ABSTRACT
This paper presents a novel strategy to integrate the lateral and longitudinal control for an autonomous vehicle based on the preview control theory. The linear longitudinal and lateral control models are derived. From the previewed path, the path profile and the velocity profile are constructed. The controllers compute the steering angle and the wheel torque from the linear vehicle model and the path information. The responses of the nonlinear vehicle with additive noises are estimated by Kalman filters. The simulation shows that under disturbances the system may maintain a good path following and speed regulating.

KEY WORDS
Preview control, Kalman filter, autonomous vehicle

1. Introduction
Recently, autonomous vehicles represent an attractive topic that is intensively investigated and tested. The autonomous vehicle running on road reduces accidents due to human errors and improves the traffic flow on highway [1]. The design of an autonomous vehicle involves many challenges. In the control aspect, strategies to regulate vehicle speed and to follow a path are important.

Falcone et al (2007) [2] proposed a model predictive controller (MPC) for an active steering system. Two different MPC were presented and compared. The first one used a nonlinear vehicle model to compute the future states of the system. The second one was the linear time-varying (LTV) MPC based on the online linearization of the nonlinear vehicle. The LTV MPC of low order overcame the difficulty in computational time and yielded good performance.

Another approach to path following control was reviewed by Cole et al [3]. The predictive and linear quadratic controllers were used to model the driver steering control. Road path was previewed relative to the vehicle longitudinal axis and it contributed to generate the steering input. The controller configurations are based on the previewed control theory that incorporated the future information to improve the current performance.

A vehicle longitudinal control algorithm was developed by Han et al [4]. First, the experiments were conducted to obtain the time gap and range clearance during vehicle following for human drivers. Further, the desired acceleration was calculated from the time to collision, the speed of the preceding vehicle and the controlled vehicle. The focus to maintain a safe distance while following the preceding vehicle speed was attained.

The combination of longitudinal and lateral control is a challenge due to the nonlinear dynamics of the vehicle. Rajamani et al [1] developed a longitudinal and lateral strategy to control vehicles in platoons. The longitudinal control system determined the desired acceleration for each car and the commands required to accelerate or decelerate. The lateral control system ensured the lane-keeping or automatically steered the vehicle to an adjacent lane. Both lateral and longitudinal control system were treated as independent processes.

In this paper, we present a novel strategy to integrate the lateral and longitudinal control for an autonomous vehicle. The design of linear lateral and longitudinal controllers is based on the preview control theory. The future information of the path is included to enhance the performance of the system. The nonlinear vehicle model that represents the real vehicle is treated as a black-box with the existence of noises. Kalman filter estimates the vehicle states and provides feedback to the linear controllers. The simulations are performed with the assumption that the necessary path information is already obtained. The overall objective of this research is to develop a comprehensive controller configuration that may be implemented to an automated vehicle.

2. Lateral Controller Design

2.1 Profile of Path
The primary information for lateral and longitudinal planning in vehicular guidance is the road shape. We assume that all the necessary information required to identify the road and the potential obstacles is available in a consistent format to the controllers. From the sensors and cameras that the vehicle is equipped with and the reference to the digital map, the road geometry may be described by the centerline using the combination of straight line segments. A real path and an observed path are illustrated in Figure 1. The real path is represented by
the curve AC while the observed path includes two lines that intersect at B.

![Figure 1. A real path and an observed path](image)

The observed path, then, is discretized by \((N+1)\) points along the longitudinal axis of the vehicle with the interval \(UT\) where \(T\) denotes the time step and \(U\) is the vehicle speed. The path information is collected in the view of the moving-vehicle framework. If the lateral deviation of the vehicle from the centerline, \(\psi(k)\) and the head angle of the vehicle, \(\psi(k)\) are detected, the absolute position of the path is derived as:

\[
Y(k + n) = y(k) + nUT\psi(k) + y_\psi(k + n)
\]

where \(y_\psi(k+n)\) is relative position of the path, \(\psi(k)\) and the curvature of the path are assumed small. The vector \(Y(k) = [Y(k) \; Y(k+1) \ldots Y(k+N)]^T\) provides the adequate information about the lateral position of the current path. When the new road is sampled, the lateral profile of the path is updated as:

\[
Y(k + 1) = D_d Y(k) + E_d Y_1(k)
\]

Where \(Y_1(k)\) is considered as the single scalar input. \(D_d\) represents the matrix \((N+1) \times (N+1)\) while \(E_d\) stands for the vector \((N+1) \times 1\) as follows:

\[
D_d = \begin{bmatrix}
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & 1 \\
0 & 0 & 0 & \ldots & 0 \\
\end{bmatrix}
E_d = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1 \\
\end{bmatrix}
\]

2.2 Linear Model of the Vehicle and the Preview Control for Lateral Direction

The vehicle model approximates the real vehicle by the equations of motion. In order to apply the linear control theory to achieve the steering angle corresponding to the desired path, a linear vehicle model is used.

The linear bicycle model providing the lateral velocity, yaw rate, lateral displacement and head angle of the vehicle may be represented as [5, 6]:

\[
\begin{bmatrix} x(k+1) \\ y(k+1) \\ \psi(k+1) \\ \psi_\omega(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \\ \psi(k) \\ \psi_\omega(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{v}{u} \end{bmatrix} \omega_\psi(k + 1)
\]

In the matrix notation, equation (4) may be written as:

\[
\dot{x}(t) = A_d x(t) + B_d \delta_{sw}(t)
\]

The input of the vehicle system as the equation (5) is the steering angle that is derived from the observed path. The path is represented by discrete data. In the preview control, the lateral path is included to the vehicle system to improve its path following capability. It is necessary to discretize the continuous vehicle system in form of lateral path. The discrete version using a sampling period of \(T\) is:

\[
x(k+1) = A_d x(k) + B_d \delta_{sw}(k)
\]

where \(A_d = e^{A_d T}\), and \(B_d = \int_0^T e^{A_d T} d\tau\) . The discretization process assumes that the steering angle input \(\delta_{sw}(t)\) to the continuous vehicle is adjusted only at times \(kT\), and that it is held constant (zero-order hold) between modifications.

The vehicle motion on the road is a closed loop system, where the primary input to the vehicle is determined by the controller. The vehicle must follow the previewed path that is available. The performance of the vehicle in the path following problem is greatly improved by introducing the future path information as system states. Combining the discrete time equation (6) for the vehicle and the equation (2) for the road path yields:

\[
\begin{bmatrix} x(k+1) \\ Y(k+1) \end{bmatrix} = \begin{bmatrix} A_d & 0 \\ 0 & D_d \end{bmatrix} \begin{bmatrix} x(k) \\ Y(k) \end{bmatrix} + \begin{bmatrix} B_d & 0 \\ 0 & E_d \end{bmatrix} \delta_{sw}(k)
\]

In the compact form, equation (6) may be written as:

\[
z_{k+1} = A z_k + B \delta_{mk} + E y_k
\]

where \(z_k\) denotes the vector \((N+5)\times1\), \(z_k = \begin{bmatrix} x(k) \\ Y(k) \end{bmatrix} \), the matrix \((N+5)\times(N+5)\), \(A = \begin{bmatrix} A_d & 0 \\ 0 & D_d \end{bmatrix} \), \(B\), the vector \((N+5)\times1\), \(B = \begin{bmatrix} B_d \\ 0 \end{bmatrix} \). \(E\), the vector \((N+5)\times1\),

\[
E = \begin{bmatrix} 0 \\ E_d \end{bmatrix}
\]

The associated performance index is defined as the quadratic function [7]:

\[
J_y = \frac{1}{2} z_N^T P_N z_N + \frac{1}{2} \sum_{k=1}^{N-1} (Q_k z_k + \delta_{mk}^T R_{mk} \delta_{mk})
\]

where \(Q_k = C^T Q_k C\). \(Q_k = \begin{bmatrix} q_y & 0 \\ 0 & q_y \end{bmatrix}\), with \(q_y\) and \(q_y\) represent the weighting factors of lateral path and head angle, respectively. \(C\) denotes the matrix \(2 \times (N+5)\),

\[
C = \begin{bmatrix}
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 1 & \frac{1}{uT} & -\frac{1}{uT} & 0 & 0 & \ldots & 0
\end{bmatrix}
\]

By changing the weighting factors \(q_y\) and \(q_y\), the priorities in the cost function is defined.

The optimal problem is described by the performance index as the equation (9) and the constraint as the equation (8). Solving the problem means to determine the
optimal control sequence $\delta_{m0}$, $\delta_{m1}$, ..., $\delta_{mN}$ to minimize $J_c$. The optimal steering angle at the instant $k$ is:

$$\delta_{mk} = -K_m z_k$$  \hspace{1cm} (10)

where $K_m = (B^T P_{k+1} B + R_m)^{-1} B^T P_{k+1} A$. The matrix $P$ is computed from the Riccati difference equation defined as:

$$P_k = A^T \left[ P_{k+1} - P_{k+1} B (B^T P_{k+1} B + R_m)^{-1} B^T P_{k+1} \right] A + Q_y$$  \hspace{1cm} (11)

\section{3. Longitudinal Controller Design}

\subsection{3.1 Velocity Profile of Path}

Road is designed to ensure the continuity of its curvature [8]. The maximum curvature is determined by the centrifugal force depending on the recommended speed. Figure 2 illustrates a method to create the curvature for an observed path that includes line segments.

![Figure 2. The observed path and the curvature](image)

The curvature of a straight line is zero ($\kappa = 0$) while the curvature of a circle is constant ($\kappa = \frac{1}{R}$). To create a smooth transition between the line and the circle, a clothoid is used. The clothoid curvature increases from zero to $\kappa$, for the entrance of the curve or decreases from $\kappa$, to zero for the exit of the curve. The lengths ($l_1$ and $l_2$) of the path with varying curvatures and constant curvatures are chosen in the consideration of the speed limitation.

The speed of the vehicle is determined by the path curvature. The experiments conducted by Lee et al [9] reveals that normally, the speed of the vehicle never reaches the maximum speed corresponding to the limit of the centrifugal force. The drivers tend to choose the most comfortable speeds for their own that are lower than the limitation. In general, the lower the curvature, the higher speed is taken. For instance, the vehicle generic speed profile for the observed path is constructed as in Figure 3.

![Figure 3. The velocity profile of the path](image)

When new information of the path is available, the acceleration profile is updated as:  \hspace{1cm} (12)

$$a(k+1) = M_{iJ} a(k) + N_d a(k)$$

where $a(k)$ is considered as the single scalar input. $M_{iJ}$ represents the matrix $(N+1) \times (N+1)$ while $N_d$ stands for the vector $(N+1) \times 1$ as follows:

$$M_d = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad N_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$  \hspace{1cm} (13)

\subsection{3.2 Linear Model of the Vehicle and the Preview Control for Longitudinal Direction}

The linear model of a vehicle for the longitudinal motion is illustrated in Figure 4. The three degrees of freedom are the motion in the forward direction and the rotations of two wheels. The forces, $F_{sf}$ and $F_{sr}$ that drive the vehicle with a velocity $V$ are applied at the contact patch of the tires and the road surface. These forces are generated due to the slips between the wheels and the ground. For the simplicity, the longitudinal forces are approximated as the linear relations with respect to the difference of the vehicle velocity and wheel velocity as follows:

$$F_{sf} = C_{sf} (R \omega_f - V)$$

$$F_{sr} = C_{sr} (R \omega_r - V)$$  \hspace{1cm} (14)
Combining equations (15) and (16) and rearranging while the braking torque is applied on both front and rear wheels, the tracking torque is only applied on the front wheels that results in $F_{sf} > 0$. If the vehicle is braking, both braking force on front and rear wheel are negative, $F_{sf} < 0$ and $F_{sr} < 0$.

The vehicle motion is governed by the Newton second law as:

$$ m\ddot{V} = F_{sf} + F_{sr} $$  \hspace{1cm} (15)

The motions of the two wheels are represented by the equations:

$$ I_{sf}\dot{\omega}_f = -F_{sf}R + T_f $$
$$ I_{sr}\dot{\omega}_r = -F_{sr}R + T_r $$  \hspace{1cm} (16)

Combining equations (15) and (16) and rearranging yields:

$$ \begin{vmatrix} \dot{V} \\ \dot{\omega}_f \\ \dot{\omega}_r \end{vmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix} \begin{bmatrix} V \\ \omega_f \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$  \hspace{1cm} (17)

where:

$$ a_{11} = -\left(\frac{C_{sf} + C_{sr}}{m}\right), \quad a_{12} = \frac{C_{sf}R}{m}, \quad a_{13} = \frac{C_{sr}R}{m} $$
$$ a_{21} = \frac{C_{sf}R}{I_{sf}}, \quad a_{22} = -\frac{C_{sr}R^2}{I_{sf}} $$
$$ a_{31} = \frac{C_{sf}R}{I_{sr}}, \quad a_{33} = -\frac{C_{sr}R^2}{I_{sr}} $$
$$ b_{22} = \frac{1}{I_{sf}}, \quad b_{33} = \frac{1}{I_{sr}} $$

In the matrix notation, equation (17) may be written as:

$$ \dot{\xi}(t) = F\xi(t) + G\omega_{wh}(t) $$  \hspace{1cm} (18)

The discrete version of equation (18) using a sampling period of $T$ is:

$$ \xi(k+1) = F\xi(k) + G\omega_{wh}(k) $$  \hspace{1cm} (19)

To apply the preview control in the longitudinal direction, the acceleration profile is incorporated to the vehicle states. The system can be represented as:

$$ \begin{bmatrix} \ddot{\xi}(k+1) \\ a(k+1) \end{bmatrix} = \begin{bmatrix} F_d & 0 \\ 0 & M_d \end{bmatrix} \begin{bmatrix} \ddot{\xi}(k) \\ a(k) \end{bmatrix} + \begin{bmatrix} G_d \\ 0 \end{bmatrix} T_{wh}(k) + \begin{bmatrix} 0 \\ N_d \end{bmatrix} a_{wh}(k) $$  \hspace{1cm} (20)

In the compact form, equation (20) may be written as:

$$ s_{k+1} = Fs_k + GT_{wh,k} + Na_{wh,k} $$  \hspace{1cm} (21)

where $s_k$ denotes the vector $(N+4) \times 1$, $s_k = \begin{bmatrix} \ddot{\xi}(k) \\ a(k) \end{bmatrix}$, $F$ is the matrix $(N+4) \times (N+4)$, $F = \begin{bmatrix} F_d & 0 \\ 0 & M_d \end{bmatrix}$, $G$ is the matrix $(N+4) \times (N+4)$, $G = \begin{bmatrix} G_d \\ 0 \end{bmatrix}$, $N$ is the vector $(N+4) \times 1$, $N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

The associated performance index is defined as the quadratic function [8]:

$$ J_N = \frac{1}{2} s_k^T P_N s_N + \frac{1}{2} \sum_{k=0}^{N/4} s_k^T Q_s s_k + T_{wh,k}^T R_{wh,k} T_{wh,k} $$  \hspace{1cm} (22)

where $Q_s$ is the matrix $(N+4) \times (N+4)$, $Q_s = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

The torque applied to the front wheels is:

$$ T_{wh,k} = -K_{ss} s_k $$  \hspace{1cm} (23)

where $K_{ss}$ is the optimal gain for longitudinal control.

### 4. The Overall Model and Simulation

The lateral and longitudinal controllers provide the steering angle and the necessary wheel torque to the vehicle. In this paper, both linear and a nonlinear vehicle models [13, 14] including the longitudinal, lateral, yaw and roll motion is used to represent the real vehicle. The overall model of the longitudinal and lateral control is illustrated in Figure 5.

![Figure 5. The overall model of vehicle control](image-url)
torque affect to the lateral position of the vehicle. There are errors between the outputs of the linear models as internal models for controllers and the outputs of the nonlinear model presenting the real vehicle. For instances, the lateral position calculated in the lateral controller is different from the lateral position measured in the nonlinear model when both the steering angle and the wheel torque are applied.

The Kalman filter is used to estimate the states for the linear model and to eliminate additive noise due to the inputs and the measurements. The dynamics of the vehicle is subjected to disturbances. One type of disturbance is represented by the errors of equipment such as sensors, cameras and speedometers. The errors are modeled as a noise added to the observer. The other disturbances are the random external forces such as wind and fluctuated road surface. These types of disturbances directly affect the vehicle states and are also modeled as noises added to the inputs.

The lateral vehicle model can be represented as [10]:

$$\mathbf{x}_{k+1} = \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{w}_k$$  \hspace{1cm} (24)

where $\mathbf{w}_k$ is a white Gaussian noise with the zero-mean and the intensity of $W$. The lateral position observer is established as:

$$y_k = H_y \mathbf{x}_k + \eta_k$$  \hspace{1cm} (25)

where $H_y$ denotes the observer matrix, $H_y = [0 \ 0 \ 1 \ 0]$ and $\eta_k$ is a white Gaussian noise with the zero-mean and the intensity of $N_y$. The estimated state at the instant before performing the measurement is:

$$\hat{x}_k(-) = A_d \hat{x}_{k-1}(+) + B_d \delta_{m,k-1}$$  \hspace{1cm} (26)

The covariance before and after the measurement are:

$$P_k(-) = A_d P_{k-1}(+) A_d^T + A_d W_d A_d^T$$  \hspace{1cm} (27)

$$P_k(+) = P_k(-) + H_y^T N_y^{-1} H_y + N_y$$  \hspace{1cm} (28)

The estimation of the state as the feedback to the lateral controller is [11]:

$$\hat{x}_k(+)=\hat{x}_k(-)+K_{f,k}y_k-H_y\hat{x}_k(-)$$  \hspace{1cm} (29)

where $K_{f,k}$ is the Kalman gain as:

$$K_{f,k} = P_{k}(-)H_y^{T}[H_yP_{k}(-)H_y^{T}+N_y]^{-1}$$  \hspace{1cm} (30)

For the longitudinal estimation, the vehicle with disturbances is modeled as:

$$\xi_{k+1} = F_d \xi_k + G_d T_{wh,k} + A_d \mathbf{w}_k$$  \hspace{1cm} (31)

and the velocity is measured as:

$$V_k = H_x \xi_k + \eta_k$$  \hspace{1cm} (32)

where $H_x = [1 \ 0 \ 0]$, $\mathbf{w}_k$ and $\eta_k$ are white Gaussian noises with the zero-means and the intensities of $W$ and $N_y$ respectively. The mean and covariance using the Kalman filter are:

$$\hat{\xi}_k(-) = F_d \hat{\xi}_{k-1}(+) + G_d T_{wh,k-1}$$  \hspace{1cm} (33)

$$P_k(-) = F_d P_{k-1}(+) F_d^T + A_d W_d A_d^T$$  \hspace{1cm} (34)

$$P_k(+) = P_k(-) + H_x^T N_x^{-1} H_x + N_x$$  \hspace{1cm} (35)

$$K_{f,k} = P_{k}(-)H_x^{T}[H_xP_{k}(-)H_x^{T}+N_x]^{-1}$$  \hspace{1cm} (36)

$$\hat{\xi}_k(+)=\hat{\xi}_k(-)+K_{f,k}y_k-H_x\hat{\xi}_k(-)$$  \hspace{1cm} (37)

The simulation is performed to evaluate the controllers and the estimators. First, no disturbance is added to the inputs and the measurements. From the observed path, many velocity profiles may be generated. The chosen profile is in the consideration of the vehicle and road conditions. In the simulation, three velocity profiles are tested. The velocity profile 1 corresponds to the high level of acceleration and deceleration while the velocity profile 3 is for the low level of acceleration and deceleration. The total torque applied to the wheels is illustrated in Figure 6(a). The higher torque is required to follow the velocity profile 1. In Figure 6(b), three histories of the vehicle speed are recorded. The vehicle gradually accelerates and decelerates when it enters and exits the curve.

![Figure 6. Longitudinal performance](image-url)
observed path with the small lateral deviation. The difference of three trajectories is not significant.

A Kalman filter is capable of reducing the effect of noise on the vehicle state estimation. In reality, the filter is required due to the existence of disturbances affecting the position and speed measurement. On the other hand, the vehicle is a complex dynamic system, the filter is considered as an estimator that provides the approximate vehicle states based on the measured parameter. The states are necessary for the linear controller that needs feedback to calculate the next steering and torque inputs.

In the following simulation, the white Gaussian noises are added to the inputs and the measurements. The true position and the estimated position of the vehicle are illustrated in Figure 8(a). Although the measurement fluctuates due to the additive noise (±0.3m) as in Figure 8(b), the estimation is very close to the true state. The steering angle input as in Figure 8(c) is highly affected by disturbances. The large fluctuation of the steering angle comes from the corrupted noise and the noise that is partly filled by the Kalman filter.

The measurement of the vehicle speed is assumed to be corrupted by noise (±0.4m/s) as in Figure 9(b). The Kalman filter performs estimation from the noise speed measurement. It provides the estimated vehicle speed and wheel angular velocities to the linear controller. The difference between the true vehicle speed and the estimated speed is not significant as in Figure 9(a). The
wheel torque is computed from the estimated states with the presence of noise as in Figure 9(c).

![Graph](a)

![Graph](b)

![Graph](c)

**Figure 9.** Longitudinal performance with disturbances

5. Conclusion

The paper proposes a new method for the longitudinal and lateral planning that may be implemented in autonomous vehicles. The linear models of steering control and speed control are derived. The preview control is applied to enhance the performance of the vehicle when the future path is available. The Kalman filter is used to estimate the vehicle states based on the nonlinear model with disturbances. The simulation shows that the controllers perform a good path following and speed regulating.

The most important parts of the controller and the filters are the computation of control gains and filter gains. These tasks deal with an enormous amount of calculation and matrices of large dimension. For the proposed control strategy, these gains can be pre-calculated and stored in the hardware of controller. The computational time is shortened which enables the vehicle to operate real time even where the road is frequently updated. The proposed controller introduces a feasible application to autonomous vehicles.

References


