ASSESSING ROBUST STABILITY OF POWER SYSTEMS USING THE COMPLEX AND MIXED SSV

Evaristo N. Reyes, Marco A. Pérez, Arturo R. Messina.
Centro de Investigación y de Estudios Avanzados del I.P.N. Unidad Guadalajara
Av. del Bosque No. 1145, Col El Bajío, Zapopan, Jal., 45019, México
Email: noereyesp@gmail.com, [mperez, aroman]@gdl.cinvestav.mx

ABSTRACT
The aim of this work is to evaluate the robust stability in power systems by comparing the use of the complex against the mixed structured singular value (SSV). This proposal includes a percent of complex perturbations to avoid the semi-continuity in the lower bound produced by the mixed SSV and consequently it allows to assess the impact of complex uncertainty in the system. Numerical simulations are performed on a multi-machine power system to show the advantages of the method as well as the challenges to overcome.

KEY WORDS
Electromechanical oscillations, Robust stability, Structured singular value

1. Introduction

Many linear methods have been proposed to evaluate the nominal stability in power systems based on the use of the small-signal stability [1-4]. These linear techniques have demonstrated their effectiveness in several researches to study poor damping of the oscillations of synchronous machine rotors, instability, undesired steady-state network performance, etc. Furthermore these methodologies have the ability of the power system to maintain synchronism when subjected to small disturbances [22-23]. However, these linear methodologies do not model the uncertainty in the nominal system and they do not provide robust properties for a wide range of operating conditions.

In several studies, the uncertainty arises as a natural expression of unmodelled dynamics, parameter variations, nonlinear variations, etc. that can lead to instability. The investigation of closed-loop subject to modelled uncertainty is an important issue to assure robustness in stability and performance. The structured and unstructured uncertainty in power systems emerges due to load flow variations, transmission line parameters, changes in the network topology, deregulation in the sector, neglected dynamics, etc. all these factors have caused the system to be closer to its operational limits.

The structured singular value (SSV) theory was developed as a mean to determine the robust stability and performance of a linear system with parameter uncertainty. Computing the exact value of the SSV has shown to be NPhard (Non-deterministic polynomial-time hard), thus in practice is not possible to compute this value except for very low order systems. However, using scaling properties an upper and lower bound can be defined to approximate the SSV. Algorithms for computing these bounds have been documented in several researches [5-8]. An essential and sometimes underrated concept regarding to the SSV is the difference between the complex and the mixed SSV. The complex case was derived using upper and lower bounds out of real parametric uncertainties [9-10]. Unfortunately, the gap between these bounds can be conservative. In fact, the mathematical formulation of the complex SSV shows that it may be considered as a first approximation with respect to the mixed SSV.

The mixed SSV has been the focus of researches due to the fact that it incorporates real parametric uncertainties [11-14]. In the case where \( \mu \) have real parametric uncertainties, a solution is to reformulate the upper and lower bounds with new scaling matrices. However, the computational burden of the procedure grows and the mixed SSV is discontinuous. This means that the possibility of missing a point does exist. A partial solution for the discontinuity behavior of the mixed SSV is to incorporate a percent of complex uncertainty, but this will result in an increase of the uncertainty [15].

The introduction of \( \mu \) techniques to power systems has been applied in different analyses [16-18]. In short, two methodologies have been developed by researchers to represent the uncertainty in power systems. A percent representation of the structured uncertainty from the nominal linear system added to the unstructured
uncertainty. The other methodology is the polynomial representation using least square minimization derived from different operating conditions. However, the first uncertainty representation is more conservative and is limited to each perturbation in a given parameter. In addition, the process of designing controllers using $\mu$ has been implemented in different works using a $\mu$ synthesis process [19-21].

The main contributions of this paper are: The comparison between the complex and mixed SSV to evaluate the robust stability in power systems and the incorporation of a percent of complex perturbations to real parameters to avoid the semicontinuity of the mixed SSV. The rest of the paper is organized as follows: Section 2 introduces the nominal system model. Then the theory of the SSV for linear systems and the differences between complex and mixed SSV are presented in section 3. Section 4 describes the case study for a multimachine power system. Finally, the conclusion of this paper is given in section 5.

2. System Modelling

Let a finite dimensional LTI dynamical system be described by the following linear constant coefficient differential equations:

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t), \quad x(t_0) = x_0, \\
y(t) &= Cx(t) + Du(t)
\end{align*}$$

(1)

where $x(t):\mathbb{R} \rightarrow \mathbb{R}^n$ is the system state, $x(t_0)$ is the initial condition of the system, $u(t):\mathbb{R} \rightarrow \mathbb{R}^p$ is the system input, $y(t):\mathbb{R} \rightarrow \mathbb{R}^q$ is the system output, and $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times p}$, $C \in \mathbb{R}^{q \times n}$, $D \in \mathbb{R}^{q \times p}$ are state, input, output matrices, respectively.

The corresponding transfer matrix from $u(s)$ to $y(s)$ is defined as:

$$y(s) = G(s)u(s) = [C(sI - A)^{-1}B + D]u(s)$$

(2)

where $u(s)$ and $y(s)$ are in the frequency domain. The system can be described with the following notation called a realization of $G(s)$.

$$G(s) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = C(sI - A)^{-1}B + D.$$  

(3)

3. Complex and Mixed SSV

The nominal system described in (1) can be extended including uncertainty. The representation of a perturbed state space system with uncertainty is given by the next equation:

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + D\Delta u(t), \\
y(t) &= Cx(t) + Du(t) + B\Delta y(t)
\end{align*}$$

(4)

The above system can be described in Fig 1. with an upper Linear Fractional Transformation (LFT) to obtain the representation for robust performance where $u_\Delta(t):\mathbb{R} \rightarrow \mathbb{R}^u$, $y_\Delta(t):\mathbb{R} \rightarrow \mathbb{R}^y$ and $\Delta$ is due to sources of uncertainty. It is assumed that the matrix $A$ is stable, and the input $u_\Delta(t)$ is described as:

$$u_\Delta(t) = \Delta y_\Delta(t).$$

(5)

![Figure 1. Upper LFT for robust performance analysis.](image)

The uncertain closed-loop transfer function from $u(s)$ to $y(s)$ is related by an upper LFT,

$$F = F_u(N, A) = N_{23} + N_{21}A(I - N_{11}A)^{-1}N_{12}.$$  

(6)

To analyze the robust stability of (6), the system can be redraw into the $MA$-structure of Fig. 2. where $M = N_{11}$ is the transfer function from the output to the input of perturbations. The matrix $M$ can be obtained as:
\[
M = D_{y_A u_A} + C_{y_A} \left[ I - \frac{1}{s} A \right]^{-1} B_{u_A}
\]  
(7)

From
\[
\Gamma = \begin{bmatrix} A & B_{u_A} \\ C_{y_A} & D_{y_A u_A} \end{bmatrix}
\]  
(8)

Figure 2. **MA**-structure for robust stability analysis.

To study the robust stability, the mixed SSV considers three types of blocks: repeated real scalar, repeated complex scalar and full blocks. In each type of block are three nonnegative integers, \(S_c, S_e\) and \(F\). So that the total number of blocks in the system is defined by:

\[
\sum_{i=1}^{S_c} k_i + \sum_{j=1}^{S_e} r_j + \sum_{l=1}^{F} m_l = n
\]  
(9)

The block structure can be defined as:

\[
\kappa = (k_1, \ldots, k_{S_c}, r_1, \ldots, r_{S_e}, m_1, \ldots, m_F)
\]  
(10)

Hence the normalized uncertainty can be described

\[
BX_\kappa = \left\{ \Delta = \operatorname{diag} \left[ \delta_{1} I_{k_1}, \ldots, \delta_{S_c} I_{k_{S_c}}, \delta_{1} I_{r_1}, \ldots, \delta_{S_e} I_{r_{S_e}} \right], \right. \\
\left. : \sigma(\Delta) \leq 1 \right\}
\]  
(11)

The purely real case corresponds to \(S_c = F = 0\) and the purely complex case corresponds to \(S_e = F = 0\).

**Definition 1** [14]:

The mixed structured singular value, \(\mu_\kappa(M)\) of a matrix \(M \in \mathbb{C}^{m \times n}\) with respect to a block structure \(\kappa\) is defined as

\[
\mu_\kappa(M) = \min_{\Delta \in BX_\kappa} \frac{1}{\sigma(\Delta) \cdot \det(I - \Delta MA)}
\]  
(12)

With \(\mu_\kappa(M) = 0\) if no \(\Delta \in BX_\kappa\) solves \(\det(I - \Delta MA) = 0\).

For large systems the computation of the exact value of the SSV has shown to be NPhard. However, it is possible to compute upper and lower bounds to approximate this analytical problem. In [11] upper and lower bounds were expressed for the mixed case as:

\[
\max_{Q=Q_1} \rho_R(M) \leq \mu_\kappa(M)
\]

\[
\leq \inf_{D \in D_G, G \in G_k} \min_{\beta \in \mathbb{R}} \left\{ \beta : M^*DM + j[G(M - M^*) - \beta^2D] \leq 0 \right\}
\]

(13)

Similarly, for the complex SSV, \(\mu_c\), the upper and lower bounds can be written as:

\[
\max_{U \in U_c} \rho_Q(M) \leq \mu_c(M)
\]

\[
\leq \inf_{D_L, D_R} \max_{\bar{\sigma}(D_LMD_R^{-1})} \left\{ \beta \cdot M^*DM + j[GM - M^*G] - \beta^2D \leq 0 \right\}
\]

(14)

Let \(M(BX_\kappa)\) denote the set of all real-rational, proper, stable, block diagonal transfer matrices, with block structure like \(BX_\kappa\):

\[
M(BX_\kappa) = \{ \Delta \in RH_\infty : \Delta(j\omega) \in BX_\kappa \text{ for all } \omega \in \mathbb{R} \}
\]

(15)

where \(RH_\infty\) represents the space of real-rational proper transfer matrices with no poles in \(\text{Re}(s) \geq 0\). The following theorem addresses robust stability.

**Theorem 1 (Robust stability with \(\mu_\kappa\))** [14]:

Suppose that \(M(s) \in RH_\infty\). Then for all \(\Delta \in M(BX_\kappa)\) with \(\|\Delta\|_\infty \leq 1\), the perturbed closed-loop system is well-posed and stable if and only if
\[ \| \mu_\kappa (M(s)) \|_\infty := \sup_{\omega \in \mathbb{R}} \mu_\kappa (M(j\omega)) < 1 \quad (16) \]

In the case of purely real uncertainties, \( \mu_\kappa \) is not necessarily a continuous function [11-12]. These discontinuities can cause problems in the convergence of the lower bound mixed- \( \mu_\kappa \) algorithm. A method to avoid the semicontinuity of \( \mu_\kappa(M) \) is to introduce small levels of complex uncertainty into the uncertainty model, in order, to improve the convergence properties of the lower bound of \( \mu_\kappa \). This procedure can be shown in Fig. 3 where \( \alpha \) represents a scaling factor.

Figure 3. Replacing real uncertainty with real+complex uncertainty.

4. Numerical Simulations

The system shown in Fig. 4 is a sixteen machine five area with 68 buses. This system is a reduced-order model of the New England/New York interconnection. The detailed description of the system and network parameters can be found in [1].

Figure 4. Sixteen machine five area study system.

The system data have been slightly modified to introduce more perturbation and uncertainty to the nominal system. In addition, the tie line between buses 1 and 27 is open. All generators are represented by a two-axis model with a slow excitation system (IEEE-DC1A). The structured uncertainty was computed using the method described in [16-17].

The nominal modal analysis with different types of uncertainty is performed and the electromechanical oscillations identified. The results are summarized in Table 1 where the inter-area electromechanical oscillations with damping less than .05 are identified.

<table>
<thead>
<tr>
<th>Inter-area mode and damping ratio of the nominal system.</th>
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<tbody>
<tr>
<td>Linear uncertainty</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Inter-area mode</td>
</tr>
<tr>
<td>-.0536 ± 2.5061i</td>
</tr>
<tr>
<td>-.0467 ± 3.0611i</td>
</tr>
<tr>
<td>-.1485 ± 3.9229i</td>
</tr>
<tr>
<td>-.2391 ± 4.9537i</td>
</tr>
</tbody>
</table>

The nonlinear and linear structured uncertainty was computed using two parameter variations. The variation range in the reactance line connecting buses 1 and 2 is [0 p.u. -0.0822 p.u.] and the active load range is [0 p.u. – 14 p.u.].

In Fig. 5 and Fig. 6 the results evidence that the system does not have robust stability with complex SSV (dash line). On the other hand using the mixed SSV the system has robust stability (solid line). It must be noted that lower bounds are not depicted in both figures. The gap between the lower and upper bound for the complex SSV is practically zero. The lower bound of the mixed SSV is zero.

The dimensions of the nonlinear and linear uncertainty matrices are summarized in Table 2. The computational burden is notorious as the size of the uncertainties \( \Delta^{405 \times 405} \) and \( \Delta^{162 \times 162} \) is revealed for these cases.

<table>
<thead>
<tr>
<th>Size of uncertainty for robust stability ( \mu ) calculation.</th>
</tr>
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<tbody>
<tr>
<td>Description of uncertainty</td>
</tr>
<tr>
<td>Nonlinear ( \delta_1^{243 \times 243}, \delta_2^{162 \times 162} )</td>
</tr>
<tr>
<td>Linear ( \delta_1^{81 \times 81}, \delta_2^{81 \times 81} )</td>
</tr>
</tbody>
</table>
Figure 5. $\mu$ bounds with nonlinear uncertainty.

Figure 6. $\mu$ upper and lower bounds with linear uncertainty.

The comparison of the robust stability is given in Table 3, where the SSV peak values are shown for the different types of modeled perturbations. Furthermore from Table 1, Fig. 5 and Fig. 6 is evident that the peaks of the complex SSV identify inter-area and local oscillations modes, a feature that the mixed SSV lacks.

Table 3
Robust stability $\mu$ calculation with nonlinear and linear uncertainty representation.

<table>
<thead>
<tr>
<th>Description of uncertainty</th>
<th>$\mu$</th>
<th>Frequency</th>
<th>$|\mu|_{\infty}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinear</td>
<td>CSSV</td>
<td>8.4848</td>
<td>1.6082</td>
</tr>
<tr>
<td>Linear</td>
<td>CSSV</td>
<td>8.5859</td>
<td>1.3088</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>MSSV</td>
<td>2.5253</td>
<td>0.6859</td>
</tr>
<tr>
<td>Linear</td>
<td>MSSV</td>
<td>2.5253</td>
<td>0.5691</td>
</tr>
</tbody>
</table>

Fig. 7 and Fig 8 show the upper and lower bounds of the mixed SSV for different levels of uncertainty including complex perturbation. For the non-linear uncertainty Fig. 7 shows that as the amount of complex uncertainty increases the system approaches to its robust stability limit which is surpassed when this quantity reaches approximately 30%.

Figure 7. $\mu$ upper and lower bounds with complex perturbation and non-linear uncertainty.

On the contrary Fig. 8 evidences the disadvantages of using the linear perturbation modelling, since the mixed SSV does not reach the robust stability limit which makes this measure within this modelling not be the best indicator to identify this scenario.

Figure 8. $\mu$ upper and lower bounds with complex perturbation and linear uncertainty.

5. Conclusion

In this paper robust stability assessment results between complex and mixed SSV have been studied to analyze and quantify the upper bounds of the SSV. These results show
that for large power systems the modelling of the structured uncertainty can have a great impact in measuring the limits of the robust stability. Furthermore, by including complex perturbations the lower bound of the mixed SSV can change substantially. Future work should be focused to obtain reduced order models of the structured uncertainty to design robust decentralized controllers with real parameters. Finally, the proposed methodology provides a unified framework for the analysis of the complex and real parametric uncertainty cases in power systems.

References


