RUNNING OF BIPED ROBOTS WITH VARIABLE SPEED

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ABSTRACT
In this paper, a trajectory generation method that changes the running speed of a biped robot with appropriate selection of a foot placement is proposed. An acceleration of the center of mass of a biped robot is associated with the relative position of the zero moment point to the center of mass. It means that the acceleration of the robot can be controlled by changing a position of the zero moment point. In this paper, a biped robot changes the speed by changing the zero moment point based on the desired speed. The effectiveness in the performance of the proposed method are shown in computer simulations with a 3D biped robot.

KEY WORDS
Running, Biped Robots, ZMP (Zero-Moment Point), Speed Change, Running Pattern

1 Introduction

Human beings have been working to make a robot that looks and acts like human, a humanoid. To realize a humanoid robot, lots of researches in various fields are necessary. Among the fields, research on a biped robot is a key to achieving the goal.

Bipedal running is one of the challenging research topics. The study on this was started by Raibert who used a spring loaded inverted pendulum model (SLIPM) for a hopping robot [5]. The SLIPM was commonly used to generate a trajectory of biped robot running [6, 7]. Another method that generate a running trajectory of a biped robot is ZMP-based running pattern generation method proposed by Kajita [8, 9]. However, these methods are based on a constant velocity and cannot change the speed. In order to resolve this, a method for changing the running speed by applying a virtual torque was proposed in [10]. When using this method, the robot is able to change the running speed. However, the amount of speed change during a single step is limited because the ZMP has to be located in the range of a sole. Besides, a biped robot can be unstable when the ZMP moves for a large speed change.

To overcome these problems, a method for stable speed changes with an proper foot placement is proposed. In this proposed method, based on the amount of speed change, an appropriate position of the ZMP at the beginning of the single support phase is computed with respect to the supporting foot. Its extreme value is also computed to check the possibility of stretching the land foot beyond its kinematic constraints. By forcing the swing foot land such that the center of the sole is located at the position, a speed change is resulted. When using the method, the amount of the speed change allowable in a single step is larger than the previous method proposed by the authors. Moreover, running of the biped robot becomes more stable because the ZMP is located at the center, not an edge, of the sole.

Methods for trajectory generation and computing the foot placement for the desired speed change are proposed in Section 2. And, simulations of a biped robot show the effectiveness and performance of the proposed method in Section 3. Finally, the paper ends with conclusions in Section 4.

2 Running Pattern Generation

2.1 Acceleration According to the ZMP

Here, an acceleration of a biped robot according to the relative position between the ZMP and the COM is explained. Eq. (1) expresses an equation of motion of the LIPM when the ZMP is not located at the center of a sole.

\[ M \ddot{X} - M (X - p_{ZMP}) Z = M (X - p_{ZMP}) g, \]  \hspace{1cm} (1) 

where \( M \) is the weight of robot, \( X \) and \( Z \) is the sagittal and vertical position of the COM, \( p_{ZMP} \) is the position of the ZMP and \( g \) is the gravity acceleration. Eq. (1) can be expressed as

\[ \ddot{X} = (X - p_{ZMP}) \frac{g + \dot{Z}}{Z}. \]  \hspace{1cm} (1-a)
When the COM is ahead of ZMP, the acceleration is positive and a biped robot is accelerated. On the other hand, when the COM of robot is located behind the ZMP, the acceleration of COM is negative and the COM reduces the speed. By using this property, a biped robot can change a speed.

### 2.2 Vertical Trajectory

In this paper, a simple hopping mas with a spring is assumed to design a vertical motion of a biped robot. Fig. 1 illustrates a simplified model of a hopping robot with nonlinear spring.

![Figure 1. The simplified hopping robot for biped robot.](image)

The equation of motion in the vertical direction is

\[ M \ddot{Z} = f_z - Mg, \]

where \( M \) is the weight of a biped robot, \( Z \) is the vertical position of the COM, \( g \) is the gravity acceleration, and \( f_z \) is the ground reaction force. The force, \( f_z \), is determined by period of supporting phase and flight phase.

\[
f_z = \begin{cases} 
(1 + \frac{T_f}{T_s}) Mg & (0 \leq t \leq T_s) \\
0 & (T_s \leq t \leq T_s + T_f)
\end{cases}
\]

Where \( T_s \) is period of supporting phase and \( T_f \) is period of flight phase. From Eq. (2), a vertical acceleration of the COM, \( \ddot{Z} \), is obtained. Since the vertical acceleration is constant during supporting phase, a vertical trajectory of the biped robot is 2nd-order polynomial of time. The vertical trajectory is

\[ Z(t) = \frac{1}{2} \ddot{Z}_0 t^2 + \dot{Z}_0 t + Z_0, \]

where \( Z_0 \) and \( \dot{Z}_0 \) are the initial position and the initial velocity in the vertical direction during the supporting phase, respectively; and \( \ddot{Z}_0 \) is the assumed constant vertical acceleration.

### 2.3 Horizontal Trajectory

To generate the horizontal trajectory for a speed change, the biped robot is modeled as an inverted pendulum with a mass attached as shown in Fig. 2. The particles in Fig. 2 indicate the body and the swing foot. In this simplified model, it is assumed that the point \( O \) is a hinged point where the ZMP is located.

To generate a trajectory in the horizontal direction, the following equation of moment of the model is used.

\[
\dot{H}_O = \sum M_O,
\]

where \( H_O \) is the angular momentum of the robot about point \( O \) and \( \sum M_O \) is the resultant moment of external forces about point \( O \).

With two-mass model shown in Fig. 2,

\[
\vec{p}_o \times M \vec{\ddot{p}}_o + (\vec{p}_o + \vec{q}) \times m (\vec{\ddot{p}}_o + \vec{\ddot{q}}) = \vec{p}_o \times M \vec{\ddot{g}} + (\vec{p}_o + \vec{q}) \times m \vec{g},
\]

where

\[
\vec{p}_o = [X \ Y \ Z]^T,
\]

\[
\vec{p}_{ZMP} = [x_{ZMP} \ y_{ZMP} \ 0]^T,
\]

\[
\vec{q} = [x_f \ y_f \ z_f]^T,
\]

\[
\vec{g} = [0 \ 0 \ -g]^T,
\]

\[ m \] and \( M \) are the weights of the swing foot and a the body including the supporting foot respectively, \( \vec{p}_o \) is the position vector of the COM with respect to a point \( O \) and \( \vec{q} \) is the position vector of the swing foot with respect to the COM.

From Eq. (5), differential equations of \( X \) and \( Y \) are obtained.

\[
\dot{X} = \frac{(\alpha + 1)(\ddot{Z} + g) + \ddot{z}_f}{(\alpha + 1)\ddot{Z} + z_f} X + F_x(t),
\]

\[
\dot{Y} = \frac{\ddot{Z} + g}{Z} X + F_y(t),
\]
\[ \alpha = M/m, \]
\[ F_x(t) = \frac{x_f(\ddot{Z} + \dddot{z}_f) - (Z + z_f)\dddot{x}_f + g\dot{x}_f}{(\alpha + 1)Z + z_f}, \]
\[ F_y(t) = \frac{(Y + y_f)(\ddot{Z} + \dddot{z}_f + g) - (Z + z_f)(\dddot{Y} + \dddot{y}_f)}{\alpha Z}. \]

Note that Eqs. (9) and (10) are non-homogeneous. In order to solve these equations, assumptions are made. Since the vertical motion of the body and the foot are small, \( Z \) and \( z_f \) are assumed to be constant at their average values. Also, the vertical acceleration is assumed to be constant at \( \dddot{Z}_0 \). By using the assumptions, solutions of the differential equations are obtained as
\[
\begin{align*}
X(t) &= C_1 e^{w_xt} + C_2 e^{-w_xt} + X_p(t), \quad (11) \\
Y(t) &= D_1 e^{w_yt} + D_2 e^{-w_yt} + Y_p(t), \quad (12)
\end{align*}
\]
where
\[
\begin{align*}
w_x &= \sqrt{\frac{(\alpha + 1)(\dddot{Z}_0 + g) + \dddot{z}_f}{(\alpha + 1)Z + z_f}} \quad (13) \\
w_y &= \frac{\dddot{Z}_0 + g}{Z} \quad (14) \\
C_1 &= \frac{1}{2} \left( X_0 - X_p(0) + \frac{\dot{X}_0 - \dot{X}_p(0)}{w_x} \right) \quad (15) \\
C_2 &= \frac{1}{2} \left( X_0 - X_p(0) - \frac{\dot{X}_0 - \dot{X}_p(0)}{w_x} \right) \quad (16) \\
D_1 &= \frac{1}{2} \left( Y_0 - Y_p(0) + \frac{\dot{Y}_0 - \dot{Y}_p(0)}{w_y} \right) \quad (17) \\
D_2 &= \frac{1}{2} \left( Y_0 - Y_p(0) - \frac{\dot{Y}_0 - \dot{Y}_p(0)}{w_y} \right), \quad (18)
\end{align*}
\]
and \( X_p(t) \) and \( Y_p(t) \) are the particular solutions, which are determined by the form of \( F_x \) and \( F_y \). \( F_x \) and \( F_y \) depend on trajectories of the swing foot. In this paper, the swing foot trajectories of forward and vertical direction are designed as follows.

\[
\begin{align*}
x_f(t) &= A_1 t^3 + A_2 t^2 + A_3 t + A_4 \quad (19) \\
z_f(t) &= \dddot{z}_f t^2 + \dddot{z}_f_0 t + \dddot{z}_f_0 \quad (20)
\end{align*}
\]

The lateral trajectory of the swing foot in global coordinate, \( Y + y_f \), is constant during a supporting phase. The coefficients of the forward trajectory of the swing leg are determined to satisfy the four conditions: The positions and velocities of initial and end in the supporting phase. The vertical foot trajectory, \( z_f(t) \), is expressed by a 2nd-order polynomial of time that has constant acceleration. The initial position, \( \dddot{z}_f_0 \), and velocity, \( \dddot{z}_f_0 \), are final position and velocity in flight phase and the acceleration is determined by the initial position, velocity and maximum height of the swing foot. If these trajectories of the swing foot are used for trajectory generation, \( F_y \) is a 3rd-order polynomial of time and \( F_y \) is constant. Therefore, particular solution \( X_p \) and \( Y_p \) are obtained as
\[
\begin{align*}
X_p(t) &= \alpha_1 t^3 + \alpha_2 t^2 + \alpha_3 t + \alpha_4 \quad (21) \\
Y_p(t) &= \beta_1. \quad (22)
\end{align*}
\]
Here, \( X_0 \) and \( Y_0 \) denote the initial positions of the COM with respect to the ZMP. Therefore, \( |X_0| \) and \( |Y_0| \) are the distances between ZMP and COM in the X and Y direction when \( t = 0 \). Therefore, changing \( X_0 \) and \( Y_0 \) means changing the positions of the ZMP with respective to the COM. \( X_0 \) and \( Y_0 \) are to be determined for a desired speed change in the Section 2.4.

### 2.4 Foot Placement
To reach the desired speed, a proper location of the foot placement should be determined. The speed during supporting phase is obtained by differentiating Eq. (11). The speed of the robot at the end of the supporting phase, i.e. when \( t = T_s \), is obtained as
\[
\dot{X}(T_s) = w_x C_1 e^{w_xt} - w_x C_2 e^{-w_xt} + \dot{X}_p(T_s), \quad (23)
\]
where \( T_s \) is the period of supporting phase. If the desired horizontal speed at the end of the supporting phase is \( X_{d, T} \), \( X_0 \) needed for the desired speed change is obtained by Eqs. (23), (15), and (16). Thus, the running trajectory for the desired speed change is obtained by combining Eqs. (11) and (24)
\[
\begin{align*}
X_0 &= X_P(0) - \frac{e^{2w_x T_s} + 1}{w_x (e^{2w_x T_s} - 1)} (\dot{X}(0) - \dot{X}_p(0)) \\
&\quad + \frac{2w_x T_s}{w_x (e^{2w_x T_s} - 1)} (\dot{X}_d - \dot{X}_p(T_s)), \quad (24)
\end{align*}
\]
When a trajectory is generated, condition \( X_0 = -X(T) \) is typically used in order to make the trajectory repeatable, which results in a symmetric trajectory. In this paper, however, this condition is not used; therefore, the generated running trajectory is asymmetry. Too much asymmetry may result in a trajectory that exceeds the workspace of the biped robot. Thus, the maximum speed change allowable at a supporting phase is limited to prevent the biped robot from exceeding the limitation of the workspace. If the final position in a supporting phase should be less than or equal to limitation of the workspace, \( X_t \), in order not to exceed the workspace,
\[
X(t) = C_1 e^{w_x T_s} + C_2 e^{-w_x T_s} + X_p(T_s) \leq X_t, \quad (25)
\]
or
\[
X_0 \leq X_P(0) - \frac{\dot{X}(0) - \dot{X}_p(0)}{w_x} + \frac{2(X_t - X_p(T_s)) e^{w_x T_s}}{e^{2w_x T_s} + 1} \quad (26)
\]
From Eqs. (24) and (26), the maximum desired speed that the biped robot can change during the supporting phase is obtained. To change the speed of the biped robot stably, the desired speed of the biped robot should satisfy

$$\dot{X}_d \leq \frac{w_x}{e^{2w_xT_s} - 1} (X_l - X_p(T_s)) + e^{w_xT_s} (\dot{X}(0) - \dot{X}_p(0)) + \dot{X}_p(T_s).$$

If the desired speed of the biped robot satisfies Eq. (27), the proper $X_0$ for desired speed change is obtained by using Eq. (24), and the speed is changed. If the desired speed does not satisfy Eq. (27), on the other hand, the maximum desired speed calculated in Eq. (27) is used to prevent the biped robot from exceeding a workspace, and then the process of speed change is carried out again in the next supporting phase. By using this method, the biped robot can reach the desired speed through several steps even if it cannot reach the desired speed in a single step.

$$f_n = k\delta^{m_1} + c\frac{\dot{\delta}}{|\dot{\delta}|} \delta^{m_2} \delta^{m_3},$$

where $k$ and $c$ are the spring and damping coefficient, respectively, and $m_1$, $m_2$ and $m_3$ denote the stiffness, damping and indentation exponents. The friction coefficient, $\mu(v)$, is determined by the friction model shown in Fig. 4.

### Table 1. Friction parameters used in the simulations

<table>
<thead>
<tr>
<th>symbols</th>
<th>values</th>
</tr>
</thead>
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<tr>
<td>$k$</td>
<td>2,000</td>
</tr>
<tr>
<td>$c$</td>
<td>10</td>
</tr>
<tr>
<td>$m_1$</td>
<td>1.3</td>
</tr>
<tr>
<td>$m_2$</td>
<td>1.0</td>
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<tr>
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<td>1.2</td>
</tr>
</tbody>
</table>

The numerical values of the friction parameters are shown in Table 1.

When the biped robot is accelerated, the biped robot ran according to the following. During $t = 0–2$ s, the biped robot set for a run by bending its knees in order to avoid a possible kinetic singularity during the run. After that, the robot started running by tracking the trajectory generated for a constant speed. The trajectory was based on the stride of 0.1 m, the stepping period of 0.3 s and the flight time of 0.1 s. After this constant running speed, the desired running speed was changed approximately at $t = 4.6$ s. When the biped robot changed the speed, the period was changed to 0.2 s and the biped robot accelerated by 0.25 m/s at each step. After taking 4 steps in changing the running speed, the biped robot ran at 2 m/s. In this simulation, the biped robot was able to increase the speed as shown in Fig. 5(a) and ran without falling down.

In the case of deceleration. Until $t=4.6$ s, the biped robot tracked the trajectory generated for a constant speed as in the first simulation. After running at the constant speed, the biped robot starts changing its speed without changing the period of the supporting phase. The biped robot slowed down its speed at a deceleration rate of 0.1 m/s at each step. After taking 4 steps to slow down the running speed, the biped robot ran at 0.6 m/s. The deceleration of the biped robot is shown in Fig. 5(b). From this simulation, it is shown that the proposed method is useful not only in acceleration but also in deceleration.

Fig. 6 shows the desired foot placement for speed change. Fig. 6(a) shows the foot placement when the biped robot accelerates. In that case, the foot placement was shorter than the initial foot placement under a constant speed. When the biped robot is decelerated, on the other
hand, the initial foot placement with respect to the COM is larger than the initial foot placement under a constant speed as shown in Fig. 6(b). From this result, it is confirmed that the speed of the biped robot was changed by selecting a proper foot placement.

In these simulations, the biped robot was able to change its running speed by based on a foot placement. However, in both cases, the speed fluctuated significantly as shown in Fig. 5. Small foot slips and the impact from the ground due to the lack of controls such as posture and foot contact are thought to be the source of the fluctuation in the speed. Therefore, it is necessary to reduce foot slips and to regulate the ground reaction force appropriately in order to improve the performance of the proposed method further.

4 Conclusion

In this paper, a method to change the running speed of a biped robot by selecting an appropriate foot placement is proposed. By computing the position of the ZMP for the desired speed change and designing a foot trajectory to make the center of the sole of the stepping foot be located at the ZMP, a running trajectory of a biped robot for the stable speed change is computed. Besides, the amount of speed change during one step is determined to prevent a biped robot from exceeding the limitation in the workspace of the robot. The performance of the proposed method was confirmed in computer simulations with a 3D biped robot. In simulations, it was shown that the running speed of the biped robot is changed according to the position of the foot placement. The proposed method was found effective both in acceleration and in deceleration.

References


