A PROCEDURE TO DETERMINE A BOND GRAPH IN THE FREQUENCY DOMAIN

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ABSTRACT
A methodology to obtain a bond graph of a system in the frequency domain is presented. This proposed bond graph has a derivative causality assignment and additional dissipation elements. The methodology to obtain the frequency bond graph is applied to a DC motor and a passive suspension of a car modelled by bond graphs. Finally, this paper can be used to know the frequency response properties, for example high or low frequencies, of a system in a direct way.

KEY WORDS
Bond graph, frequency response, DC motor, passive suspension

1 Introduction
The bond graph technique is an energy based modelling approach, unifying symbology for phenomena from different physical domains. Bond graph was established by [1]. The idea was developed by [2] and [3] how a powerful tool of modelling.

The use of bond graph provides structured approach to system dynamics modelling. The bond graph language is used to abstract physical systems into basic elements that represent localized dynamic properties of a small part of the system.

Improvements in the response of feedback tracking control systems and disturbance rejection can be achieved in the frequency domain. There are many references to analyze the frequency response of a system using the traditional approaches, i.e., description of a system in a transfer matrix from the system or given a state space representation apply Laplace transform [4, 5]. However, systems formed by different domains of energy is complicated to apply these traditional approaches. In these cases is possible to model the system in the physical domain and determine the transfer matrix [6].

In this paper a bond graph called Frequency Bond Graph (FBG) is proposed. This bond graph allows to determine the state variables and the system output in the frequency domain in a direct way. This FBG is based on a Transformed Bond Graph (TBG) in a derivative causality assignment. In addition, this TBG is the original bond graph and new dissipation elements. The proposed approach can be useful to analyze or design robust control.

This is the first step to obtain the behavior of the system in the frequency domain in a direct way by using the proposed bond graph. The reference [6] can give the transfer function of the system modelled by a bond graph but in an indirect way.

The next and future steps are to get to know the system performance for low and high frequencies by applying the proposed methodology in this paper.

In [7] proposed an incremental bond graph can be introduced for determination of sensitivities. The μ-analysis for dealing with the robustness problem in case of structured uncertainties on parameters describing a procedure in bond graph is proposed in [8]. Hence, the proposed paper can be applied in the direction to obtain the frequency response properties using a bond graph approach in a direct way. It is possible to study bond graphs in high and low frequencies with this proposed methodology [9, 10].

The outline of the paper is as follows: Section 2 gives some basic elements of the modelling in bond graph. Section 3 proposes a junction structure of a bond graph in a derivative causality assignment. A bond graph in the frequency domain is proposed in Section 4. Section 5 applies the methodology to determine the state variables of DC motor and the system output of a passive suspension of a car in the frequency domain. Finally, Section 6 gives the conclusions.

2 Modelling in Bond Graph
The symbolic form of a Bond Graph in Integral causality assignment (BGI) of a Linear Time Invariant (LTI) system is shown in Fig. 1 [11, 12].
In the next section, a junction structure of a bond graph with a derivative causality assignment of a linear time invariant system is presented.

### 3 A Bond Graph in a Derivative Causality Assignment

A junction structure configuration of a Bond Graph with preferred Derivative causality assignment (BGD) is proposed in Fig. 2. Note that the key vectors between the junction structure and storage field are changed respect to Fig. 1.

From (3) the junction structure is given by

\[
\begin{bmatrix}
  z(t) \\
  D_{\text{ind}}(t) \\
  y(t)
\end{bmatrix} =
\begin{bmatrix}
  J_{11} & J_{12} & J_{13} \\
  J_{21} & J_{22} & J_{23} \\
  J_{31} & J_{32} & J_{33}
\end{bmatrix}
\begin{bmatrix}
  x(t) \\
  D_{\text{in}}(t) \\
  u(t)
\end{bmatrix}
\]

(11)

where the entries of \( J \) have the same properties that \( S \). Also, the key vectors of the dissipation field \( D_{\text{ind}}(t) \) and \( D_{\text{out}}(t) \) are defined of the same manner that \( D_{\text{in}}(t) \) and \( D_{\text{out}}(t) \), considering that

\[
D_{\text{out}}(t) = L_d D_{\text{ind}}(t)
\]

(12)

but they depend on the causality assignment for the storage elements and that junctions must have a correct causality assignment.

From (4) to (10) with (11) and (12), we obtain

\[
\begin{align*}
  z(t) &= A^* x(t) + B^* u(t) \\
  y(t) &= C^* x(t) + D^* u(t)
\end{align*}
\]

(13)

(14)

where

\[
\begin{align*}
  A^* &= J_{11} + J_{12} N J_{21} \\
  B^* &= J_{13} + J_{12} N J_{23} \\
  C^* &= J_{31} + J_{32} N J_{21} \\
  D^* &= J_{33} + J_{32} N J_{23}
\end{align*}
\]

(15)

(16)

(17)

(18)

being

\[
N = (I - L_d J_{23})^{-1} L_d
\]

(19)

\[
M = (I - LS_{22})^{-1} L
\]

(10)
The relationships of a system between the BGI and BGD are

\[ A^* = FA^{-1} \]  \hspace{0.5cm} (20)  \\
\[ B^* = -FA^{-1}B \]  \hspace{0.5cm} (21)  \\
\[ C^* = CA^{-1} \]  \hspace{0.5cm} (22)  \\
\[ D^* = D - CA^{-1}B \]  \hspace{0.5cm} (23)  \\

It is well known that the BGD has been used for structural controllability and observability properties of a LTI system [16] and to determine the steady state of a system [14].

4 A Bond Graph in the Frequency Domain

The problem considered in this section is determine the transfer matrix when a state variable representation is modelled by a bond graph of a system.

By taking Laplace transform throughout in (4) and (5), and by setting all initial conditions to zero, we obtain

\[ x(s) = (sI - A)^{-1}Bu(s) \]  \hspace{0.5cm} (24)  \\
\[ y(s) = C(sI - A)^{-1}B + Du(s) \]  \hspace{0.5cm} (25)  \\

In this section a procedure to get a bond graph in the frequency domain is presented. The proposed procedure is the following:

1. A bond graph in an integral causality assignment of the system is considered. This step yields the description \((A, B, C, D)\).

2. Assign the negative to constitutive relations of the storage field, then

\[ z_s(t) = -Fx_s(t) \]  \hspace{0.5cm} (26)  \\
and the state matrix is \(-A\).

3. Connect a dissipation element on the respective junction to each storage element, the value of this new element is \(s\) multiplied by the value of the given storage element where \(s\) is the Laplace operator. The state variable representation of this new bond graph called Transformed Bond Graph (TBG) is defined by

\[ x_s(t) = A_sx_s(t) + Bu(t) \]  \hspace{0.5cm} (27)  \\
\[ y_s(t) = C_sx_s(t) + Du(t) \]  \hspace{0.5cm} (28)  \\
where

\[ A_s = (sI - A) \]  \hspace{0.5cm} (29)  \\
\[ C_s = -C \]  \hspace{0.5cm} (30)  \\
The realization of this new system is \((A_s, B, C_s, D)\).

4. Obtain the bond graph in a derivative causality assignment called Frequency Bond Graph (FBG) from TBG, then the state variables in the Laplace domain are

\[ x(s) = F^{-1}B_s^*u(s) \]  \hspace{0.5cm} (31)  \\
and the system outputs in the Laplace domain are

\[ y(s) = D_s^*u(s) \]  \hspace{0.5cm} (32)  \\
where \(B_s^*\) and \(D_s^*\) are determined by (16) and (18) with FBG.

By using the relationship (21) with (29), (26) and (31) we prove (24). From (23), (29), (30) and (32) we prove (25).

Next section describes how to obtain the frequency bond graph of two examples applying the proposed methodology.

5 Examples

Example1. Consider the DC motor, the power in the armature circuit is transduced to shaft power, the field port establishes a magnetic field, which provides the coupling between electric variables and mechanical variables for the individual conductors on the rotor. Fig. 3 shows a scheme of a DC motor.

![DC Motor and its bond graph](image1)

A bond graph model in an integral causality assignment of the DC motor is shown in Fig. 4.

![BGI of the DC motor](image2)

The key vectors of the BGI are

\[ x = \begin{bmatrix} p_2 \\ p_6 \end{bmatrix}; \dot{x} = \begin{bmatrix} e_2 \\ e_6 \end{bmatrix}; z = \begin{bmatrix} f_2 \\ f_6 \end{bmatrix} \]

\[ D_{in} = \begin{bmatrix} f_3 \\ f_7 \end{bmatrix}; D_{out} = \begin{bmatrix} e_3 \\ e_7 \end{bmatrix}; u = e_1 \]
the constitutive relations

\[ L = \text{diag}\{R_a, b\} \]  \hspace{1cm} (33)

\[ F = \text{diag}\left\{ \frac{1}{L_a}, J \right\} \]  \hspace{1cm} (34)

and the junction structure is

\[ S = \begin{bmatrix}
  0 & -n & -1 & 0 & 1 \\
  n & 0 & 0 & -1 & 0 \\
  1 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0
\end{bmatrix} \]  \hspace{1cm} (35)

The transformed bond graph is shown in Fig. 5. Here, each storage element has a causal loop with the new resistor and the value of each storage element is the negative of the original value, with this the state matrix of this system is \((sI - A)\).

The key vectors of the frequency bond graph are

\[ D_{\text{ind}}^s = \begin{bmatrix}
  e_3 \\
  e_7 \\
  f_8 \\
  f_9
\end{bmatrix}^T \]

\[ D_{\text{outd}}^s = \begin{bmatrix}
  f_2 \\
  f_6 \\
  e_3 \\
  e_7 \\
  f_8 \\
  f_9
\end{bmatrix}^T \]

and the constitutive relation is

\[ L_d^s = \text{diag}\left\{ \frac{1}{R_a}, b, sL_a, sJ \right\} \]  \hspace{1cm} (36)

the junction structure of the frequency bond graph is

\[ \begin{bmatrix}
  f_2 \\
  f_6 \\
  e_3 \\
  e_7 \\
  f_8 \\
  f_9
\end{bmatrix} = \begin{bmatrix}
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  -1 & 0 & 0 & -n & -1 & 0 \\
  0 & -1 & n & 0 & 0 & -1 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
  e_2 \\
  e_6 \\
  f_3 \\
  f_7 \\
  e_8 \\
  e_9
\end{bmatrix} \]

By substituting (36) and (37) into (16), we have

\[ B_s^* = \frac{1}{\Delta} \begin{bmatrix}
  J & s + b \\
  n & J
\end{bmatrix} u(s) \]  \hspace{1cm} (38)

where \(\Delta = JL_a s^2 + S (JR_a + bL_a) + bR_a + n^2\).

From (31), (34) and (38), the state variables response in the frequency domain is given by

\[ x(s) = \frac{1}{\Lambda} \begin{bmatrix}
  s + b \\
  n
\end{bmatrix} u(s) \]  \hspace{1cm} (39)

where

\[ \Lambda = s^2 + \frac{s}{L_a} + \frac{bR_a + n^2}{JL_a} \]  \hspace{1cm} (40)

Example 2. Let us consider the mechanical system shown in Fig. 7 corresponding to the suspension of a car. The bond graph representation is shown in Fig. 7 b). The input vector is composed of a flow source and two effort sources associated with gravity. The inertial, capacitive and resistive elements associated with masses, springs, stiffness, damping coefficients are linear.

The key vectors of the frequency bond graph are

\[ \begin{bmatrix}
  q_2 \\
  p_6 \\
  q_9 \\
  p_{12}
\end{bmatrix}^T \]

\[ \begin{bmatrix}
  f_2 \\
  e_6 \\
  f_9 \\
  e_{12}
\end{bmatrix} \]

and the state vector are

\[ \begin{bmatrix}
  q_2 \\
  p_6 \\
  q_9 \\
  p_{12}
\end{bmatrix} ; \begin{bmatrix}
  f_2 \\
  e_6 \\
  f_9 \\
  e_{12}
\end{bmatrix} ; \begin{bmatrix}
  e_2 \\
  f_3 \\
  f_7 \\
  e_8 \\
  e_9 \\
  f_1
\end{bmatrix} ; \begin{bmatrix}
  f_1 \\
  e_4 \\
  e_{11}
\end{bmatrix} \]

This bond graph is of the order 4 and the key vectors are

\[ x = \begin{bmatrix}
  q_2 \\
  p_6 \\
  q_9 \\
  p_{12}
\end{bmatrix}, x = \begin{bmatrix}
  f_2 \\
  e_6 \\
  f_9 \\
  e_{12}
\end{bmatrix}, z = \begin{bmatrix}
  e_2 \\
  f_3 \\
  f_7 \\
  e_8 \\
  e_9 \\
  f_1
\end{bmatrix}, u = \begin{bmatrix}
  f_1 \\
  e_4 \\
  e_{11}
\end{bmatrix} \]
where \( f \) is the velocity and \( e \) is the force in each element of the mechanical system; \( q_2 \) and \( q_9 \) are the translational displacements in \( C : 1/k_1 \) and \( C : 1/k_2 \) respectively; \( p_6 \) and \( p_{12} \) are the translational momentums \( I : m \) and \( I : M \), respectively; \( f_1 \) is the input velocity and \( e_4 \) and \( e_{11} \) are the gravity force under \( m \) and \( M \) respectively.

The constitutive relations for the elements are
\[
F = \text{diag}\left\{ k_1, \frac{1}{m}, k_2, \frac{1}{M} \right\} \tag{41}
\]
\[
L = b \tag{42}
\]

The junction structure of the system is given by
\[
S = \begin{bmatrix}
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{43}
\]

Fig. 8 shows the transformed bond graph of the mechanical system obtaining \((sI - A)\).

![Fig. 8. TBG of the mechanical system.](image)

We assign derivative causality to the transformed bond graph to get the frequency bond graph, which is shown in Fig. 9.

![Fig. 9. Frequency bond graph of the suspension.](image)

The key vectors of the dissipation field for the frequency bond graph are
\[
D_{\text{ind}}^s = \begin{bmatrix} f_8 & e_{13} & f_{14} & f_{15} & e_{16} \end{bmatrix}^T
\]
\[
D_{\text{outd}}^s = \begin{bmatrix} e_8 & f_{13} & e_{14} & e_{15} & f_{16} \end{bmatrix}^T
\]

with the constitutive relation given by
\[
L_s^s = \text{diag}\left\{ b, \frac{s}{k_1}, sM, \frac{s}{k_2} \right\} \tag{44}
\]

and the junction structure is
\[
J_{11}^s = \begin{bmatrix} 0 & 1 & 0 & 1 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
-1 & 0 & -1 & 0
\end{bmatrix}; J_{13}^s = \begin{bmatrix} 0 & -1 & -1 \\
1 & 0 & 0 \\
0 & 0 & -1 \\
0 & -1 & 1 & 0
\end{bmatrix}
\]
\[
J_{22}^s = \begin{bmatrix} 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 \\
0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 1
\end{bmatrix}; J_{13}^s = \begin{bmatrix} 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1
\end{bmatrix}
\]
\[
J_{12}^s = \begin{bmatrix} 0 & 0 & 1 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 1
\end{bmatrix}; J_{31}^s = \begin{bmatrix} 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]
\[
J_{32}^s = \begin{bmatrix} 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}; J_{33}^s = 0 \tag{45}
\]

From (18), (32), (41) and (45), the system output in the frequency domain is
\[
y(s) = \frac{1}{\Delta} \left[ s^2 Mk_1 \quad s^3 M \quad -s^3 m \quad -sk_1 \right] u(s) \tag{46}
\]
where
\[
\Delta = s^4 + s^3 \left( \frac{b}{M} + \frac{b}{M} \right) + s^2 \left( \frac{k_1}{m} + \frac{k_2}{M} + \frac{k_2}{m} \right) + s \frac{bk_1}{mM} + \frac{k_1 k_2}{mM} \tag{47}
\]

6 Conclusion

The frequency response of a system in a bond graph approach is presented. This approach proposes a procedure to construct a bond graph in a derivative causality assignment and additional dissipation elements. Hence, this new bond graph gives the responses of the state variables and transfer matrix in a direct way. In robust control analysis of systems modelled by bond graphs, it is possible that, this approach can have a great potential.
References


