ABSTRACT
This paper addresses the design of an explicit model predictive controller for the outlet water temperature control of a solar cooling plant system that uses a field of Fresnel collectors. Solving multi-parametric quadratic programs (mpQP) to obtain explicit solutions to constrained model predictive control (MPC) problems enables the offline design before the MPC controller is implemented, e.g., on a standard programmable logic controller. The simulation result shows the feasibility of the proposed control design for such a system. This is implementable on industrial application-based hardware platforms where computational and memory resources are limited.

KEY WORDS
Constrained solar cooling plant; Model predictive control; Piecewise affine controller.

1 Introduction
During the last decades solar energy has been experienced a great impulse. The driving factor is the need of reducing the environmental impact caused by the use of fossil fuels, [1]. The experimental solar trough plant of ACUREX at the PSA of Almería has been used as a testbench for advanced control strategies, [2, 3]. In general, the control goal in solar systems is to maintain the outlet temperature of the field at a desired set-point in spite of disturbances. Solar plants are affected by multiple disturbance sources such as solar radiation, inlet temperature, optical efficiency etc. Conventional control strategies cannot cope with the complex dynamics and multiple disturbance sources. Advanced control techniques are needed to obtain a good performance in controlling solar systems.

One of the application of solar energy is solar cooling systems. In recent years there has been substantial growth in the requirements of cooling systems in commercial and industrial buildings in order to provide greater comfort when thermal load due to the extensive use of personal computers and the use of lighting is dramatically increased, [4]. The coincidence between peak demand and peak production (solar maximum) is a remarkable advantage of the solar cooling plants compared to other applications of solar energy. The utilization of solar energy for cooling can be achieved by using solar collectors that produce heat to medium or high temperature and power an absorption machine to supply conditioning air.

In low-power solar system, Fresnel solar concentrator systems are a new technology intending to make it in direct competition to the parabolic trough type systems. In such systems it is important to keep the inlet temperature to the absorption machine at a specific value. In this work we propose to use a model predictive control (MPC) strategy for controlling the outlet temperature while taking into account the constraints on flow level. We describe an explicit MPC formulation in such a way that an explicit form of the entire control law can be computed off-line by solving a multi-parametric quadratic program (mpQP), [5], thus making the MPC controller implementable by means of evaluating the piecewise linear function within the control unit.

The paper is organized as follows. Section 2 is devoted to the formal description of the dynamic model of the particular Fresnel collector field-type solar cooling system along with the outlet temperature control problem formulation. In Section 3, the outlet temperature control by means of the constrained explicit MPC formulation is described. Section 4 presents a simulation result that demonstrates the viability of the proposed control scheme. Finally, concluding remarks and future work are presented.
LiBr+ water absorption machine which produces refrigeration using heat pressurized water (145-165 °C). Its nominal cooling power is of 174 kW and it has a theoretical COP of 1.34.

- **PCM storage tank**: It is a tank of 18 m long and a diameter of 1.31 m. The PCM storage tank stores energy in latent heat form in order to deliver it when the Fresnel collector field does not reach the minimum temperature to power the absorption machine.

- **Fresnel solar collector field**: The Fresnel collector field consists of 11 rows of linear Fresnel collectors with a total reflective surface of 352 m². The collectors concentrate solar radiation onto an absorption metal tube measuring 64 m long as shown in Fig. 2 heating up the pressurized water that circulates through the tube.

The reader is referred to [6] for a complete description of the solar cooling plant. The control system of the plant is integrated into the building automation system comprising various controllers as shown in Fig. 3. Although it is possible to implement a control strategy on a computer within the network, it is necessary, from a stability and security of the system perspective, to implement the controller within the building control units. Therefore, if we want to develop a constrained MPC controller, it is necessary to obtain an explicit solution to the MPC optimization problem that suits the control unit with computational limitations.

### 2.1 Modeling

In this section, a mathematical model of the Fresnel collector field cooling system is derived. In general, two approaches to model such a system are classified as the concentrated parameter model and the distributed parameter model. The concentrated parameter model provides a lumped description of the Fresnel collector field, [4]. The variation in the internal energy of the collector can be described as, [7]

\[
C \frac{dT}{dt} = n_0SI - qP_{cp}(T - T_{in}) - H_1(T_m - T_a), \tag{1}
\]

where \( T \) denotes the outlet temperature on the collector, \( I \) the effective solar radiation, \( T_{in} \) the inlet temperature, \( T_a \) the ambient temperature, \( T_m \) the mean inlet-outlet temperature, \( q \) the flow level, \( n_0 \) the optical efficiency, \( S \) the effective surface, \( C \) the specific thermal capacity of the fluid, \( P_{cp} \) a term that accounts for the product and quotient of characteristic magnitudes (areas, thermal capacities, etc.), and \( H_1 \) the global thermal losses coefficient.

Due to the variation of \( H_1 \) with temperature, we have that

- \( H_1(T_m - T_a) = h_{t1}(T_m - T_a) + h_{t2}(T_m - T_a)^2 \)
- \( P_{cp}(T - T_{in}) = p_0(T - T_{in}) + p_1(T - T_{in})^2. \)
Then,
\[
C \frac{dT}{dt} = n_0SI - q \left[ p_0T - T_{in} \right] + p_1(T - T_{in})^2 \\
- h_{l1}(T_m - T_a) + h_{l2}(T_m - T_a)^2 = Cf(t) \quad (2)
\]
and
\[
T(t + \Delta t) = T(t) + \Delta t \cdot f(t). \quad (3)
\]
Note that the variables \( T_a, T_{in}, q, T, I_d \), where \( I_d \) represents the direct solar radiation, can be obtained from the monitoring system.

The advantage of a distributed parameter model, besides the consideration of spatial distribution, is a better modeling of the metal-fluid heat transmission. In this paper, a distributed parameter model described by the following two PDE system equations shown in [7] is used.
\[
\rho_m C_m S_m \frac{\partial T_m}{\partial t} = I K_{opt} n_o G - H_l G (T_m - T_a) - l_p H_t (T_m - T_f) \quad (4a)
\]
\[
\rho_f C_f S_f \frac{\partial T_f}{\partial t} + \rho_f C_f q \frac{\partial T_f}{\partial t} = l_p H_t (T_m - T_f), \quad (4b)
\]
where \( m \) and \( f \) subindices stand for metal and fluid, respectively. The system parameters and their units are listed in Table 1. The computation of optical efficiency \( K_{opt} \) requires the knowledge of multiple factors such as the mirror reflectivity, the metal tube absorbance, the shape factor etc. The computation of the geometric efficiency \( n_o \) is performed using complex mathematical formulas described in [4]. The PDE system can be solved by dividing the metal and fluid into 64 segments measuring 1 m long. The chosen integration step is 0.5 sec.

Equation (5) shows the dependence of \( H_l \) on \( T_m \) and \( q \), [8]. As previously stated, pressurized water is used as heat transfer fluid. Its density \( \rho_f \) and specific heat coefficient \( C_f \) have been obtained as polynomial functions of \( T_f \) ((6)–(7)).

\[
H_v = 1.34 \times 10^{-4} T_f^4 - 7.79 \times 10^{-2} T_f^3 + 18.73 T_f^2 \\
- 2573.11 T_f + 4.10 \times 10^5, \quad (5)
\]
\[
H_t = H_v q^{0.8}, \quad (5a)
\]
\[
\rho_f = -2.55 \times 10^{-3} T_f^2 - 0.202 T_f + 1003.92, \quad (6)
\]
\[
C_f = 5.17 \times 10^{-7} T_f^4 - 1.57 \times 10^{-4} T_f^3 + 2.77 \times 10^{-2} T_f^2 \\
- 1.63 T_f + 4207.40. \quad (7)
\]
The thermal losses coefficient has also been obtained using experimental data from the collector field and can be expressed as
\[
H_l = 0.001297(T_m - T_a) - 0.028585. \quad (8)
\]

Figure 4 shows the validation between distributed parameter model (Toutdist), the lumped parameter model (Toutconc), and the real temperature data (Tout). It can be observed that the distributed parameter model reproduces better the plant evolution.

### 2.2 Control Objective

The main objective is to obtain an appropriate control law for the solar cooling system. Within the context, we
develop an explicit MPC controller that must be implementable using a resource-constrained control unit with respect to processing and storage. The controller determines the optimal input increment for adjusting the flow level inside the solar field so that the outlet temperature at the end of the hot water circuit can reach a given reference value, while satisfying operational constraints.

3 Explicit MPC for Solar Plant Temperature Control

The dynamical model, lumped parameter, describing the solar plant temperature control process derived in the previous section is now used to facilitate an explicit model-based predictive control formulation. The model has shown to reproduce the system behavior that yields a trade-off between accuracy and complexity of the model. [4]. MPC is well-known and basically a heuristic for assigning optimal current and future control inputs by on-line minimization of the difference between reference sequence and future output sequence predicted using a selected process model. Only the optimal current input is then applied to the plant and the optimization is repeated with the new measured values and on the new control and prediction horizons, shifted one-step ahead. Solving the optimization problem on-line can be critical when applying the MPC to the practical solar cooling system through the use of existing constrained hardware platforms such as a programmable logic controller, Johnson Controls FX15 Controller, that is installed in the networked control system (see Fig. 3).

3.1 Explicit MPC

We now consider the reference tracking problem, i.e., the problem of driving the output (outlet temperature) $y$ to track a given reference signal $r \in \mathcal{R}^p$ by adjusting the control input (flow level) $u$ under the control input and control increment constraints. For the current $x$, the constrained MPC solves the following optimization problem such that the optimal control increment $\Delta u$ is found at each sampling instant, where the cost function to be minimized is given by

$$
\mathcal{J}(u, r, y(k)) = \sum_{j=0}^{N_u-1} \left[ y(k+j|k) - r(k) \right]^T Q \left[ y(k+j|k) - r(k) \right] + \sum_{j=0}^{N_a-1} \Delta u(k+j)^T R \Delta u(k+j),
$$

and $u \triangleq \left[ \Delta u(k)^T, \ldots, \Delta u(k+N_u-1)^T \right]^T$, $x(k+j|k)$ is the predicted state at time step $k$, $N_y$ and $N_u$ are the prediction and control horizons, and $Q \geq 0, R > 0$.

The MPC optimization problem (9) can be written in the standard quadratic program (QP) form, [9], and as shown in [5] such an MPC QP problem can be transformed into

$$
\min_u \left\{ \mathcal{J}(u, \theta(k)) = \frac{1}{2} u^T H u + \theta(k)^T F^T u \right\}
$$

s.t. $G u \leq W + S \theta(k),

(11)

where $u$ is defined as in (10), and $\theta$ is the vector of parameters defined as

$$
\theta(k) = [x(k), u(k-1), r(k)]^T.
$$

The MPC mpQP problem (11) is solved explicitly, off-line, for all the feasible values of $\theta$ of interest. This results in obtaining the solution $u^*(\theta)$ in the form of a continuous piecewise-affine function defined over a polyhedral partition in the $\theta$-space represented as

$$
\Delta u(k) = f(\theta(k)) = \left\{ \begin{array}{ll}
K_1 \theta(k) + k_1, & \text{if } \theta(k) \in \Theta_1 \\
K_2 \theta(k) + k_2, & \text{if } \theta(k) \in \Theta_2 \\
& \vdots \\
K_{N_{rej}} \theta(k) + k_{N_{rej}}, & \text{if } \theta(k) \in \Theta_{N_{rej}},
\end{array} \right.
$$

(12)

with a polyhedral partition $\mathcal{P} = \{ \Theta_1, \ldots, \Theta_{N_{rej}} \}$, where the polyhedral sets are represented by linear inequalities (hyperplanes),

$$
\Theta_i = \{ \theta(k) | L_i \theta(k) \leq l_i \}, \quad i = 1, \ldots, N_{rej}.
$$

(13)

Here, $K_i$ and $k_i$ are respectively the control gain and offset for each region, and $N_{rej}$ is the number of regions. Consequently, the on-line temperature control algorithm is reduced to a look-up table: the region associated with the current state $\theta$ is first determined, and then the optimal control law valid for that region is applied.

3.2 Controller Design

In order to simplify the design we take into account the plant as a nominal system, and assume that $T_{in}, I, T_a$ are constant at $130.17^\circ C$, $900 \text{ Wm}^{-2}, 38^\circ C$, respectively. The nonlinear model (2) is linearized around such an operating point $T_{in,0}, I_0, T_{a,0}$. Then, by choosing the sample time
\( T_s = 20 \) sec and using ZOH method we can get an equivalent discrete-time model as follows.

\[
x(k + 1) = 0.9647 x(k) - 0.1093 u(k) \quad (14a) \\
y(k) = x(k). \quad (14b)
\]

Note that when considering the tracking problem formulation (9) the previous input \( u(k - 1) \) is introduced to the prediction model as an additional internal state and the input increment \( \Delta u \) the input degree-of-freedom. This is the way to add an integrator in the control loop, [5].

The tuning parameters used for deriving the explicit MPC controller are as follows: \( N_y = 10, N_u = 3, Q = 2, \) and \( R = 0.01. \) The constraints on control input and input increment are given as \( 2 \leq u \leq 13 \) and \( |\Delta u| \leq 2, \) respectively.

The solution of the MPC mpQP problem, obtained from the model (14), the tuning parameters and the constraints, provides a polyhedral partition over the \( \theta \)-space, consisting of 16 regions, [10]. The algorithm that finds the optimal control increment \( \Delta u^*(0) \) is restricted to the following look-up table that depends on the current \( \theta, \)

\[
\begin{align*}
\text{if} & \begin{bmatrix} 2.78 & -0.43 & -2.92 \\ -2.78 & 0.43 & 2.92 \\ 1.29 & 0.11 & -1.25 \\ -2.78 & -0.07 & 2.92 \\ -1.49 & 0.04 & 1.67 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 5.5633 \\ -0.8561 \\ -5.8398 \\ \end{bmatrix} \theta(k) \\
\text{else if} & \begin{bmatrix} 0 & -0.17 & 0 \\ 1.48 & -0.33 & -1.59 \\ 1.20 & -0.31 & -1.30 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 0.18 * 10^{-14} \\ -0.18 * 10^{-14} \\ \end{bmatrix} \theta(k) + \frac{2}{2} \\
\text{else if} & \begin{bmatrix} 0 & 0.11 & 0 \\ -1.20 & 0.31 & 1.30 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 0.18 * 10^{-14} \\ 0.18 * 10^{-14} \\ \end{bmatrix} \theta(k) + \frac{2}{2} \\
\text{else if} & \begin{bmatrix} -0.33 & 0 & 0.36 \\ 0 & -0.09 & 0 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 0 & -1 & 0 \\ \end{bmatrix} \theta(k) + 13 \\
\text{else if} & \begin{bmatrix} 0 & 0.25 & 0 \\ 2.37 & 0.04 & -2.52 \\ 1.96 & 0 & -2.12 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 0 & -1 & 0 \\ \end{bmatrix} \theta(k) + 2 \\
\text{else if} & \begin{bmatrix} 0 & -0.11 & 0 \\ 0 & 0.09 & 0 \\ -0.44 & 0.04 & 0.47 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} -1 \\ 1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} -0.18 * 10^{-14} \\ 0 & -0.09 * 10^{-14} \\ \end{bmatrix} \theta(k) + 2
\end{align*}
\]

\[
\begin{align*}
\text{else if} & \begin{bmatrix} 0 & 0.1667 & 0 \\ 0 & -0.25 & 0 \\ 6.58 & -0.85 & -7.00 \\ 7.14 & -0.66 & -7.73 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ -1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} -0.18 * 10^{-14} \\ 0 & -0.09 * 10^{-14} \\ \end{bmatrix} \theta(k) + 2 \\
\text{else if} & \begin{bmatrix} 0.33 & 0 & -0.36 \\ -0.36 & 0 & 0.39 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} -1 \\ 1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 0 & -1 & 0 \\ \end{bmatrix} \theta(k) + 13 \\
\text{else if} & \begin{bmatrix} 0 & 0.09 & 0 \\ 0 & -0.5 & 0 \\ -4.10 & 0.91 & 4.39 \\ 5.40 & -1.38 & -5.85 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 0.18 * 10^{-14} \\ 0 & 0.18 * 10^{-14} \\ \end{bmatrix} \theta(k) + 2 \\
\text{else if} & \begin{bmatrix} 0 & -0.5 & 0 \\ 0 & 0.17 & 0 \\ -1.41 & -0.02 & 1.50 \\ 1.42 & 0.01 & -1.52 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 0 & -1 & 0 \\ \end{bmatrix} \theta(k) + 4 \\
\text{else if} & \begin{bmatrix} 0 & 0.5 & 0 \\ -2.37 & 0.30 & 2.52 \\ 1.96 & -0.18 & -2.12 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ -1 \\ 1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} -0.18 * 10^{-14} \\ 0 & -0.09 * 10^{-14} \\ \end{bmatrix} \theta(k) + 2 \\
\text{else if} & \begin{bmatrix} 4.10 & -0.91 & -4.39 \\ -1.48 & 0.33 & 1.59 \\ -1.42 & -0.01 & 1.52 \\ -1.29 & -0.11 & 1.25 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ \end{bmatrix}, \\
\Delta u^*(0) &= \begin{bmatrix} 4.35 & -0.96 & -4.66 \\ \end{bmatrix} \theta(k) + 0.94 \\
\text{else if} & \begin{bmatrix} 1.20 & -0.31 & -1.30 \\ -5.40 & 1.38 & 5.85 \\ 0 & 0.09 & 0 \\ 0.44 & -0.04 & -0.47 \\ -1.96 & 0.18 & 2.12 \\ -2.78 & 0.43 & 2.92 \\ \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ \end{bmatrix}
\end{align*}
\]
then

\[ \Delta u^*(0) = \begin{bmatrix} 0 & 0 & 0.89 \times 10^{-15} \end{bmatrix} \theta(k) + 2 \]

else if

\[ \begin{bmatrix} 2.37 & -0.30 & -2.52 \\ -6.58 & 0.85 & 7.00 \\ -2.37 & -0.04 & 2.52 \\ 1.49 & -0.04 & -1.67 \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \]

then

\[ \Delta u^*(0) = \begin{bmatrix} 6.97 & -0.89 & -7.41 \end{bmatrix} \theta(k) - 0.94 \]

else if

\[ \begin{bmatrix} -1.20 & 0.31 & 1.30 \\ 0 & -0.25 & 0 \\ -7.14 & 0.66 & 7.73 \\ 2.78 & -0.43 & -2.92 \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \]

then

\[ \Delta u^*(0) = \begin{bmatrix} 0 & 0 & 0.89 \times 10^{-15} \end{bmatrix} \theta(k) - 2 \]

else if

\[ \begin{bmatrix} 0 & 2.50 & 0 \\ -1.96 & 0 & 2.12 \\ 2.7817 & 0.0719 & -2.92 \end{bmatrix} \theta(k) \leq \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \]

then

\[ \Delta u^*(0) = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \theta(k) + 2 \]

else (problem is infeasible)

\[ \Delta u^*(0) = 0 \]

end if

4 Simulations

The simulations performed in this section have been set up through the use of the model (4). Figure 5 depicts the evolution of outlet temperature and flow level dynamics in each iteration, comparing the performance of the proposed controller with a PID. The PID parameters have been selected as follows: \( K_p = 0.3, \ K_i = 0.02, \) and \( K_d = 0.1. \)

In general, the control of solar plants becomes more difficult at low flow, [11], because the delay increments and the plant dynamics become highly nonlinear. The test shows that the performance at medium flow is quite similar, but when working at low flow water flow the MPC controller outperforms the PID controller. It can also be seen that when the plant is running under \( T_{in,0}, I_0, T_{a,0} \) the explicit MPC controller can keep well the outlet temperature at a given set of different reference values. Also, the input constraints are satisfied throughout the simulation.

5 Conclusion

This paper has presented an explicit MPC approach to address the outlet temperature control problem encountered in a Fresnel collector field-type solar cooling system. The scheme we have described demonstrates that the constrained MPC problem of the temperature control problem can be solved by an mpQP to acquire an explicit solution. The simulation result shows that a desired response is obtained while the constraints are satisfied. The proposed design is beneficial to the implementation of the constrained MPC on the control unit within the existing building automation system.

References


