength. Antennas gain factor can be included in a single term $G(\theta_{TX}, \theta_{RX}) = G_{TX}(\theta_{TX}) \cdot G_{RX}(\theta_{RX})$, which includes both effects. For the simplicity in the following discussing it is assumed that antennas are isotropic with unity gain, that is, $G(\theta_{TX}, \theta_{RX}) = G_{TX}(\theta_{TX}) = G_{RX}(\theta_{RX}) = 1$ (see Fig. 3).

The channel coefficient $h_{21}$ can be described in a similar way as follows:

$$
h_{21} = G_{TX}(\theta_{TX}) \cdot G_{RX}(\theta_{RX}) \cdot A(d_{21}) \cdot e^{-j2\pi d_{2}/\lambda}.
$$

Assuming the simplifications described above, (2) and (3) becomes respectively: $h_{11} = A(d_{11}) \cdot e^{-j2\pi d_{1}/\lambda}$ and $h_{21} = A(d_{21}) \cdot e^{-j2\pi d_{2}/\lambda}$. In consequence, the received signals can be described as: $s_{11}(t) = s_1(t) \cdot A(d_{11}) \cdot e^{-j2\pi d_{1}/\lambda}$ and $s_{21}(t) = s_1(t) \cdot A(d_{21}) \cdot e^{-j2\pi d_{2}/\lambda}$. The received signals can now be used in (1) to compute the correlation coefficient. The terms in the denominator in (1) can be calculated as follows:

$$
E\{|s_{11}(t)|^2\} = E\{|s_1(t) \cdot A(d_{11}) \cdot e^{-j2\pi d_{1}/\lambda}|^2\} = A^2(d_{11}) \cdot E\{|s_1(t)|^2\} .
$$

![Figure 2. Antennas’ physical disposition: one transmit and two receive antennas.](image-url)
Assuming that the transmitted signal $s_{11}(t)$ has unitary energy, i.e., $E\{|s_{11}(t)|^2\} = 1$, it is finally obtained:

$$E\{|s_{11}(t)|^2\} = A^2(d_{11}) .$$ \hfill (5)

A similar development can be performed on the term $E\{|s_{21}(t)|^2\}$ which deals to:

$$E\{|s_{21}(t)|^2\} = A^2(d_{21}) . \hfill (6)$$

The numerator in (1) can be developed assuming similar considerations. Under the assumption that $s_{11}(t)$ has unitary energy, i.e., $E\{|s_{11}(t)|^2\} = 1$, the numerator in (1) finally becomes:

$$E\{s_{11}(t) \cdot s_{21}(t)\} = A(d_{11}) \cdot A(d_{21}) \cdot e^{-j2\pi(d_{11} - d_{21})/\lambda}. \hfill (7)$$

Finally, by substituting (5), (6) and (7) into (1) the correlation coefficient is given by:

$$\rho = e^{-j2\pi(d_{11} - d_{21})/\lambda}. \hfill (8)$$

The result obtained in (8) can be used in line of sight situations. However, wireless channels requires scattered environments to be taken into consideration. Considering the scattered environment represented in Fig. 4, where $d_{11}$ represents the scatters’ mean distance from transmit antenna #1 to receive antenna #1, and $d_{21}$ represents the scatters’ mean distance from transmit antenna #1 to receive antenna #2. The angle $\phi$ represents the mean angle of arrival of the scatters to both receive antennas (assuming far field communication conditions). The angle $\xi$ represents the angle deviation of concrete scatters (in Fig. 4 $\xi_\nu$ from $\phi$). The distance $d$ is the receive antennas separation. Then, the distance difference between mean scatter received at both antennas is $\Delta d = d_{21} - d_{11} = d \cdot \cos(\phi)$. For an arbitrary scatter the distance difference which affects the correlation coefficient is given by $\Delta d = d \cdot \cos(\phi + \xi)$, where $\xi$ is a random variable that must be appropriately modeled.

In order to select an appropriate model for $\xi$ it is considered that most of the scatters are concentrated around the mean scatter and the probability of receiving scatters for a certain angle $\xi$ follows a normal distribution with mean $\mu = 0$ and standard deviation $\sigma_\xi$, i.e., $N(0, \sigma_\xi)$, whose probability distribution function is given by:

$$p(\xi) = \frac{1}{\sqrt{2\pi} \sigma_\xi} e^{-\frac{\xi^2}{2\sigma_\xi^2}}. \hfill (9)$$

In consequence the numerator in (1) is given by:

$$E\{s_{11}(t) \cdot s_{21}(t)\} = A(d_{11}) A(d_{21}) E\{e^{-j2\pi \Delta d/\lambda}\}. \hfill (10)$$

with

$$E\{e^{-j2\pi \Delta d/\lambda}\} = E\{e^{-j2\pi \delta_{\lambda} \cdot \cos(\phi + \xi)}\}. \hfill (11)$$

where $\delta_{\lambda} = d/\lambda$. Substituting (10), (5) and (6) in (1) the correlation coefficient is obtained:

$$\rho(\phi, \xi, \delta_{\lambda}) = E\{e^{-j2\pi \delta_{\lambda} \cos(\phi + \xi)}\}. \hfill (12)$$

Having considered $\xi$ a normal distributed random variable, the computation of the expectation in (12) becomes:

$$\rho(\phi, \xi, \delta_{\lambda}) = \int_{-\infty}^{\infty} e^{-j2\pi \delta_{\lambda} \cos(\phi + \xi)} p(\xi) d\xi. \hfill (13)$$

The term $\cos(\phi + \xi)$ in (13) can be developed as

$$\cos(\phi + \xi) = \cos(\phi) \cos(\xi) - \sin(\phi) \sin(\xi). \hfill (14)$$

In the case $\xi$ is small enough (as can be assumed in the model), then $\cos(\xi) \approx 1$ and $\sin(\xi) \approx \xi$, and finally $\cos(\phi + \xi) = \cos(\phi) - \xi \sin(\phi)$ is obtained. Now, substituting this result in (13), it follows:

$$\rho(\phi, \xi, \delta_{\lambda}) = e^{-j2\pi \delta_{\lambda} \cos(\phi)} \int_{-\infty}^{\infty} e^{j2\pi \delta_{\lambda} \xi \sin(\phi)} p(\xi) d\xi \hfill (15)$$
Finally, (15) becomes

$$\rho(\phi, \sigma_\xi, d_\lambda) = e^{-i/2 \pi d_\lambda \cos(\phi)} e^{-2(\pi d_\lambda \sin(\phi) \sigma_\xi)^2}. \quad (16)$$

Equation (16) reveals that the phase of the correlation coefficient is given by the angle of arrival $\phi$ and the antennas separation $d_\lambda$, while the magnitude of the correlation coefficient depends on both the angle of arrival and the variance $\sigma_\xi^2$ of the scatters' spread angle $\xi$, as well as the receive antennas separation $d_\lambda$.

Once obtained the equation describing the receive antennas correlation coefficient as a function of the angle of arrival $\phi$, the antennas separation $d_\lambda$ and the angle spread $\sigma_\xi$, the variation of the coefficients for the different parameter sets can be analysed to determine the dependence strength. Fig. 5 – 7 represent the correlation coefficient magnitude as a function of the angle of arrival $\phi$, the antennas separation $d_\lambda$ and the angle spread $\sigma_\xi$. Analysing a fixed angle of arrival $\phi$ as well as angle spread $\sigma_\xi$, the magnitude of the correlation coefficient changes with the receive antennas separation. As expected, the larger the antennas separation the lower the magnitude of the correlation coefficient. Furthermore, for low angle spread values the magnitude of the correlation coefficient is high because under that condition the scatters arrive the receive antennas into a narrow angle. As the angle spread increases the correlation coefficient magnitude decreases.

## 3 MIMO Channel Correlation

It is quite common to assume that the coefficients of the $(n_R \times n_T)$ channel matrix $H$ are independent and Rayleigh distributed with equal variance. However, in many cases correlations between the transmit antennas as well as between the receive antennas can’t be neglected. The way to include the antenna signal correlation into the MIMO channel model for Rayleigh flat-fading like channels is given by [4] and results in

$$\text{vec}(H) = R_{\text{HH}}^* \cdot \text{vec}(G) \quad (17)$$

where $G$ is a $(n_R \times n_T)$ uncorrelated channel matrix with independent, identically distributed complex Rayleigh distributed elements and vec$(\cdot)$ being the operator stacking the matrix $G$ into a vector column-wise. The matrix $R_{\text{HH}}$ describing the correlation within the channel coefficients $h_{\nu,\mu}$ (with $\nu = 1, \ldots, n_R$ and $\mu = 1, \ldots, n_T$) is defined as

$$R_{\text{HH}} = E \{ \text{vec}(H) \cdot \text{vec}(H)^* \} \quad (18)$$

with vec$(H)$ resulting exemplarily for the considered $(2 \times 2)$ MIMO system in

$$\text{vec}(H) = \begin{pmatrix} h_{1,1} \\ h_{2,1} \\ h_{1,2} \\ h_{2,2} \end{pmatrix}. \quad (19)$$

Assuming that the correlation introduced by the antenna elements at the transmitter side is independent from the correlation introduced by the antenna elements at the receiver side, the correlation matrix can be defined over the transmit side correlation matrix $R_{\text{TX}}$ as well as the receiver side correlation matrix $R_{\text{RX}}$. In this case the matrix $R_{\text{HH}}$ results in

$$R_{\text{HH}} = R_{\text{TX}} \otimes R_{\text{RX}} \quad (20)$$

where $\otimes$ represents the Kronecker product. For the exemplarily investigated $(2 \times 2)$ MIMO system, the receiver side correlation matrix $R_{\text{RX}}$ is given by

$$R_{\text{RX}}^{(2 \times 2)} = \begin{pmatrix} \rho_{1,1}^{(\text{RX})} & \rho_{1,2}^{(\text{RX})} \\ \rho_{2,1}^{(\text{RX})} & \rho_{2,2}^{(\text{RX})} \end{pmatrix} = \begin{pmatrix} 1 & \rho^{(\text{RX})} \\ \rho^{(\text{RX})} & 1 \end{pmatrix} \quad (21)$$
and describes the correlation between the receive antennas \( m \) and \( n \), independent from the transmit antenna \( k \). The receive side correlation coefficient between the receive antennas \( m \) and \( n \) can be calculated as follows

\[
\rho_{m,n}^{(RX)} = E\{h_{m,k} \cdot h_{n,k}^*\}. \tag{22}
\]

It should be taken under consideration that the value of the correlation coefficient depends on the reference antenna. That is, the correlation coefficient between antenna \( n \) and antenna \( m \) is given by:

\[
\rho_{n,m}^{(RX)} = E\{h_{n,k} \cdot h_{m,k}^*\} = \rho_{m,n}^{(RX)}. \tag{23}
\]

Hence in the correlation matrix the symmetric elements with respect to the main diagonal are complex conjugated. This relationship is due to the sign change when computing the distance difference between antennas with different antenna reference.

In this work it is assumed that no correlation between the antennas at the transmitter side appears. Under this assumption the transmitter side correlation matrix \( \mathbf{R}_{TX}^{(2 \times 2)} \) results in

\[
\mathbf{R}_{TX}^{(2 \times 2)} = \begin{pmatrix} \rho_{1,1}^{(TX)} & \rho_{1,2}^{(TX)} \\ \rho_{2,1}^{(TX)} & \rho_{2,2}^{(TX)} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{24}
\]

The transmitter side correlation matrix \( \mathbf{R}_{TX}^{(2 \times 2)} \) describes the correlation between the transmit antennas \( k \) and \( \ell \), independent from the receive antenna \( m \). The transmitter side correlation coefficient between the transmit antennas \( k \) and \( \ell \) is obtained as

\[
\rho_{k,\ell}^{(TX)} = E\{h_{m,k} \cdot h_{m,\ell}^*\}. \tag{25}
\]

Finally, the overall correlation matrix \( \mathbf{R}_{HH} \) with the elements

\[
\mathbf{R}_{HH}^{(2 \times 2)} = \begin{pmatrix} \rho_{1,1,1,1} & \rho_{1,1,1,2} & \rho_{1,2,1,1} & \rho_{1,2,1,2} \\ \rho_{1,1,2,1} & \rho_{1,1,2,2} & \rho_{1,2,2,1} & \rho_{1,2,2,2} \\ \rho_{2,1,1,1} & \rho_{2,1,1,2} & \rho_{2,2,1,1} & \rho_{2,2,1,2} \\ \rho_{2,1,2,1} & \rho_{2,1,2,2} & \rho_{2,2,2,1} & \rho_{2,2,2,2} \end{pmatrix}
\]

results in

\[
\mathbf{R}_{HH}^{(2 \times 2)} = \begin{pmatrix} 1 & \rho_1^{(RX)} & 0 & 0 \\ \rho_1^{(RX)} & 1 & 0 & 0 \\ 0 & 0 & 1 & \rho_2^{(RX)} \\ 0 & 0 & \rho_2^{(RX)} & 1 \end{pmatrix}. \tag{26}
\]

Therein, the elements \( \rho_{k,\ell,m,n} \) of the overall correlation matrix \( \mathbf{R}_{HH} \) are given by the following equation

\[
\rho_{k,\ell,m,n} = E\{h_{m,k} \cdot h_{n,\ell}^*\} = \rho_{k,\ell}^{(TX)} \cdot \rho_{m,n}^{(RX)}. \tag{28}
\]

For the performance analysis two different MIMO configurations are studied: Within the \((2 \times 2)\) MIMO system, the receiver-side correlation matrix \( \mathbf{R}_{RX} \) is given according to (21), whereas in the \((4 \times 4)\) MIMO system it is assumed that correlation appears only between neighbouring antennas of the uniform linear array. In this particular case, the receiver-side correlation matrix is given by

\[
\mathbf{R}_{RX}^{(4 \times 4)} = \begin{pmatrix} 1 & \rho_1^{(RX)} & 0 & 0 \\ \rho_1^{(RX)} & 1 & 0 & 0 \\ 0 & 0 & 1 & \rho_2^{(RX)} \\ 0 & 0 & \rho_2^{(RX)} & 1 \end{pmatrix}. \tag{29}
\]

The \((4 \times 4)\) MIMO system is an extension of that shown in Fig. 4 where receive antennas correlation was depicted. It is assumed that antennas are uniformly distributed in a linear array and they are numbered in increasing order, i.e., \#1 \#2 \#3 \#4, resulting in the receiver side correlation matrix described in (29) where just neighbour antennas correlation was considered.

Taking (20) into account, the equation (17) can be expressed as

\[
\vec{\mathbf{H}} = \mathbf{G}^{1/2} \cdot \vec{\mathbf{G}} \tag{30}
\]

with

\[
\mathbf{G}^{1/2} = \left( \mathbf{R}_{TX}^{1/2} \otimes \mathbf{R}_{RX}^{1/2} \right). \tag{31}
\]

Therein, \( \mathbf{G}^{1/2} \) is the square-root of the overall correlation matrix \( \mathbf{G} \). Here, it is worth noting that the square root of the matrix is obtained by using the Cholesky decomposition.

When considering a non-frequency selective SDM (space division multiplexing) MIMO link composed of \( n_T \) transmit and \( n_R \) receive antennas, the system is modelled by

\[
\mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{w}. \tag{32}
\]

In (32), \( \mathbf{u} \) is the \((n_R \times 1)\) received vector, \( \mathbf{c} \) is the \((n_T \times 1)\) transmitted signal vector containing the complex input symbols and \( \mathbf{w} \) is the \((n_R \times 1)\) vector of the additive, white
Gaussian noise (AWGN) having a variance of $U_R^2$ for both the real and imaginary parts. The interference between the different antenna’s data streams, which is introduced by the non-diagonal channel matrix $H$, requires appropriate signal processing strategies. A popular technique is based on the singular value decomposition (SVD) [5] of the system matrix $H$, which can be written as $H = S \cdot V \cdot D^H$, where $S$ and $D^H$ are unitary matrices and $V$ is a real-valued diagonal matrix of the positive square roots of the eigenvalues of the matrix $H^H H$ sorted in descending order. The SDM MIMO data vector $c$ is now multiplied by the matrix $D$ before transmission. In turn, the receiver multiplies the received vector $u$ by the matrix $S^H$. Thereby neither the transmit power nor the noise power is enhanced. The overall transmission relationship is defined as

$$y = S^H (H \cdot D \cdot c + w) = V \cdot c + \tilde{w}.$$  (33)

Here, the channel matrix $H$ is transformed into independent, non-interfering layers having unequal gains.

When applying the proposed system structure, the SVD-based equalization leads to different weighted AWGN channels, where the weighting factor $\sqrt{\xi_{\ell,k}}$ represents the positive square roots of the eigenvalues of the matrix $H^H H$ (Fig. 10). The number of readily separable layers is limited by $\min(n_T, n_R)$.

As the singular values decomposition is used to prepend post-processing of the system signals in order to avoid inter-antenna interferences, the impact of antennas correlation on the singular values has been analysed (see Fig. 8–9). By applying the SVD, the MIMO channel can be described as multiple independent SISO channels (so-called layers) with different gains (given by the corresponding singular values). For the same noise power at the receive antenna, the larger the singular value the higher is the SISO channel reliability. The ideal situation is when all singular values are equal. The apparition of predominant layers (high valued singular value) is accompanied by weak layers (low valued singular value). As highlighted in Fig. 8 and 9, the difference between the smallest and the largest singular value increases as the antennas correlation increases. The antennas correlation effect increases the probability of having predominant layers.

4 Quality Criteria and associated Power Allocation Solutions

Analysing M-QAM modulation, the bit-error probability is given by:

$$P_t = \frac{2}{\log_2(M)} \left(1 - \frac{1}{\sqrt{M}}\right) \text{erfc} \left(\frac{\varphi}{\sqrt{2}}\right).$$  (34)

Therein, the argument $\varphi = U_A^2/U_R^2$ of the complementary error function is obtained by taking the half-vertical eye-opening $U_A$ and the noise power per quadrature component

\footnote{The transpose and conjugate transpose (Hermitian) of $D$ are denoted by $D^T$ and $D^H$, respectively.}

$U_R^2$ at the detector input into account [5]. Using singular-value decomposition (SVD), the whole MIMO system is transferred into a number of weighted AWGN-channels (layers). Thereby, the number of easily separable MIMO-Layers is given by the minimum number of antennas at both sides the transmitter as well as the receiver side. Having a parallel transmission over a number of weighted AWGN-channels, the quality on each layer can be obtained by taking the half-vertical eye-opening and the noise power per quadrature component at the detector input into account (Fig. 10). The half-vertical eye-opening per MIMO layer $\ell$ and per transmitted symbol block $k$ results in

$$U_A^{(\ell,k)} = \sqrt{\xi_{\ell,k}} \cdot U_{s_\ell} ,$$  (35)

where $U_{s_\ell}$ denotes the half-level transmit amplitude assuming $M_\ell$-ary QAM and $\sqrt{\xi_{\ell,k}}$ represents the positive square roots of the eigenvalues of the matrix $H^H H$. The average transmit power $P_{s_\ell}$ per MIMO layer $\ell$ determines the half-level transmit amplitude and is given by

$$P_{s_\ell} = \frac{2}{3} U_{s_\ell}^2 (M_\ell - 1).$$  (36)

Activating $L \leq \min(n_T, n_R)$ MIMO layers, the overall transmit power $P_s = \sum_{\ell=1}^L P_{s_\ell}$ can be calculated.

The layer-specific bit-error probability at the time $k$ is obtained as

$$P_{s_{(\ell,k)}} = \frac{2}{\log_2(M_\ell)} \left(1 - \frac{1}{\sqrt{M_\ell}}\right) \text{erfc} \left(\frac{U_A^{(\ell,k)}}{\sqrt{2} U_R}\right).$$  (37)

The aggregate bit-error probability at the time $k$, taking the
Taking all activated MIMO layers into account, the overall block transmit power per symbol block \( k \) is obtained as

\[
P_{s,PA}^{(k)} = \sum_{\ell=1}^{L} P_{s,PA}^{(\ell,k)} .
\]  

(41)

With (39) the layer-specific bit-error probability at the time \( k \) changed to

\[
P_{l,PA}^{(\ell,k)} = \frac{2}{\log_2(M_{\ell})} \left( 1 - \frac{1}{\sqrt{M_{\ell}}} \right) \erfc \left( \frac{U_{PA}^{(\ell,k)}}{\sqrt{2}U_R} \right) .
\]  

(42)

In order to find the optimal set of PA parameters minimizing the overall BER, i.e., \( \sqrt{p_{l,k}} \), the Lagrange multiplier method is used. Using the Lagrange multiplier method, the cost function \( J(p_0, \cdots, p_{N_b-1}) \) may be expressed as

\[
J(\cdots) = \frac{1}{\sum_{\nu=1}^{L} \log_2(M_{\nu})} \sum_{\ell=1}^{L} \log_2(M_{\ell}) P_{s,PA}^{(\ell,k)} + \lambda \cdot B ,
\]  

(43)

where \( \lambda \) denotes the Lagrange multiplier. The parameter \( B \) in (43) describes the boundary condition

\[
B = \sum_{\ell=0}^{L} \left( P_{s,\ell} - P_{s,PA}^{(\ell,k)} \right) = 0
\]  

(44)

and

\[
\sum_{\ell=0}^{L} P_{s,\ell} (1 - p_{\ell,k}) = 0 .
\]  

(45)

Without PA, a natural choice is to opt for a scheme that shares the transmit power uniformly between the number of activated MIMO layers, i.e. \( P_{s,\ell} = P_s/L \). In this case, the boundary condition simplifies to

\[
B = \frac{P_s}{L} \sum_{\ell=0}^{L} (1 - p_{\ell,k}) = 0 .
\]  

(46)

Following this equation the transmit power coefficients have to fulfill the following equation \( \sum_{\ell=0}^{L} p_{\ell,k} = L \). Differentiating the Lagrangian cost function \( J(p_{1,k}, p_{2,k}, \cdots, p_{L,k}) \) with respect to the \( p_{\ell,k} \) and setting it to zero, leads to the optimal set of PA parameters. For finding the optimal set of PA parameters, of computer algebra system such as Matlab or Maple comes in handy.

In order to study the effect of PA thoroughly, fixed channel profiles as shown in Table 1 are investigated.
In order to study the effect of PA two different time-invariant channel profiles are studied. For comparison reason, the channel profile CM-1 is oriented on the uncorrelated channel with $\vartheta = 0.125$ and the channel CM-2 on the correlated one, with $\vartheta = 0.037$. The difference in the layer-specific fluctuations is described by the parameter $\vartheta = \sqrt{\xi_{\ell,k}^2} / \sqrt{\xi_{\ell,k}^2}$. In general, the BER performance is affected by both the layer-specific weighting factors $\sqrt{\xi_{\ell,k}}$ and the QAM-constellation sizes $M_\ell$. Assuming a fixed date rate the resulting layer-specific QAM constellations are highlighted in Table 2.

Table 2. Investigated QAM transmission modes

<table>
<thead>
<tr>
<th>throughput</th>
<th>layer 1</th>
<th>layer 2</th>
<th>layer 3</th>
<th>layer 4</th>
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<td>0</td>
<td>0</td>
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<td>4</td>
<td>0</td>
<td>0</td>
</tr>
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<td>8 bit/s/Hz</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 12 shows the bit-error rate with optimal PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Tab. 2 and transmitting 8 bit/s/Hz over channel CM-1.

Analysing the channel profiles of Table 1, the resulting BER curves without and with optimal PA are highlighted in Fig. 12 and 13. From the BER curves it can be seen, that PA can be helpful in combination with an appropriate number of activated MIMO to minimize the overall BER. In all situations, activating all MIMO layers does not necessarily lead to the best BERs.

Next to the optimal PA, a suboptimal solution is investigated, which concentrates on the argument of the complementary error function. In this particular case the argument

$$p_{\ell,k} = \frac{(\ell,k)}{\sqrt{\xi_{\ell,k}}}$$

is assumed to be equal for all activated MIMO layers per data block $k$, i.e., $p_{\ell,k} = \text{constant}$ $\ell = 1, 2, \ldots, L$. The power to be allocated to each activated MIMO layer $\ell$ and transmitted data block $k$ can be shown to be calculated as follows:

$$p_{\ell,k} = \frac{1}{\sum_{\nu=1}^{L} \xi_{\nu,k}}$$. (48)

Here, for each symbol of the transmitted MIMO symbol vector the same half vertical eye opening of

$$U_{PA}^{(\ell,k)} = \sqrt{p_{\ell,k} \cdot \xi_{\ell,k}}, U_{s,\ell} = \left[ \sum_{\nu=1}^{L} \frac{1}{\xi_{\nu,k}} \right]^{1/2}$$

can be guaranteed $\ell = 1, \ldots, L$, i.e.,

$$U_{PA}^{(\ell,k)} = \text{constant} \quad \ell = 1, 2, \ldots, L$$

When assuming an identical detector input noise variance for each channel output symbol, the above-mentioned
equal quality scenario is encountered. The obtained BER curves are shown in Fig. 14.

5 Results

In this work a \((4 \times 4)\) MIMO system with antennas correlation was studied. In order to show the distribution of the layer-specific characteristic properly, the CCDF (complementary cumulative distribution function) is used (see Fig. 15). It can be noticed the different effect on strong and weak layers. The antennas correlation increases the probability of having layers with larger values (see layers \(\sqrt{\xi_1}\) and \(\sqrt{\xi_2}\)) and increases for weak layers the probability of having lower values (see layers \(\sqrt{\xi_3}\) and \(\sqrt{\xi_4}\)). As investigated by the pre-defined channels CM-1 and CM-2, PA can be helpful in combination with an appropriate number of activated MIMO in order to minimize the overall BER. In doing so, activating all MIMO layers did not necessarily lead to the best BERs. The results obtained by PA, can be adopted to the here investigated correlated MIMO configuration.

6 Conclusion

Among the several factors affecting the performance of a MIMO link by limiting the possibility of reaching the maximum theoretical potential, the transmit and receive side antennas array’s correlation due to its proximity is one of the most impacting factors significantly degrading the system performance. Compared to the uncorrelated channel and as demonstrated by computer simulations, the probability of having predominant layers increases. The influence of layers with high weighting factors becomes even stronger whereas the influence of layer with low weighting factors diminishes. In consequence appropriate strategies must be adopted in order to improve the MIMO system performance. Our results show that not necessarily all layers might be used for the data transmission even when the wave propagation between the different pairs of transmit and receive antennas is affected by correlation.

References


