ABSTRACT
A web-based learning system is a convenient vehicle to integrate structured learning materials with interactive graphics, which connects symbolic and graphic representations through direct manipulation by students. However, it often needs some mechanisms to attract students’ attention when the system is used at extra curricular activities. As the attraction, we adopted a virtual game played by 40 students in class and combined it with the learning system of 3D vector equations. The game is a tournament of battles fought among teams of avatars. And the winner of a battle is calculated by an operation of 3D characteristics vectors of avatars and the battlefield. The students who want to win the battles try to collect training points for their avatars by doing exercises on the learning system. Configuration and preliminary effect of the combined system are presented in this talk.

KEY WORDS
Software and hardware systems for web-based education, Linear algebra, Conceptual understanding, Virtual game, and Interactive graphics

1. Introduction

In mathematics both analytic and visual reasoning are important. Analytic reasoning is based on the use of symbolic representations and construction of logical inference chains. Instead, visual reasoning is based on visual interpretations of mathematical concepts. Visualization makes it possible to perceive abstract mathematical objects through senses. Visual representations can be considered more concrete than analytic ones, because they are based on external objects. Analytic reasoning is often exact and detailed, but visual reasoning is needed to reveal wider trends of the whole problem solving process and holistic features of the problem situation. Often visualization acts as a map showing a direction for the reasoning process [1]. But according to Stylianou and Dubinsky [2], many students have difficulties in analyzing visual representations, and therefore, they cannot utilize them in problem solving. Interactively changing graphics have a potential to change this situation by connecting visual and symbolic representations closely together to help deepening students’ conceptual understandings of mathematics. For example, if a student is given the opportunity to manipulate a 3D graphic object in virtual space by changing parameters of the vector equation, he recognizes more easily the relation how the parameters influence on the graphic object through his experiment. Because such interactive manipulation of 3D graphic objects is difficult to trace on chalkboards or on papers, it is an advantage of 3D graphics displayed on computer screens. However, students need additional explanation how a change in symbolic expression transforms the graphic object because they are not accustomed to such direct link used in the lectures of linear algebra. The explanation must include 2D similar examples and 3D special cases where the symbolic expressions include fewer independent variables. Each piece of explanation must be linked closely to the graphics and to each other, so students could go back and forth between the pieces to build up their personal ideas.

A web-based system becomes the natural solution to integrate both interactive graphics and additional explanation linking them together in a system. The web-based learning system also needs something to attract students’ attention when the system is used at extra curricular activities. Novice students tend to avoid unfamiliar learning activities if there is nothing that interests them. Students would show interest if there was rich opportunity to share their ideas and emotions among friends when they encountered a new mathematics content to learn.

In this paper, we tried to attract students’ attention by a virtual game, where 40 students in a class participate and share their experiences, to the web-based learning. Students could be motivated to use the learning system hoping to perform better in the game. The game itself works as an introduction to vector operations, and shows an actual application of 3D vectors in the real world.

2. Deepening Students’ Understanding in 3D Vector Equations

The web-based learning system has been developed to assist our new teaching approach for improving poor performance of our students in linear algebra, especially
thought the knowledge is well known to the public, and the discovered knowledge is connected to rich personal experience. Such knowledge is usually more structured, and robust, and thus valuable to the students.

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3. A Virtual Game

3.1 Game as an Application

We have originally developed the virtual game as the introductory application of 3D vector equations. When students have some experience of handling and observing graphical images of vectors in an actual application, they are expected to feel reality in vectors, have big pictures of them, and are ready to learn the abstract concepts and methods. As an introductory application, the game does not show any symbolic representations. Instead, it focused to show visual relations between graphical objects. A virtual game should appeal to most young Japanese students who have been playing video games regularly. 3D vectors are introduced as the means to explain the rule of the virtual game. The students who want to perform well in the game will analyze the rule and find the patterns and underlying mechanisms, which are the mathematical ideas and methods we want to teach them. In the virtual game, teams of students’ avatars compete each other in a tournament of different kind of battles, for example, boxing, solving puzzles, or singing competitions. The characteristics of battlefields and avatars are expressed with 3D vectors. Avatars have graphic images, for example animals or robots, and characteristics vectors...
as shown in Figure 1. Every student designs his own avatar by selecting the avatar-type from four candidates and adjusting its characteristics vector. An operation of those vectors decides the winning and losing of a battle, and result of the battle is also shown graphically. Because the winner of each battle is decided by vector operations and shown in 3D graphic images, students are accustomed to use vectors before they learn the mathematical definition of vectors.

![Figure 1. Designing an avatar for the game](image1)

In Figure 2, graphical images of the competing avatars are shown in the upper left corner, characteristics vectors of the avatars and the battlefield are shown in the upper right corner, and two lists of characteristics vectors, which belong to the avatars of both teams, are shown at both lower corners. The buttons, shown at the lower central part of the screen, control the game.

![Figure 2. Displaying on-going virtual game](image2)

In operation, we divide 40 students into eight groups, and let them design their own avatars in the computer laboratory before the lesson. They select a graphic image from four candidates and adjust the characteristics vectors. Because length of the vectors is fixed by the system, with a small fluctuation to avoid completely the same values, students are only allowed to change the directions of the vectors as shown in Figure 1. By selecting the directions of their own avatars, students are accustomed to three elements of vectors and their influence on the direction of the vectors.

In a lesson, the on-going battle is projected onto the big screen, and all the students in class observe the game. Two teams are called at a battle, and each selects the right avatar for the battle from their member lists. Winner/loser of a battle is displayed in the upper right corner of the screen using arrows and their projected shadows. Waiting teams and already-lost teams observe the on-going battles, analyze the vector field, and try to predict the winner or to plan their own tactics in the next battles.

3.2 System Configuration of the Game

The virtual game is conducted with three computers, the server and two terminals, connected by a wireless network. Mathematica or Mathematica Player Pro [8] must run on each machine. The server has a database that holds all the information related to the battles and avatars, which have characteristics vectors and special items obtained by training points accumulated on the learning system. Latest data of the avatars must be retrieved from the learning system’s database just before the battle. The server program, which the instructor operates, retrieves the avatars’ data, and manages the game. It also projects the on-going result to a large screen in the classroom through a data projector.

Terminals are used by the two teams in a battle to examine the characteristics of the battlefield and their candidate avatars, to select the player, and consider if they use one of the special items in this battle or keep it for the coming battles. Terminal programs have connections to the database on the server, read and rewrite the data through the wireless network.

3.3 Expected Effect of the Game

Because the game is conducted in a classroom, all the students can immediately share their impression and discuss their ideas with classmates, which rarely happens in traditional Japanese mathematics lessons. Through the experience of designing avatars, and joining and observing battles, students will gradually be accustomed to graphical and symbolic representations of 3D vectors and their operations. Thinking about how the winners/losers are decided naturally leads students to think about qualitative nature of vector operations. And they become ready to start learning 3D vector equations after the game.

4. Interactive Worksheets on the Web

4.1 Content of the Worksheets

Our web-based learning system embeds interactive
worksheets for showing the close link between graphical and symbolical representations of mathematical functions and vector equations with their simultaneous changes [9]. For users to be able to reshape or reposition graphic objects, the worksheets contain interface components called Sliders.

Figure 3 shows an example how a slider is used in the worksheets. If a student clicks and drags the circular knob along the slider located at the left side of the screen, the blue dotted circle shown on the right side changes its radius r accompanying two vectors attached to it. The purple normal vector is elongated or shrunken with the radius without changing its direction. The blue parallel vector changes its starting point without changing its direction and length. From the simultaneous change of the graphic objects, student is expected to understand that the purple vector represents radius of the circle, the blue vector always shows the direction of the circle’s tangent line, and two vectors are always perpendicular to each other.

After dragging the knob for a while, some students recognize that there is also a red line on the screen. The blue vector is always parallel to this line, and the purple vector is always perpendicular to the line. The graphic objects imply that we could decide the position and direction of the red line as we define it as a tangent line to the circle, and actually the only graphic object we need is the purple vector: normal position vector to the line. Many students need additional explanation other than the dragging experience, of course, but the actual experience helps them to recognize the relation among the graphic objects more easily. We also would like to point that it also becomes easier to explain the relation symbolically if students understand the relation among graphic objects earlier. In the case of Figure 3, we could express the red line as \( ax + by = d^2 \) and the dotted circle as \( x^2 + y^2 = r^2 \).

The radius \( r \) of the circle is adjustable with the slider, and the line becomes a tangent line of the circle when \( r = d \). Figure 4 shows an extension of the approach to express a plane \( ax + by + cz = d^2 \) in 3D space. It shows a plane in the middle of the screen and its vector equation under it. The vector equation has two vectors connected by a dot on the left-hand side, which means an inner product of the two vectors, and a number \( d^2 \) on the right-hand side. Again the direction of the normal vector is fixed, and its length \( d \) between the plane and the origin is adjustable with the slider. When a students change \( d \) with the slider, the plane shifts the position without changing its direction, and the vectors and number \( d^2 \) of the equation changes all simultaneously. It also shows a position vector \( x \) which points any point on the plane as the blue arrow. By moving the 2D slider, a student can confirm that the end point of the blue arrow never leaves the plane even if the position vector change the three parameters vigorously.

Figure 4. Displaying normal vector to a plane in 3D space

Figure 5 shows an example exercise of relating 3D graphic objects to symbolic expressions. In this worksheet the target is given as a red plane; a graphic object, and the position and direction of the movable blue plane is adjustable with four sliders on the left side of the screen.

Each of four sliders is related to a parameter of the equation \( ax + by + cz = D \) and the equation is displayed just under the graphic objects. It intends to show the difference of three parameters \( a, b, c \) on the left-hand side of the equation and the parameter \( D \) on the right-hand side. The left-hand side parameters change the direction of the plane but the parameter \( D \) does not affect the direction. It
only slides the plane along the blue arrow, which direction is decided by the left-hand side parameters. Students with little prior knowledge of graphic-to-symbolic relation should need many trials and errors to fit the blue plane to the red target. Special cases where the target plane is perpendicular or parallel to one of the axes are included in the exercise because they are relatively easier targets to adjust but become the key to understand the relation of the parameters and direction of the plane. The worksheet has two activities; matching the plane to the target graphically, and finding the values of parameters. However the latter activity is still just the confirmation because current 3D interface cannot display enough graphical information for the students to decide the parameters precisely, which is realized in 2D interface. Instead it displays the values just next to the sliders so the students can answer the values.

### 4.2 Technology Supporting the Worksheets

Interactive worksheets are built as Computable Document Format (CDF) files programmed with Mathematica language and embedded in webpages of the learning system. Terminal PCs of the learning system may be operated by Windows, Mac OS, or Linux and require Wolfram’s CDF players [8] installed as the plug-in to web-browsers for running the interactive worksheets. The worksheets can also run as stand-alone programs on a machine with the CDF player installed to it. As described in previous chapters, CDF files provide several interface components: buttons, pull-down menus, and sliders, for facilitating users to manipulate the graphic objects in worksheets interactively. On the other hand, it cannot have any means to communicate with the server directly. For the communications between the terminals and the server, we have to use usual HTML forms instead.

### 5. Combining the Game and Worksheets

#### 5.1 New Functionality of the Combined System

Virtual game was a modest success. It attracted students’ attention, conjured heated discussion between the students, and made 3D vectors a familiar learning material to them. However the rule of the game was too simple to analyze the winning tactics, and there were few space left to devise sophisticated ones. On the other hand, only a few students used the interactive worksheets as their extra curricular works, mainly because the worksheets’ educational values are unknown to the students. This is another reason why we decided to combine the game and interactive worksheets together. To do this we introduced the function to train avatars through exercises on the interactive worksheets. When a students use one of the worksheets in the learning system, training point is accumulated to his account, the added point depends on the amount and difficulty of the exercises. With the accumulated training points, the student can improve his own avatar by simply stretching the length of its characteristics vector, thus increase its general strength, or purchasing special items that can be used to add a power optimized to some type of battles. Both actions increase the avatar’s winning possibility at battles but in different ways, so the aggressive students may have more options to devise their winning tactics at the game. A team increases its winning potential at the tournament as many team-members train their avatars through the learning system.

Figure 6 shows how a student collects a training point. When he completes a worksheet, it displays a completion code of 36 digits, which includes user ID, date, duration of the exercise, and his performance at the tasks. When he sends the completion code to the server via the HTML form, a program on the server decodes the code, evaluates his performance, issues a training point, and adds it to his account.

With the combined learning system, we also changed our lesson plan to include two games; the first game is conducted at the first lesson just as the introduction, and the second game is done after a few months later for the opportunity where students could show their trained avatars to compete each other. The term between the two games is the training period where students use the interactive worksheets on the learning system to collect training points for their avatars.

#### 5.2 Preliminary Effect of the Combined System

We have introduced the virtual game in 2011 as the introduction to 3D vectors and the combined system in 2012. Students, especially slow learners, reflected positively to the combined learning activities of virtual game in class with a series of interactive manipulating experience using web-based learning system.

Table 2 shows the preliminary effect of the virtual game and combined system on students’ learning activities in the term. The students of 2011 showed higher efforts indexes for the pencil-and-paper based exercises in which students are requested to explain the difference of vector equations for lines and planes in 3D space. The efforts
index increased especially for the advanced exercise, in which many slow learners had given up easily in previous year. The students of 2012 also increased their final passing-rates for the advanced exercise. More slow learners continued their effort on the exercise and fewer students gave up during the term. The virtual game and combined system seem to have encouraged the students to learn 3D vector equations without giving up.

Table 2. Students’ activities at out-of-class exercises (Step 6) on 3D vector equations

<table>
<thead>
<tr>
<th>Activity of Step 1</th>
<th>Academic year (Spring semester)</th>
<th>Final passing-rate*</th>
<th>Efforts index **</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2010</td>
<td>Basic</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Advanced</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>2011</td>
<td>Basic</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Advanced</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>2012</td>
<td>Basic</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Advanced</td>
<td>1.05</td>
</tr>
</tbody>
</table>

*1 In step 6 of our lesson, pencil-and-paper exercise, students who didn’t get the passing mark at in-class quizzes are given additional opportunity to solve the problems at out-of-class exercises under the supervision of teaching assistants. Well-explained answer sheets at the exercises receives passing mark.

*2 Students are allowed to challenge the same problem in out-of-class exercises until they finally succeed. Efforts index becomes 1.0 if all the candidate students tried to solve the problem once in one of the out-of-class exercises. Success or fail does not matter. Lower efforts index than 1.0 means that some students gave up to tackle the problem.

However, we don’t know yet if these motivated students understood the concept deeper as we have not observed any positive influence of the virtual game or combined system on the students’ test scores. We still need to use the combined system and observe the students’ activities further, and to analyze their performance on comparable problems at term-end examinations or INCT achievement tests.

Current system also has limitations. Connectivity between the game system and learning system is rather awkward because of the hybrid structure of a Mathematica/Database management system (DBMS) for the game and a web-based DBMS system for interactive learning. It works fine with our current lesson plan, where the game is conducted only in a classroom equipped with a data projector and wireless LAN. But we prefer if the virtual game also functions on the web-based system. Then we could extend the game-based activities outside the classroom.

CDF, which current learning system depends on, operates rather heavily and the system needs fast-running machines for user terminal PCs as we implement more sophisticated functionality with longer source code. Worksheets sometimes stopped running when we used an older machine as the user terminal. We should divide the functions into several parts and separate a big worksheet into smaller ones to avoid such inconvenience.

6. Conclusion

We have combined a virtual game and web-based learning system to attract students’ attention to the interactive learning activities on the system. The virtual game has designed as an actual application of 3D vectors. And the interactive learning activities have developed to deepen students’ conceptual understanding of 3D linear algebra, which was the least performed mathematics content taught in engineering colleges in Japan.

The virtual game and combined system have motivated students to learn 3D vector equations with more effort, although the positive effect on test scores are not yet observed. We could say at least that it is a good introduction to rather abstract learning of 3D vector equations and their operations.

Interactive worksheets on the Web have a potential to replace a part of pencil-and-paper based exercises with more students’ focus on the relation among symbolic, descriptive and graphic expressions. They must also be effective on the learning of functions and their graphs, where the relation among symbolic, descriptive and (2D) graphic expressions plays vital role.

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