ROBUST CASCADE CONTROL OF ELECTRIC MOTOR DRIVES USING PI OBSERVERS

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ABSTRACT
This paper presents a new practical control structure for electric motor drives under parameter uncertainty. Since the conventional PI-type controller requires accurate information on the motor parameters and load conditions to achieve the desired performance, a reduced-order observer-based perturbation estimator has been proposed with the PI controller to preserve the nominal performance of the conventional method. Analysis on the robust performance of the perturbation observer is presented by using the singular perturbation theory. Both computer simulations and experimental results verify that the proposed controller can be used effectively as an additional compensator to the PI-type cascade scheme for electric motor drives.

KEY WORDS
Robust control, PI observer, Electric motor drive, Cascade control

1 Introduction

In a large number of industrial applications the PI (proportional-integral) controller has played a dominant role thanks to its reliable performance and relatively easy implementation. Conventional PI-type controllers have been widely applied to motor drives and shows robustness against modeling errors [1, 2]. However, parameter uncertainty is one of main obstacles to obtain the desired nominal performance of the control system. This paper presents a new practical control structure for robust control of electric motor drives to preserve the nominal performance of the conventional PI-type cascade controller under parameter uncertainty.

A conventional controller design for motor drives consists of a cascade configuration of two PI controllers. One is for speed control and the other for current control [2]. Since the scheme considers two separate first-order systems, the desired closed-loop dynamic specifications can be satisfied by using two PI controllers if the system parameters are exactly known. Another benefit of the cascade configuration is that the design process allows for the logical saturating function to simply consider the limits on input voltage and the current that can be allowed to flow in the armature windings.

In order to alleviate the performance degradation owing to the parameter uncertainties, this paper presents a perturbation estimator-based compensation method by constructing two simple reduced-order PI observers (PIO’s) that are robust against external disturbances [3]. Since the design process of the observers are decoupled from each other, the PIO for the current control loop does not use the information of the mechanical part of the system. The observer for the speed control loop uses neither electrical parameters nor unknown load torque $T_L$.

For electric motor control systems Luenberger observer-based estimation schemes have been tried to cope with unknown load torque disturbances [4, 5, 6, 7]. Additional feedforward compensation using the estimation has enhanced robust transient performance against parameter uncertainties as well as the slowly varying disturbance load torque $T_L$.

Since the load torque observer has been utilized without rigorous analysis on the robust property against parameter uncertainties, however, this paper presents an analysis on the robust performance of the observer by using the singular perturbation theory [8]. Transfer functions of the augmented system show the robust properties of the proposed approach in the steady state. The result can be also extended to the position control problem as well.

The proposed controller design process can be divided into two stages. In stage one, nominal cascade PI controllers are designed using technical optimum scheme [1] to achieve the control objectives in the absence of uncertainties. In stage two, the reduced-order PI observers are constructed to reserve the nominal performance of the cascade control approach.

Computer simulations and experimental results show the effectiveness of the proposed method compared to classical PI controllers. The main feature of the proposed algorithm is that it can cope with the parameter uncertainties as well as the disturbances by using two simple perturbation estimators. Whilst the advanced control schemes, e.g. $H_\infty$ approach, are often restricted in practice by their complex configurations and multiple tuning parameters, the proposed method can be easily implemented in practical applications e.g. electric power steering system [9, 10] due to its low dynamic order and robust property.
2 Preliminaries

2.1 System Model

This paper deals with the current and velocity control of a DC motor represented by
\[ v_a = R_a i_a + L_a \frac{di_a}{dt} + K_b \omega_m, \]
\[ \tau = K_e i_a = J_m \frac{d\omega_m}{dt} + B_m \omega_m + T_L, \]
where \( v_a \) is the input voltage; \( i_a \) is the armature current; \( R_a \) and \( L_a \) represent the armature resistance and inductance, respectively; \( \omega_m \) is the rotor angular velocity; \( K_b \) is the back-EMF (Electromotive Force) constant; the torque \( \tau \) is related to \( i_a \) by the torque constant \( K_t \); \( J_m \) and \( B_m \) represent the rotor inertia and the friction coefficient, respectively; \( T_L \) implies the unknown load torque. In (1) the back-EMF is supposed to be proportional to the velocity \( \omega_m \) [11]:
\[ e_b = K_b \omega_m. \]

Since all industrial processes are subject to modeling errors, this paper assumes parameter uncertainties in (1)-(2).

2.2 Cascade Control & Technical Optimum

A conventional cascade control system consists of an inner current control loop and an outer speed control loop. And both control loops typically contain PI controllers. The outer-loop controller (PI1) generates the current reference \( i_a^* \) for the inner-loop control (see Fig. 1). The inner-loop manipulates the input voltage \( v_a \) to control the current \( i_a \) to tracks the reference \( i_a^* \). This paper designs the PI controllers by using the technical optimum scheme [1, 2].

With the decoupling term of the back-EMF \( e_b \), the PI controller for the current loop can be designed as follows:
\[ v_a = \left( K_{pc}s + K_{ic} \right) (i_a^* - i_a) + K_b \omega_m \]
where \( s \) is the Laplace variable and \( i_a^* \) is the reference input. When \( K_{pc} = \omega_e L_a \) and \( K_{ic} = \omega_e R_a \), the closed-loop system (1)-(4) is obtained by
\[ \frac{i_a(s)}{i_a^*(s)} = \frac{\omega_e}{s + \omega_e} \]
where \( \omega_e \) is the desired bandwidth of the current loop (see Fig. 1).

Similarly, for the speed control loop, the control input yields the following transfer function
\[ \frac{\omega^*_m(s)}{\omega_m^*(s)} = \frac{\omega_s}{s + \omega_s} \]

\[ \omega^*_m(s) = \omega_s \]

Figure 1. Cascade control system

where the PI controller gains \( K_{ps} = \omega_s J_m / K_t, \)
\( K_{is} = \omega_s B_m / K_t \); and \( \omega_s \) is the desired bandwidth of the outer-loop system.

In the cascade control approach the value of \( \omega_s \) is assumed to be much smaller than that of \( \omega_e \), and (5) can be regarded as one for the outer-loop controller design.

Though the control inputs (4) and (6) have compact forms and are easy to implement, they require rather exact values of the system parameters and unknown disturbance \( T_L \).

3 Main Results

In order to remove the effect of the disturbance \( T_L \) in (2), a load torque observer has been considered. Since the load torque varies very slowly in practice, it is supposed to be an unknown constant [4, 5, 6, 7]. By using the additional integral of the estimation error to the Luenberger observer, the PI (or extended) observer has been successfully applied to estimate the disturbance (see e.g. [3, 12, 13, 14] for more details on PI observer). This section investigates whether the additional feedforward compensation using the PI observer can enhance the robust transient performance against parameter uncertainties as well as the slowly varying disturbance.

3.1 Design of PI Observer

We first show that the dynamic feature of the system in Fig. 2 with the estimation \( \hat{d} \) is similar to that of the nominal system of Fig. 3 if the gain of the proposed observer has sufficiently large positive value. The parameters \( a_n \) and \( b_n \) in Fig. 3 are the nominal values of the real parameters \( a_r \) and \( b_r \) of Fig. 2, respectively.

The system in Fig. 2 can be represented by
\[ \dot{y} = -a_r y + b_r (u - \hat{d}) \]

Figure 2. Feedforward compensation using PI observer
where the control input \( u = u_r + \hat{d} \). To construct a PI observer the above equation is rewritten as
\[
\begin{bmatrix}
\hat{y}
d
\end{bmatrix} = \begin{bmatrix}
-a_n & -b_n \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\hat{y} 
d
\end{bmatrix} + \begin{bmatrix}
b_n \\
0
\end{bmatrix} u
\tag{9}
\]
where the nominal values are used and the disturbance \( d \) is supposed to be an unknown constant \([4, 5, 6, 7]\).

Since the system (9) is observable w.r.t. the output \( y \) \([15]\), the reduced-order observer can be designed to estimate the disturbance \( d \) as follows:
\[
\dot{\hat{d}} = l(d - \hat{d})
= l \left[ \frac{1}{b_n} (-\hat{y} - a_n y + b_n u) - \hat{d} \right]
\tag{10}
\]
where the observer gain \( l > 0 \). Then, we can obtain
\[
\dot{\hat{d}} = \frac{d}{s + l} \hat{d}
\tag{11}
\]
where \( s \) is the Laplace variable.

The estimation \( \hat{d} \) is used for a feedforward compensation as shown in Fig. 2. If there is no parameter uncertainty, i.e., \( a_n = a_r \) and \( b_n = b_r \), the effect of \( d \) can be successfully attenuated from (8) because \( \hat{d} \) converges to the real value.

In order to implement the observer (10) without using the unmeasurable state \( \hat{y} \), a new variable \( \xi \) is defined as
\[
\xi = \hat{d} + \frac{l}{b_n} y, \text{ or, } \dot{\hat{d}} = \xi - \frac{l}{b_n} y.
\tag{12}
\]

Using (12) we can obtain the following equation
\[
\dot{\xi} = \frac{l}{b_n} (-a_n y + b_n u - b_n \hat{d})
= -l \xi + \frac{l}{b_n} (-a_n y + l) y + l u.
\tag{13}
\]

In the next subsection we show that the reduced-order observer (13)-(12) can deal with parameter uncertainties as well as the unknown load torque by using the singular perturbation theory.

### 3.2 Robust Performance Using PIO

To investigate the robust performance of the load torque observer against parameter uncertainties the system equation is transformed into the standard singular perturbation form \([8]\). Using (8) the equation (10) is rewritten as
\[
\dot{\hat{d}} = l \left[ \frac{1}{b_n} (-\hat{y} - a_n y + b_n u) - \hat{d} \right]
= l \frac{1}{b_n} \left( a_r y - b_r (u_r + \hat{d} - d) - a_n y + b_n u_r \right).
\tag{14}
\]

Hence, the singular perturbation form can be given by
\[
\begin{cases}
\dot{\hat{y}} = -a_r y + b_r (u_r + \hat{d} - d) \\
\dot{\hat{d}} = \frac{a_r}{b_n} y - b_r (u_r + \hat{d} - d) - \frac{a_n}{b_n} y + u_r
\end{cases}
\tag{15}
\]

The boundary-layer system is stable when the signs of \( b_r \) and \( b_n \) are equal. Then, the quasi-steady-state solution of (15) satisfies
\[
-a_r y + b_r (u_r + \hat{d} - d) = -a_n y + b_n u_r.
\tag{16}
\]

The above equation enables us to derive the quasi-steady-state system as follows:
\[
\dot{y} = -a_n y + b_n u_r.
\tag{17}
\]

Since the quasi-steady-state system is the nominal system without the load torque disturbance and parameter uncertainties, it can be said that the system (8)-(13) behaves like the nominal system as \( l \to \infty \).

### 3.3 Transfer Functions of Augmented System

We can find the transfer function \( G_{yu_r}(s) \) from \( u_r \) to \( y \) when \( d = 0 \). The augmented system with the PIO is given by
\[
\begin{cases}
\dot{\hat{y}} = -(a_r + l \frac{b_r}{b_n}) y + b_r \xi + b_r u_r \\
\dot{\xi} = -l \frac{b_r}{b_n} y + l u_r
\end{cases}
\tag{18}
\]

The transfer function is obtained by
\[
G_{yu_r}(s) = \frac{b_r (s + l)}{s \left[ s + (a_r + l \frac{b_r}{b_n}) \right] + l \frac{b_r}{b_n}}.
\tag{19}
\]

Note that the DC gain of (18) is \( G_{yu_r}(0) = b_n/a_n \). This is the same as that of the nominal transfer function.

To find the transfer function \( G_{yd}(s) \) from \( d \) to \( y \), we consider the augmented system when \( u_r = 0 \):
\[
\begin{cases}
\dot{\hat{y}} = -(a_r + l \frac{b_r}{b_n}) y + b_r \xi - b_r d \\
\dot{\xi} = -l \frac{b_r}{b_n} y.
\end{cases}
\tag{20}
\]

It can be shown that the transfer function is given by
\[
G_{yd}(s) = \frac{-b_r s}{s \left[ s + (a_r + l \frac{b_r}{b_n}) \right] + l \frac{b_r}{b_n}}.
\tag{21}
\]

Note that the transfer function has one zero at the origin, which means the steady-state response to the constant disturbance yields zero.

The next proposition summarizes the result on the PI observer-based compensation method of the previous subsections.

**Proposition 1** In the presence of the load torque disturbance and parameter uncertainties at the same time, the load torque observer (13) renders the augmented system to behave like the nominal system of Fig. 3 when \( l \gg 1 \).
3.4 Proposed Controller

According to a common practice in motor drive controller designs, this paper employs the cascade control structure composed of two separate PI controllers (PI₁ and PI₂). However, the conventional PI-type controllers (4) and (6) require the accurate information regarding the motor parameters and load conditions. In order to counteract the performance degradation owing to parameter uncertainties and to preserve the main advantages of the conventional cascade approach at the same time, we present a new compensation method by adopting the simple first-order disturbance estimator (13) that is robust against parameter uncertainties.

In Fig. 4 the estimations \( \hat{d}_1 \) and \( \hat{d}_2 \) are obtained from two different first-order PI observers (PIO₁ and PIO₂), respectively. To construct the reduced-order observers we consider the following system model from (1)-(2).

\[
\begin{align*}
\dot{v}_a &= \hat{R}_a i_a + \hat{L}_a \frac{di_a}{dt} + \hat{d}_2, \\
\hat{K}_t i_a &= \hat{J}_m \frac{d\omega_m}{dt} + \hat{B}_m \omega_m + \hat{K}_t \hat{d}_1
\end{align*}
\]  

(22)  

(23)

where \( \hat{R}_a, \hat{L}_a, \hat{K}_b, \hat{K}_t, \hat{J}_m \) and \( \hat{B}_m \) represent the nominal values of the system parameters. We can refer to Tab. 1 that compares two control loops for (13). It is noted that the disturbance \( \hat{d}_2 \) contains the back-EMF term \( (K_t \omega_m) \).

The proposed controller design process can be divided into two stages. In stage one, nominal cascade PI controllers are designed by using the technical optimum scheme to meet the desired performance specifications without uncertainties. In stage two, the reduced-order PI observers are constructed to preserve the nominal performance of the cascade control approach.

4 Experiments

This section tests the performance of the proposed controller through computer simulations and laboratory experiments. All the computer simulations have been performed with Matlab/Simulink. The real system parameters for the simulations are listed in Tab. 2. Fig. 5 shows a one-link
manipulator for the experimental tests of the DC motor controller. In experiments the digital controllers are implemented using a TI DSP TMS320F28335 and the sampling frequency is 40 kHz.

![Figure 6. Simulation results for current loop (proposed: w/ PIO)](image)

(a) $\hat{R}_a = 0.6$

(b) $\hat{R}_a = 0.3$

(c) $\hat{R}_a = 0.1$

Figure 6. Simulation results for current loop (proposed: w/ PIO)

4.1 Current Control

We first consider the inner-loop current control problem. To test the transient response the reference is a square wave ($\pm 1[A]$). The gains of the controller $PI_2$ have been designed with $\omega_c = 1000$. The results of Fig. 6 show that the back-EMF compensation is necessary for the current control to achieve the control objective. Based on the parameters of Tab. 2, the gain of PI observer for inner-loop($PIO_2$) is chosen as $l = 10000$. The proposed method achieves much faster rise times for both positive and negative step demands. To verify the robust performance against parameter uncertainties various nominal values of $\hat{R}_a$ are tried e.g. 0.6, 0.3, and 0.1. Fig. 6 shows the improved performance of the proposed method compared to the conventional PI controller.

The experimental results are shown in Fig. 7. We can see that the proposed approach has been successfully combined with the conventional PI controller to maintain the robust performance against parameter uncertainties. Both results ensure that the proposed algorithm achieves the control objective preserving the transient response without steady state errors.

![Figure 7. Experimental results for current loop (proposed: w/ PIO)](image)

(a) $\hat{R}_a = 0.6$

(b) $\hat{R}_a = 0.3$

(c) $\hat{R}_a = 0.1$

Figure 7. Experimental results for current loop (proposed: w/ PIO)
4.2 Velocity Control

Performance of the proposed controller for the velocity control has been tested through various computer simulations. The gains of the controller \( PI_1 \) have been designed with \( \omega_s = 200 \). Using the parameters of Tab. 2 the PI observer for velocity control loop \( (PIO_1) \) can be designed. The observer gain \( l = 1000 \) in the simulation.

To verify the robust performance against parameter uncertainties the parameters \( R_a = 0.6 \) and \( J_m = 2 \times \dot{J}_m \) are used in the simulations. The load torque \( T_L = 0.1 \) was applied at 0.05[sec].

When the reference \( \omega^*_m = 200 \), the results in Fig. 8 show that the proposed method preserves the nominal performance when both of the PI observers \( (PIO_1 \) for outer-loop and \( PIO_2 \) for inner-loop) are used with the conventional PI controllers.

We can see that the proposed approach has been successfully combined with the conventional PI controller to maintain the robust performance against parameter uncertainties. Both results ensure that the proposed algorithm achieves the control objective preserving the transient response without steady state errors.

5 Conclusion

A new practical control structure for robust control of electric motor drives has been proposed to preserve the nominal performance of the conventional PI-type cascade controller under parameter uncertainty. A perturbation estimator using the reduced-order PIO has been combined with the PI controller to enhance the robustness.

The PIO-based unknown load torque observer has been widely employed for motor drive controllers without explicit analysis on the robust performance against parameter uncertainty. This paper presents an analysis on the robust performance of the reduced-order PIO by using the singular perturbation theory.

Through computer simulations and experimental tests it has been shown that the proposed PIO-based controller can be used effectively as an additional compensator to the conventional PI-type cascade controllers. Using the proposed method for electric motor drives a real uncertain plant is forced to behave like the nominal one.

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References


