STRUCTURAL OPTIMIZATION OF POWER FLEXTENSIONAL PIEZOELECTRIC ACTUATOR USING FE MODEL AND GENETIC ALGORITHM

Sergey Shevtsov, Igor Zhilyaev*, Vladimir Axenov
Mechanical Engineering Lab, South Center of Russian Academy, Rostov-on-Don, Russia
*(zhilyaev@mail.com)

ABSTRACT
This paper concerns a structural optimization problem for a flextensional piezoelectric actuator which consists of the high power piezoelectric stack and polymeric composite shell intended for amplification of the stroke. The principal drawback of the piezoelectric transducers is a very small stroke at relatively high operating force. In order to supply the required stroke some amplification means are used. The considered flextensional transducer allows obtaining needed stroke amplification, but because of counteracting forces the initial actuator’s stroke significantly reduced. The main aim of this article is the optimization of the actuator design to simultaneously provide sufficient stroke and stiffness, allowing counteract external loads. To do this, we parameterize the shape of the amplifier’s shell by the rational Bezier curves, which parameters (coordinates and weights of the control points) are changed iteratively by genetic algorithm according to the fitness function value calculated by the finite element model of the transducer with varied geometry of shell.

KEY WORDS
Active control, Piezoelectric actuator, Structural optimization, Finite element modeling, Genetic algorithm.

1. Introduction
The modern helicopters have made a significant progress in performance, reliability, handling qualities, and efficiency, but still have the drawbacks as noise and high vibrations generated by the main rotor due to blade vortex interaction, high Mach numbers at the advancing blade, etc. Due to these troubles the field of active control of helicopter rotor blades has raised a consequent interest in the last 30 years, and most attempts to solve the problem of noise and vibrations reduction associated with active control of the rotor blades.

The first theoretical studies for the rotor vibration reduction have been implemented by higher harmonic control (HHC) which is based on actuators located below the swashplate, to enforcing oscillations of fixed frame with frequencies \( k\Omega, \quad k = 1, n_b \), where \( \Omega \) is the rotor angular frequency, and \( n_b \) is the total number of blades [1, 2]. The other solution than HHC is Individual Blade Control (IBC), which is based on actuators in the rotating frame, where actuators change the aerodynamic properties of each blade in the real time [1-3]. Both technologies work with a higher harmonic excitation of the blade pitch at the blade root, which is a very energy-consuming, and also can lead to the blades excitation on a first twisting vibration mode [4]. With an active trailing edge (ATE) flap the excitation is not introduced at the blade root, but localized at a distance 75-90% of the blade radius. ATE concept can be implemented in the form of flexible or turned trailing edge flap (see Fig. 1), and as the locally morphing airfoil [5].

The Actively Controlled Flaps are generally 15% of a chord length and most known structures are driven by the power piezoelectric transducers. Due to the high operating forces and frequency, smart materials such as piezoelectric actuators are well adapted for this task, but relatively large displacements require some sort of mechanical amplification of these devices movement. Because of the very small displacements created by the piezoelectric devices some design solutions to amplify the stroke of PZT actuators are proposed [5, 6].

The most important requirements to the piezoelectric actuator for the active flap design are the following:
- high force and large displacement in compact sizes;
- high resolution within the micrometre range;
- very short response time below 1 ms;
- life time greater than \( 10^{10} \) cycles;
- low voltage supply below 150V DC, no backlash and no play;
- low power consumption when static;
- severe environment compatibility (broad temperature and moisture ranges).

Among the known structural concepts of actuators most adoptable properties had the flextensinal actuator which consist of the high power pezolectric stack and shell which supply the stroke amplification. These actuators provide a high stiffness and relatively large displacements. On the other hand, their mass is not optimized and the metallic shell implies a large penalty on the mass. In order to improve the mass of these flextensional actuators a carbon composite is used instead of metal for the frame [5, 6]. At operating mode the
aerodynamic forces are transmitted to the actuator through the levers. These forces deflect the flap in opposite direction to its active deflection. Because of acting aerodynamic forces a higher flap deflections require larger and therefore heavier actuators, but additional mass is needed to equilibrate the center of mass of the blade close to active flap mounting.

So, we need to optimize the electro-mechanical properties of the considered flextensional actuator.

2. FE modeling of actuator with traditional elliptic shell

As polymeric composite material for the shell is less dense, we have investigated only this case. We adopted glass fiber epoxy polymeric composite with the longitudinal Young modulus $3 \cdot 10^{10}$ Pa and density 1850 kg/m$^3$. Technology of the shell manufacturing includes the winding of unidirectional glass-fiber tape on a mandrel with appropriate shape, then curing in autoclave until the complete solidification. The size of the finished shell should provide a prestress for the assembled piezoelectric stack for a double-side acting actuator.

Stack was constructed of a multilayer piezoelectric ceramic PZT-5H, the thickness of each layer, polarized in thickness, was 0.5 mm. Driving electric potential was taken as 400 V. Geometry of a typical FEM model is presented on a Fig. 2.

In our numerical experiments we used shells with the thickness 8 mm and 12 mm; major semi-axis was always 100 mm, whereas the small semi-axis taking values 40, 50, 70 mm.

The FEM static analysis was performed as follows. After applying of the driving potential the stack lengthened, deforming the shell and causing it to contract in the vertical direction (see Fig. 3). Then to the executive surfaces of the shell (small quadrilateral planes) the gradually increasing tensile force was applied. When the structure is loading the values of operating stroke, applied reaction force and deformation of piezoelectric stack were monitored. As soon as the executive surfaces displacement returns to zero, the blocking force is recorded. Some simulations results are presented on a Fig. 3, where a designation, for example (40-100-80), means the major semi-axes - least semi axes - thickness of shell, respectively.
One can see from the graphs (Fig. 4, a, d) that the total compliance of actuator is determined by the shell, not PZT stack that has a greater stiffness. Indeed, total displacement of actuator (40-100-8) at the loading force 2 kN consist of 0.23 mm, whereas deformation of PZT stack is equal to 0.01 mm only. As might be expected more thick shell has more stiffness, but less free stroke.

Dependence of the actuator’s parameters on the ratio of the elliptic shell axes is also essential. The more shallow shells provide greater amplification, but less ability to counteract external loads. This fact is illustrated on a Fig. 5, where dependencies of the amplification ratio and blocking force for the different shell dimensions are presented.

These preliminary simulation results show that actuator stroke depends on the PZT stroke and on the shell compliance. And significant shell compliance is due to non-uniform stress state of material in the different zones of the shell. Actuator ability to maintain the stroke at external loading depends on the shell properties only. Hence, the shell structural optimization can be very useful.

### 3. Structural optimization of the shell

The basic element of the optimization process is a good way to choose the design parameterization [7, 8]. Optimization of the shell can be performed by varying the shape of the generatrix and thickness distribution of the
shell. As the objective function is advisable to choose the operating stroke $h_{op}$ at the given external load $F_{ext}$. To describe the shape of the generatrix with the necessary flexibility we use the rational Bezier curves because these lines are inherent for the used FEM software Comsol Multiphysics. According to these assumptions the one-quarter of the symmetrical shell is presented as two branches, each of which consists of three 3rd order Bezier curves (see Fig. 6). The rational Bezier curve of n'th order defined by the (n+1) control points $P^i$ is described as

$$R(u) = \sum_{i=0}^{n} w_i B_n^i (u) P^i / \sum_{i=0}^{n} w_i B_n^i (u),$$

where $B_n^i (u)$ are the Bernstein polynomials, defined as

$$B_n^i (u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i},$$

$w_i, i=0,1...n$ are the weights of the control points, and $u$ is parameter runs through the values from 0 to 1.

At optimization process, external dimensions of the shell remain unchanged, so, only shape of the generatrices is optimized. In order to supply $C^1$ continuity at the connection points (denoted by $\times$) it is necessary to impose additional constraints on the location of the control points, which are adjacent to the point connection of two curves. If $P^c$ is the coordinates of connection point, $P^{lc}$ is a coordinates vector of a point adjacent to the left-to-point connection, then coordinates vector of a point adjacent to the right-to-point connection is

$$P^{rc} = 2P^c - P^{lc}.$$

The Fig. 6 shows the equality of the corresponding edges of the control polygon. The placement on the vertical (horizontal) lines of points adjacent to the terminal points (denoted as $\times$) of the generatrices allows to automatically satisfy condition of $C^1$ continuity due to symmetry of the shell (see Fig. 6). So, we have 21 degree of freedom for the points coordinates, and 18 degree of freedom for the weights, total number of DoFs is 39.

All control points positions are constrained by the system of inequalities

$$\begin{align*}
X_i &> X_{(i-1)}; \quad i \in [1; (n-1)] \\
Y_i &< Y_{(i-1)}; \quad i \in [2; n]
\end{align*}$$

by the relationship (4) for connection points by the equalities

$$\begin{align*}
X_0 &= 0; \quad Y_0 = b^{in, out} \\
X_1 &= b^{in, out} \\
Y_0 &= a^{in, out} \\
X_{n-1} &= b^{in, out}
\end{align*}$$

where the dimensions of the external generatrix $a^{out}, b^{out}$ are fixed, but the dimensions of the internal generatrix are satisfied by the relationships

$$\begin{align*}
a^{out} > a^{in} &= fixed \\
b^{out} > b^{in} &= varied
\end{align*}$$

Figure 6. Representation of the shell’s profile by the 3rd order rational Bezier curves

All weights also are constrained by the system of inequalities. Each of the three Bezier curves, which form the generatrix, is described by its own equation (1). But to ensure the smoothness of generatrix, end points of one curve and the initial point of the connecting curve have the same weight. So, we have the following system of constraints for weights

$$\begin{align*}
w_0 &= 1 \\
0 &< w_i < 5; \quad i \in [1; n]
\end{align*}$$

After a number of numerical simulations some additional restrictions are introduced to narrow the range of the coordinates of control points

$$\begin{align*}
\sum_{k=1}^{i} \xi(k, a^{in, out}) - 0.25 \xi(i, a^{in, out}) &< X_i < \\
\sum_{k=1}^{i} \xi(k, a^{in, out}) + 0.25 \xi(i, a^{in, out}) &< Y_i < \\
\xi(i, a^{in, out}) &= \frac{a}{372} \left[ 64 - (i-1)^2 \right]
\end{align*}$$

where $i = 1...n-1$

These constraints allow agreeing the density of the control points with the curvature of the generatrix, and thereby significantly reducing the search area. This is very important for a problem with large number of DoFs. Similar optimization problems can be successfully solved only by the genetic algorithm. Therefore we have used the Genetic Algorithm Toolbox MATLAB which has the advanced optimization means and direct access to the FEM computation. To reduce the computational cost our FEM model was very simplified. Due to symmetry of the shell we consider one-quarter of actuator’s structure.
All materials properties and displacement are considered as linear. Quadrilateral FEM mesh consists of near 500 elements. FEM model work in the Structural Mechanics - Piezoelectric mode. At each iteration step GA Toolbox rebuild the shell geometry, remesh it, and perform the static analysis. Calculated value of the actuator stroke is passed to the GA Toolbox, which changes the value of the design variables according to the constraints. All computation flowcharts were controlled by MATLAB which in turn refers to modules that performs the algorithm and finite element analysis. These modules are implemented as standard MATLAB’s m-files. Most important GA settings are: population size – 20, elite count – 4, crossover – scattered, mutation – adaptive feasible, hybrid function – “fminsearch”.

Due to limitations constrained by the whole design of the flap structure all exterior dimensions of the shell were assumed as: 172×70×30 mm, dimensions of PZT stack were 140×24×16 mm, and D-like aluminum insert has plane and cylindrical (R=10.5 mm) contact surfaces.

Besides these design limitations, it was necessary to take into account some technological limitations. This is cylindrical surface of the shell contacting with the aluminum insert [6]. Because the composite shell is manufactured by winding it should be avoided whenever possible concave outer surface of the shell.

4. Results and discussion

Parameterization of the shell was performed in two ways, tentatively called "Bezier-Bezier" and "Cassini-Bezier".

Given the symmetry of the shell its inner and outer form is described by three smoothly connected third orders rational Bezier curves. Each curve is defined by four points and by four weights, but with the design constraints on the size, the thickness of the shell in the end sections. Smooth connection of curves is ensured by coordinates of six points and nine weights of each generatrices. On each step optimization algorithm performed the 2D finite element calculation and determination of the actuator stroke under load.

Optimized shell geometry is presented on a Figure 7. Because the outer surface of the shell was quite complicated and difficult in the manufacture we parameterize the outer generatrix by a simpler curve.

The exterior generatrix was made as an oval Cassini having more preferred form than the ellipse. Internal generatrix was made as three smoothly linked rational Bezier curves, as described above. The layout of control points and weights for Bezier curves and parameters of external oval Cassini is optimized. The resulting geometry is shown on a Fig.7. The total computer elapsed time for the optimization process (~530 epochs) was about 6 hours on i5 computer.

The final calculations (see Fig. 8) were performed on three-dimensional models (see e.g. Fig.9) based on the results of 2D models optimization. These results showed the following. The greatest stroke under load provides shell Bezier-Bezier, next the Cassini-Bezier shell. The worst result is for elliptical. Best feature of the force transmission on the stack has the Bezier-Bezier shell. Loading actuator by the force 60 kgf create the force 230 kg compressing the stack, while for the elliptical shell this compression is 280 kgf. Weights of the fully assembled actuators are respectively 683, 643, 618 grams. Comparison manufacturability compared three designs shows the opposite: the most technologically elliptical shell, worst - Bezier-Bezier shell.

The interesting result of this optimization is the fact that the most effective Bezier-Bezier and Cassini-Bezier designs are very similar to the mechanism with four stiff rods connected by the revolute joints in the form of parallelogram. These rods are thicker shell’s wall with high bending and tensile stiffness.

The optimized Cassini-Bezier shape was chosen as “winner” and used in the real world structure as the source for CAD design of technological equipment (see Fig. 10).

Figure 7. Comparing of three different types of the shell’s geometry
To provide an actively controlled flap installed on a helicopter rotor blades which reduce vibrations level an optimization method for flexextensional piezoelectric actuator has been developed and tested.

The proposed method is based on the representation of the shape of polymeric composite shell by the rational Bezier curves, whose parameters (coordinates and weights of the control points) are the design variables for the optimization algorithm.

The optimization process is controlled by the genetic algorithm toolbox MATLAB, which access a finite element model of a device, calculates the objective functional (the stroke of actuator against external load), and modifies design variables to achieve the optimum.

The effectiveness of the proposed method is examined by comparison with the prototype structure of elliptic composite shell.

The obtained results prove the usefulness of the developed optimization methodology and its attractiveness not only for the aircraft applications but also in many technical areas.

**Acknowledgements**

The authors wish to acknowledge the National Science Council of Taiwan (NSC99-2923-E-022-001-MY3), Russian Foundation for the Basic Research (Grants 10-08013300, 12-08-31350 mol_a) and Russian National Helicopter Center.

**References**


[8] X. Huang, Y.M. Xie, *Evolutionary topology optimization. Methods and applications* (John Wiley and Sons Ltd., )