A MATHEMATICAL MODEL FOR SEQUENTIAL BATCHING PROCESSING MACHINES SCHEDULING PROBLEM IN TFT-LCD PROCESS

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ABSTRACT
In high-tech manufacturing, to enhance production efficiencies, batch processing machines which can process several jobs simultaneously are commonly used for long-processing time operations. In thin film transistor liquid crystal display (TFT-LCD) manufacturing processes, four sequential batch processing workstations are involved in the cell assembly process which is mainly used to assemble TFT substrates and color filters. Four sequential batch operations make the batch scheduling problem become more complicated. In addition, no existing solution algorithms are shown in previous research work. Consequently, in this paper, we propose a mixed integer programming (MIP) model for four sequential batching processing workstations in which we consider the constraints of limited waiting time, maximal batch size, and setup time between two different product types. In the proposed MIP model, to obtain the detail schedule for four sequential batching processing workstations, we consider the objective function is to minimize total setup times of bottleneck workstation on the premise of satisfying the throughput target. To demonstrate the applicability of the MIP model, a real-world example taken from a cell assembly shop floor in a TFT-LCD factory is provided.

KEY WORDS
TFT-LCD, batch scheduling problem, mixed integer programming.

1. Introduction
In high-tech manufacturing factories, batch processing machine can process several jobs simultaneously in order to enhance production efficiencies, particularly, for those long processing time operations. The thin film transistor liquid crystal display (TFT-LCD) manufacturing process mainly involves TFT array process, cell assembly process, and module assembly process. The purposes of the cell assembly process contain that TFT and color filter (CF) substrates are assembled, liquid crystal (LC) is injected into the cell gap, between TFT and CF substrates, and then the injection opening is sealed. The manufacturing process operations involve grinding, polyimide print, rubbing, assembly, seal bake, vacuum anneal, liquid crystal injection, end seal, after end seal, scribe, polarizer, and cell test. In a practical cell assembly factory, the processing steps ahead of assembly operation are referred to as front-end. On the other hand, those operations behind the assembly are referred to as back-end. In the back-end of cell assembly process, the LC injection operation usually spends long time, around four to five hours, to inject the LC slowly. Thus, LC injection operation is a typical batch processing workstation and the critical machine in the cell assembly process.

The cell assembly process includes four sequential batch operations involving seal bake, vacuum anneal, liquid crystal injection, and end seal. One essential characteristic of the sequential batch processing workstation is that the panel must begin its next operation step within a given time period after being completed its previous batch operation. In TFT-LCD factories, participants call the constraint is limited waiting time that is used to maintain the good display quality and to prevent the panels are scrapped due to quality issues. For example, the maximum waiting time between vacuum anneal and LC injection is 240 minutes and the maximum waiting time between LC injection and end seal is 360 minutes. Once the waiting time of a batch is greater than the limited waiting time, then the jobs in that batch should be scrapped or reworked. The maximum waiting time constraint indeed increases the complexity of the scheduling problem among the four sequential batch processing workstations. Thus, a appropriate scheduling result for the scheduling problem is very important.

In the four sequential batch processing workstations, LC injection is the bottleneck. It belongs to the incompatible product family type batch processing machine scheduling problem in which the jobs with same product type can be processed simultaneously. In this paper, we consider a batch processing machine scheduling problem with four sequential batch processing
workstations. In this scheduling problem, unequal ready time, setup time, and maximal batch size are also considered. The sequential batch processing machine problem involving multiple-stage parallel machines is more complicated than the classical batch processing machine problem. Consequently, it is very important to develop a mathematical model for the scheduling problem since that can make more effective decisions for the industrial participators.

2. Literature Review

In existing research work, there have been numerous investigations dealing with single stage batch processing machine scheduling problem. Little literature has been done on the multiple stage parallel batch processing machines scheduling problem. Batch processing machine scheduling problems are commonly categorized into two types: incompatible product family and compatible product family. It should be noted that the batch processing machine investigated in this paper is the incompatible product family. In the batch processing machine scheduling problem with incompatible product family, one batch is formed from the jobs clustered in the same product family. In this problem, the batch processing times are dependent on their jobs product family in that batch. Existing research works have been investigated solution procedures for the single batch processing machine in order to obtain solutions exactly or approximately. Uzsoy [1] is the first one to solve the single batch processing machine scheduling problem with incompatible job families for the objective function of minimum total completion time and makespan. Mehta and Uzsoy [2], Dobson and Nambimadom [3], and Tangudu and Kurz [4] also studied the single batch processing machine with the incompatible product family characteristic for various objective functions. In addition, for the parallel batch processing machines scheduling problems, Uzsoy [1,5], Koh et al. [6,7], Mönch et al. [8,9], Castro and Novais [10], Tai and Lai [11], and Pearn et al. [12] are shown procedures to solve the batch processing machine scheduling problem with incompatible product family.

In addition, the other batch processing machine scheduling problem with compatible product family in which it is assumed that jobs belonging to different product families can be processed simultaneously. In the scheduling problem, batch processing time is dependent on the longest job processing time in that formed batch. Lee et al. [13] is the first researchers to address the problem arising in a burn-in oven of the final test in the semiconductor burn-in operation. Existing research works on the batch processing machine scheduling problems with compatible product family characteristics include single batch processing machine scheduling problem (Uzsoy [1], Erramilli and Mason [14], Kashan et al. [15], and Chou et al. [16]) and the parallel batch processing machine scheduling problem (Chang et al. [17], Van Der Zee [18,19], Chung et al. [20]). However, to date, there has been relatively little research conducted on the scheduling problem with sequential batch processing machine workstations, unequal ready time, setup time, and maximal batch size which are considered in this paper.

3. A Mixed Integer Programming Model

In this paper, we present a mixed integer programming model for the complicated four sequential batch processing workstations. In the mixed integer programming (MIP) model we proposed, we consider limited waiting time, setup time, unequal ready time, and maximal batch size constraints and use the model to solve the detail schedule for the four sequential batching machines scheduling problem. Before we perform the mathematical model, we should collect the arrival time of the first batch workstation for each job and calculate the appropriate planning slot and check whether the capacity of the four batch workstations can process the coming jobs in the planning horizon.

In the following, we present a mixed integer programming model to solve the batch scheduling problem based on the planning slot. The objective of this MIP model is to minimize the total setup times of bottleneck workstation on the premise of satisfying the throughput target. Several assumptions of this model are justified in the following.

- The batch machines in each workstation are identical.
- The production quantities for each product type are given.
- The ready time of each batch is predetermined.
- The processing time of each workstation is known and fixed.
- Transportation times among the four sequential batch workstations are ignored.

Before the mathematical model is presented, the notations used in the formulation are listed below.

Indices:

- $i$: product index, $i = 1, 2, \ldots, I$.
- $g$: workstation index, $g = 1, 2, \ldots, G$.
- $m$: machine index in workstation $g$, $m = 1, 2, \ldots, M_g$.
- $t$: planning slot, $t = 1, 2, \ldots, M$.
- $BN$: bottleneck workstation among the four sequential batch operations.

Parameters:

- $b_{g}^{\text{MAX}}$: maximal batch size for machines in workstation $g$.
- $D_{i,g}$: target output of product $i$ in workstation $g$ in planning horizon.
- $M_g$: the number of machine in workstation $g$.
- $S_{t,g}$: setup time of the machines in workstation $g$.
- $\text{Cap}_{g,m}$: available capacity of $m$ in workstation $g$.

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$r_{i,t}$ : number of arriving batch of product $i$ in time $t$.

$S$ : the length of the planning slot.

"Planning slot" is used as the unit for the following parameters.

$P_{i,g}$ : processing time of product $i$ in workstation $g$.

$P_{i,g}^w$ : processing time of product $i$ in bottleneck workstation $g^{BN}$.

$Q_g$ : maximum waiting time between workstations $g$ and $g+1$.

Decision variables:

$x_{i,g,m,t}$ : production quantity of product $i$ on machine $m$ in workstation $g$ at the time $t$.

$\alpha_{i,g,m,t} \in \{0,1\}$, for all $i$, $g$, and $m$.

The mathematical formulation of the investigated scheduling problem is shown as follows.

Objective function:

$$\text{Min} \sum_{i=1}^{I} \sum_{g=1}^{G} \sum_{m=1}^{M} \beta_{i,g,m}$$

subject to

$$x_{i,g,m,t} \leq M \times \alpha_{i,g,m,t}, \quad \text{for all } i, g, m, \text{ and } t,$$  

$$x_{i,g,m,t} \leq P_{i,g}^\text{MAX}, \quad \text{for all } i, g, m, \text{ and } t,$$  

$$\sum_{i=1}^{I} \sum_{g=1}^{G} \sum_{m=1}^{M} (\alpha_{i,g,m,t} \times P_{i,g}) + \sum_{g=1}^{G} (\beta_{i,g,m} \times ST_g) \leq \frac{\text{Cap}_{g,m}}{S},$$

$$\text{for each } g \text{ and } m,$$  

$$\sum_{g=1}^{G} \sum_{m=1}^{M} x_{i,g,m,t} = D_{i,t}, \quad \text{for each } i \text{ and } t,$$  

$$\sum_{k=1}^{K} x_{i,g,m,k} \geq \sum_{k=1}^{K} x_{i,g,m+1,k}, \quad \text{for each } i, g, \text{ and } t,$$  

$$\sum_{k=1}^{K} x_{i,g,m,k} \geq \sum_{k=1}^{K} x_{i,g+1,m,k}, \quad \text{for each } i, g, \text{ and } t,$$  

$$\text{for each } g \text{ and } m,$$  

$$\sum_{i=1}^{I} \sum_{g=1}^{G} \sum_{m=1}^{M} \alpha_{i,g,m,t} \leq 1, \quad \text{for each } g, m \text{ and } t,$$  

$$\sum_{k=0}^{K} \alpha_{i,g,m,t+k} \leq 1, \quad \text{for all } i, g, m, \text{ and } t,$$  

$$\sum_{m=1}^{M} \sum_{k=1}^{K} x_{i,g,m,k} \leq \sum_{k=1}^{K} r_{i,k}, \quad \text{for each } i \text{ and } t,$$  

$$\beta_{i,g,m} \times M \geq \sum_{i=1}^{I} \alpha_{i,g,m,t}, \quad \text{for each } i, g, \text{ and } m,$$  

$$x_{i,g,m,t} \geq 0, \quad \text{for all } i, g, m, \text{ and } t,$$  

$$\alpha_{i,g,m,t} \in \{0,1\}, \quad \text{for all } i, g, m, \text{ and } t.$$  

The objective function (1) states that the number of setup on bottleneck workstation is minimal. The objective criterion can avoid unnecessary setup in order to deal with other unexpected situations. Constraint (2) indicates the relationship between $x_{i,g,m,t}$ and $\alpha_{i,g,m,t}$. If product $i$ is assigned to machine $m$ in workstation $g$ in planning slot $t$ ($\alpha_{i,g,m,t} = 1$), then $x_{i,g,m,t} \leq M$ is true with $M$ being a big digit. Constraint (3) ensures that the quantity of production on machine $m$ in workstation $g$ in planning slot $t$ always less than the maximal batch size for each workstation $g$ ($P_{i,g}^{MAX}$). Constraint (4) ensures the sum of the processing time and setup time for all products is less than available machine capacity. The planning slot is used here for the time unit. Constraint (5) assures that the sum of the production volume in each planning slot is greater than the output target of the whole planning horizon.

Constraint (6) is a flow constraint between the sequential batch processing workstations. It ensures that one batch of product $i$ should be processed at least $P_{i,g}^w$ on workstation $g$. When the batch complete on workstation $g$, it then can be processed on workstation $g+1$. Thus, the production volume of product $i$ on workstation $g$ in planning slot $t$ must be greater than the sum of the production volume of product $i$ on workstation $g+1$ in planning slot $t+P_{i,g}^w$. Constraint (7) is a maximum waiting time constraint for the cell assembly process. Constraint (8) ensures that each machine can only process at most one product in each planning slot. Constraint (9) indicates that within the time period $P_{i,g}^w$, there are only one product type $i$ can be produced in machine $m$ when the machine $m$ in the workstation $g$ are planned to produce product type $i$. Since only one product type $i$ can be processed in the machine, the value of the summation is not greater than one. Constraint (10) ensures the production volume of all machines in workstation 1 always less then total amount of arriving jobs for each planning slot. Constraint (11) is a contingent constraint. That is, if product $i$ is assigned to on machine $m$ in workstation $g$ during whole planning horizon ($\alpha_{i,g,m,t} = 1$), then at least one setup should be required for product $i$ on machine $m$ in workstation $g$ ($\beta_{i,g,m} = 1$). Constraint (12) ensures that the production volume is greater than zero. Finally, constraints (13)-(14) indicate the $\alpha_{i,g,m,t}$ and $\beta_{i,g,m}$ are both binary variables.

4. Illustrative Example

To demonstrate the applicability of the scheduling model for the four sequential batch processing workstations, a
The maximal waiting time between Vacuum anneal and LC injection is 240 minutes and maximal waiting time between LC injection and end seal is 360 minutes. In the case, the number of planned jobs are 5, 9, and 6 for product A, B, and C, respectively. The ready times of the 20 jobs for the first batch processing workstation are shown in Table 2.

Table 2
The ready times on the first batch processing workstation for the 20 jobs.

<table>
<thead>
<tr>
<th>Job ID</th>
<th>Ready time (min)</th>
<th>Job ID</th>
<th>Ready time (min)</th>
<th>Job ID</th>
<th>Ready time (min)</th>
<th>Job ID</th>
<th>Ready time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>67</td>
<td>B_4</td>
<td>228</td>
<td>B_4</td>
<td>327</td>
<td>C_5</td>
<td>491</td>
</tr>
<tr>
<td>B_1</td>
<td>108</td>
<td>C_3</td>
<td>250</td>
<td>C_4</td>
<td>370</td>
<td>A_4</td>
<td>529</td>
</tr>
<tr>
<td>C_1</td>
<td>154</td>
<td>B_4</td>
<td>267</td>
<td>B_4</td>
<td>387</td>
<td>B_8</td>
<td>554</td>
</tr>
<tr>
<td>B_2</td>
<td>168</td>
<td>A_3</td>
<td>289</td>
<td>A_2</td>
<td>411</td>
<td>C_6</td>
<td>606</td>
</tr>
<tr>
<td>A_2</td>
<td>189</td>
<td>C_3</td>
<td>310</td>
<td>B_2</td>
<td>449</td>
<td>B_9</td>
<td>644</td>
</tr>
</tbody>
</table>

In this case, we use the highest common factor as the planning slot. In the example, the planning slot is 20 minutes. In addition, the processing time, setup time, maximum waiting time, and ready time can be redefine as Table 3 and Table 4. The integer programming model is implemented using the software ilog OPL 3.5.1 to solve the sequential batch processing machines scheduling problem with 20 jobs in which the related processing information and ready time information described in Tables 1, 3, and 4. For the four sequential batch processing machines scheduling problem with four sequential batch workstations, eight machines, three product type, and 20 jobs, the mathematical model contains 6964 variables and 15319 equations. The run time is 32.48 CPU seconds on an Intel Pentium 4 3.0G Hz PC. The solution of the integer programming model requires three number of setup. Table 5 shows the output of the decision variable \( X_{i,g,m,t} \). In order to condense the presentation and make the presentation more clearly, we only show the non-zero value of the decision variables.

Table 5
Computational results for decision variables \( X_{i,g,m,t} \).

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
<th>variable</th>
<th>value</th>
<th>variable</th>
<th>value</th>
<th>variable</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_{Ph,1,2,15}</td>
<td>3</td>
<td>X_{Ph,3,3,73}</td>
<td>6</td>
<td>X_{Ph,4,2,99}</td>
<td>2</td>
<td>X_{Ph,3,3,72}</td>
<td>6</td>
</tr>
<tr>
<td>X_{Ph,1,1,32}</td>
<td>6</td>
<td>X_{Ph,3,1,102}</td>
<td>5</td>
<td>X_{Ph,4,1,101}</td>
<td>1</td>
<td>X_{Ph,4,3,1,97}</td>
<td>6</td>
</tr>
<tr>
<td>X_{Ph,1,1,52}</td>
<td>5</td>
<td>X_{Ph,3,1,97}</td>
<td>6</td>
<td>X_{Ph,4,3,1,93}</td>
<td>2</td>
<td>X_{Ph,4,1,103}</td>
<td>2</td>
</tr>
<tr>
<td>X_{Ph,1,2,24}</td>
<td>6</td>
<td>X_{Ph,3,2,108}</td>
<td>3</td>
<td>X_{Ph,4,1,112}</td>
<td>2</td>
<td>X_{Ph,4,1,112}</td>
<td>2</td>
</tr>
<tr>
<td>X_{Ph,2,1,51}</td>
<td>6</td>
<td>X_{Ph,4,1,187}</td>
<td>2</td>
<td>X_{Ph,4,1,115}</td>
<td>2</td>
<td>X_{Ph,4,1,115}</td>
<td>2</td>
</tr>
<tr>
<td>X_{Ph,2,1,72}</td>
<td>5</td>
<td>X_{Ph,4,1,191}</td>
<td>1</td>
<td>X_{Ph,4,1,118}</td>
<td>2</td>
<td>X_{Ph,4,1,118}</td>
<td>2</td>
</tr>
<tr>
<td>X_{Ph,2,2,79}</td>
<td>6</td>
<td>X_{Ph,4,1,196}</td>
<td>2</td>
<td>X_{Ph,4,1,122}</td>
<td>1</td>
<td>X_{Ph,4,1,122}</td>
<td>1</td>
</tr>
<tr>
<td>X_{Ph,2,1,94}</td>
<td>3</td>
<td>X_{Ph,4,1,198}</td>
<td>1</td>
<td>X_{Ph,4,1,139}</td>
<td>2</td>
<td>X_{Ph,4,1,139}</td>
<td>2</td>
</tr>
</tbody>
</table>

In Table 5, the variable \( X_{Ph,1,2,15} \) implies a batch of product type B is processed on machine 2 in workstation 1 at 15th time slot. The number of jobs in that batch is 3. In addition, \( X_{Ph,1,1,32} \) implies a batch of product type B is processed on machine 1 in workstation 1 at 32nd time slot. The number of jobs in that batch is 6.

In our mathematical model, since we apply the binary variable for the setup time consideration, the exact time for the setup can not be obtained. Consequently, when we obtain a solution from the mathematical model,
we should further check the reasonability of the mixed production line (β_{i,g,m} > 1). In other words, we should check whether setup time is greater than the reserved capacity in bottleneck workstation. In the case we investigated, the machine 2 only produce product type B in the bottleneck workstation (g=3). Thus, it is a dedicated production line. However, the machine 1 is assigned to produce product types A and C. There are two setups are involved in the planning horizon. It can be called mixed production line. In the computational results in the case, the number of setup is not greater then he reserved capacity, so the obtained solution can be applied for in-plant applications.

5. Conclusion

In this paper, we considered a four sequential batch processing machines scheduling problem with the practical constraints of unequal ready times, setup time, and maximal batch size. The scheduling problem has many real-world applications, particularly, for the cell assembly manufacturing process. In this paper, we have formulated the scheduling problem as a mixed integer linear programming model and consider the planning slot. To demonstrate the applicability of the proposed mathematical model in real situations, we consider a real-world case of a cell assembly shop floor at a TFT-LCD factory in Taiwan. The computational results showed that the proposed MIP model can obtain the solution within reasonable amount of computation time.

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