MODELLING A DROP TEST OF A LANDING GEAR USING A HYBRID BOND GRAPH

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ABSTRACT
A landing gear is a good example of a highly nonlinear multidisciplinary system. Using hybrid modelling, nonlinear behaviour can be abstracted to discontinuous behaviour. Bond graphs are an established and intuitive method for constructing mathematical models of multiphysics dynamic systems. The Hybrid Bond Graph is therefore used here to demonstrate the modelling of a landing gear drop test. A distinction is drawn between structural and parametric discontinuities, represented using controlled junctions and elements. Causally static and dynamic bond graphs are compared. A mixed Boolean mathematical model is obtained from the causally dynamic graph.

KEY WORDS
Bond Graph, Physical Model, Hybrid Model, Hybrid Bond Graph.

1. Introduction
The ‘drop test’ is a standard engineering test whereby the oleo and gear assembly is dropped vertically onto a surface, and both static loads and dynamic response measured. As industry moves towards virtual prototyping and using models for parts specification, there is a business need for a virtual drop test. This is a typical engineering system modelling challenge, demonstrating the limitations of much of the software modelling techniques and tools commercially available.

Firstly, the system is multidisciplinary i.e. it comprises several subsystems across multiple engineering domains (electrical control, fluid power, and mechanical multi-body dynamics) which must be modelled with equal rigor and work together. Secondly, the aircraft landing gear exhibits a range of nonlinear behaviours. Engineers typically linearise systems prior to analysis, and many software products force the user to do so. However, in aircraft systems in particular, there are a great many cases where linearisation is not appropriate. Furthermore, several components on aircraft are unique and cannot be modelled by the standard library parts available in some commercial software packages (for example, tyre models). A high level of user-control and visibility of the underlying mathematical relations is essential. Thirdly, systems models are used for a range of applications which must be supported. A Conceptual Model may be required early in the design phase before all parts have been specified or tested. Later in the design phase, ‘real-time’ proper models may be needed for Model- or Hardware-in-the-Loop testing or online health monitoring. An overly detailed, slow-running system model is therefore not always desirable.

Bond graphs were selected for this task because they lend themselves to mechatronic models by considering power flow across components of all engineering domains. They have a high level of user control, and also allow the user to identify algebraic loops and constraints graphically, and derive information from the model structure, facilitating troubleshooting and efficient simulations. It is assumed the reader is familiar with bond graph technique, and the standard textbook by Karnopp et al is recommended \cite{1}.

Hybrid Models are models which contain both continuous and discontinuous behaviour. They allow very stiff, nonlinear relationships to be abstracted to discontinuities, which can aid simulation.

Hybrid Bond Graphs are an active topic of research. There are several methods for constructing a Switched or Hybrid Bond Graph, none of which have reached common usage over the others: a full literature review is outside the scope of this paper. The most widely used methods for incorporating an ideal discontinuity in a bond graph are the use of switched sources \cite{2, 3} and controlled junctions \cite{4}, which both yield a causally dynamic bond graph. Controlled elements have also been proposed \cite{5} which differ to those described here. Non-ideal switching can be expressed via a modulated resistance or a modulated transformer attached to a resistance, and this method has been used most recently to generate causally static models and a unique equation for the system \cite{6}. Margetts et al \cite{7} review the current state of the art, and propose using...
controlled junctions to intuitively model structural discontinuities. A dynamic causality notation provides increased engineering insight, and a unique mixed-Boolean equation for the system is generated.

This paper demonstrates how controlled junctions might be used for the case study of a landing gear drop test, and introduces a controlled element for 'parametric switching.'

2. Method

2.1 A High-Level Bond Graph of the Landing Gear

A Landing Gear is similar in principle to the standard Quarter Car Model used extensively in Automotive Engineering. The difference is that aircraft are not always in contact with the ground, and they usually use an oleo strut in place of the mechanical suspension (spring and damper mounted in parallel) found on a typical car. A spring-mass-damper diagram is shown in Figure 1, and a high level model is presented in Figure 2. A rigid body with mass and weight equal to the effective vertical load and inertia effects of the aircraft fuselage is attached to the upper end of the gear. The gear is assumed to act as a lumped mass with a centre of gravity coincident with that of the wheel. The oleo strut is attached via a [common effort] 0-junction, because it is known that there is common effort and a difference in velocities across the strut. The behaviour of the tyre and its contact with the ground, and of the oleo strut will be covered in sections 2.2 and 2.3. This model is implemented in the commercial software package 20Sim.

![Figure 2: A High Level Bond Graph of a Landing Gear](image)

2.2 Structural Discontinuities: Contact with the Ground

Contact is a well known modelling and simulation challenge. It gives a variable topology system, which can lead to dynamic causal assignment and/or a change in the size of the underlying equations. The use of a controlled junction for elastic contact has been addressed by several authors and is included in the 20Sim documentation [8]. Essentially it is a bespoke coded element where the displacement of the gear and ground are compared to establish whether contact occurs. The effort and flow on each incident bond is then defined according to whether the contact ‘condition’ is true or false. Controlled 0-Junctions – denoted X0 in this paper - act as regular 0-Junctions in one state (ON) and as sources of zero effort on each incident bond in the other (OFF). This effectively inhibits flow and disconnects the surrounding structure. The ideal causal assignment for the model consequently

![Figure 3: The Tyre & Contact Model, with Dynamic Causality Notation](image)
changes with commutation. In this case, the tyre resistance prevents the dynamic causality from propagating throughout the model significantly, which means that the model can be simulated with a standard commercial software package. The model is shown in Figure 3 with the dynamic causality notation proposed by Margetts et al [7].

In this model the tyre stiffness and resistance were given typical approximate linear values.

2.3 Parametric Discontinuities: The Oleo Strut

A typical oleo strut consists of a chamber filled with a fluid and gas mixture (which acts as a fluid spring), and an orifice plate which controls the rate at which the strut is compressed (adding damping). The use of ‘2-stage’ oleo struts is commonplace (especially for the main landing gear of heavier aircraft) where there are two chambers of fluid and the second becomes active after a ‘breakover’ load is reached. Damping is provided by an orifice plate, which is in series with the fluid.

The constitutive equation for the fluid compliance varies depending on the volume of fluid displaced (i.e. the stroke of the oleo). When the aircraft is in the air, the oleo is fully extended and no load is applied: the piston rests on its end stops. When the aircraft first touches down, the oleo compliance is that of the fluid in the primary chamber. As load on the oleo increases, it reaches a breakout point and the second chamber starts to compress. There are therefore three modes of operation. The compliant modes are modelled by equation 1.

\[
P_1 = \frac{P_0 V_0^{\gamma}}{(V_0 - \int Q dt)^{\gamma}}
\]

Each mode of operation could be described by a standard compliance element in bond graph notation, and activated by a controlled junction. In the ‘rigid’ mode the strut simply reacts the weight of the gear and could be represented as a modulated effort source. A ‘tree’ arrangement of controlled junctions is shown in Figure 4. The controlled 1-junctions – denoted X1 – become a source of zero flow when OFF. Switching coefficients \( \lambda_n \) are mutually exclusive.

Since the switching coefficients are mutually exclusive, and the causality on the bond connected to the rest of the system is static, the ‘tree’ could be concatenated into a single controlled element as proposed in Figure 5. The constitutive equation for this element is given by equation 2.

\[
e = \lambda_1 \Phi_{C1}^{-1} \left( \int f \cdot dt \right) + \lambda_2 \Phi_{C1}^{-1} \left( \int f \cdot dt \right) + \lambda_3 \Phi_{C3}^{-1} \left( \int f \cdot dt \right)
\]

It is worth noting that the resistance provided by the orifice plate is also non-linear, in this case modelled using the standard equation for an orifice.

\[
Q = C_f A_o \sqrt{\frac{2 \Delta P}{\rho}}
\]

Figure 4: A 2-Stage Oleo represented by a ‘tree’ of Bond Graph Elements.

Figure 5: A 2-Stage Oleo represented by a Controlled Element

2.4 Determination of the State Equations

For standard bond graphs, a junction structure matrix may be found and used to derive the state equations for the system [9]. Margetts et al [7] extended this method for a Hybrid Bond Graph incorporating controlled junctions, to obtain a mixed Boolean implicit state equation.

Some of the elements in this case study have nonlinear constitutive relations, rendering any Linear Time-Invariant (LTI) assumptions invalid. However, in this case the method for deriving the implicit state equations can still be used with nonlinear functions in place of linear coefficients.

The bond graph of the full model, including the detail of the contact and 2-stage oleo compliance, is given in Figure 6. A transformer (TF-element) with piston area as its modulation coefficient is inserted to formalise the transition between mechanical and hydraulic domains.

Inspection of this graph yields the junction structure equation which relates all inputs and outputs, and this is turn can be used to derive the implicit state equations (4). The state variables are the inputs to each storage element.
It can be seen that the switching term for the contact manifests in the equations, clearly disconnecting the ground velocity from the gear. The tyre resistance and compliance also cease to have any effect on the gear when it is not in contact with the ground, which is consistent with expectation. The switching terms inside the controlled element representing the oleo fluid compliance do not manifest in the equations: they are contained inside the compliance function.

Figure 6: Complete Model of the Gear, Control Signals omitted for clarity

Deriving the state equations yields a mathematical model which can be transported to other modelling environments for simulation purposes. This would be necessary if the model is to be used as part of a larger programme or on a Model-in-the-loop apparatus which uses a specific code.

In Matlab, for example, a descriptor state-space model object can be defined from the explicit or implicit state and output equations (provided all elements have linear relationships). Some code would be required to establish the mode of operation at each time-step and hence values of the Boolean switching parameters $\lambda_n$. The nonlinear nature of this model means that nonlinear techniques must be used rather than relying on the LTI functionality in most common software packages.

$$p_{ac} = -\frac{A^2}{M_{ac}} p_{ac} \Phi_{ac_{coffee}} (p_{ac}) - A \Phi_{c_{ac_{coffee}}} (q_{ac_{coffee}}) - \frac{A^2}{M_{wheel}} p_{gear} \Phi_{ac_{coffee}} (p_{gear}) + W_{ac}$$

$$q_{ac_{coffee}} = \frac{1}{AM_{ac}} p_{ac} - \frac{1}{AM_{wheel}} p_{gear}$$

$$p_{gear} = -\frac{A^2}{M_{wheel}} p_{ac} \Phi_{ac_{coffee}} (p_{ac}) - A \Phi_{c_{ac_{coffee}}} (q_{ac_{coffee}}) - \frac{A^2}{M_{wheel}} p_{gear} \Phi_{ac_{coffee}} (p_{gear}) + R_{ac_{coffee}} p_{gear} + \frac{1}{M_{wheel}} p_{gear} + \lambda \Phi_{c_{ac_{coffee}}} (q_{ac_{coffee}}) + W_{gear}$$

$$q_{tyre} = \frac{1}{M_{wheel}} q_{gear} + \nu_{ground}$$

3. Results & Discussion

The model was populated with approximate data for a typical aircraft and a simulation was run in 20Sim, first with a single stage oleo, then a 2-stage oleo model. The oleo was sized for the applied load, and the two-stage oleo is consequently smaller than the single-stage. Since the model does not relate to any production aircraft, the results are assessed against subjective engineering experience.

Initially the model did not run due to the highly nonlinear constitutive relationship of the oleo plate. Linearising this element relieved this problem. The model could be improved by implementing a more representative piecewise-continuous relationship, which could be formalised by a XR-element (in the same way that the piecewise-continuous fluid compliance was formalised using a XC-element).

The results are presented in Figure 7 and Figure 8 respectively. The single stage oleo exhibits a classic ring-down response as expected. The 2-stage oleo has similar behaviour, but evidence of ‘breaking out’ can be seen as discontinuities in the signals. Again, this is consistent with experience. There is ‘chattering’ in the results, which is a consequence of the way discontinuity is defined in the model: the oleo mode changes are ‘jump discontinuities.’ These slow the simulation significantly, and defining the discontinuities without a ‘jump’ (i.e. by modelling the isothermal and polytropic phases of the damper compression/extension separately to minimise the ‘jump’ between modes of operation) would reduce the chattering.

The 2–Stage oleo strut displaces less fluid than the single-stage variant, and hence has a shorter stroke. This means that a similar dynamic response can be achieved with a smaller and lighter oleo unit.

The state equations (4) can be used to inspect the system a little further. The parametric switching relationship used to describe the fluid compliance is contained in the nonlinear compliance function. Only the switching coefficient $\lambda$ denoting contact with the ground affects the form of these equations by multiplying terms by 1 or 0.
4. Conclusion

A Bond Graph of a typical aircraft landing gear has been built. Using controlled junctions the contact with the ground can be modelled. The nonlinear 2-Stage oleo can also be represented using a ‘tree’ of bond graph structure with controlled junctions, which can be concatenated into a controlled element.

The state equations for the system are obtained, which feature a Boolean switching parameter to represent contact with the ground. The switching behaviour for the piecewise-continuous oleo strut is contained within the nonlinear compliance function for that element.

Results were obtained in 20Sim by creating some bespoke elements. It is possible to transfer the state equations to other modelling environments.

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References


Figure 7: Simulation of the Drop Test, Single Stage Oleo

Figure 8: Simulation of the Drop Test, Two Stage Oleo