ROBUST FAULT-TOLERANT FINITE-TIME STABILIZATION OF SWITCHED SYSTEMS WITH ASYNCHRONOUS SWITCHING

Ronghao Wang¹, Jianchun Xing¹, Ping Wang¹, Qiliang Yang¹, Zhe Cao² ³

1. College of Defense Engineering, PLA University of Science and Technology
   Nanjing, China

2. State Key Laboratory of Precision Measurement Technology and Instruments, Tsinghua University
   Beijing, China

3. Electrical Engineering Department, University of California, Los Angeles
   Los Angeles, USA
   wrh1985@aliyun.com

ABSTRACT
In this paper, we investigate the robust fault-tolerant finite-time stabilization problem of switched system with asynchronous switching. The asynchronous switching idea originates from the fact that switching instants of the controllers lag behind or exceed those of subsystems. The attention is focused on designing a robust fault-tolerant asynchronously switched controller that guarantees the finite-time stability of dynamic system. At first, on the basis of finite-time stability theory and multiple Lyapunov functions approach, a finite-time stabilizable condition related to dwell time is established. Secondly, the problem of fault-tolerant control for switched systems with asynchronous switching is investigated and a fault-tolerant asynchronously switched controller is designed to guarantee finite-time stability of the switched systems. Furthermore, the design method of feedback controller is proposed to ensure robust finite-time stability of the closed-loop system for all admissible uncertainties and actuator fault. Finally, a numerical simulation shows the effectiveness of the obtained results.

KEY WORDS
Finite-time; asynchronous switching; switched system; average dwell-time; fault-tolerant

1. Introduction
A switched system consists of a finite number of subsystems and a switching signal that orchestrates the switching between them. A different switching law would give rise to different dynamic response and steady state characteristics of the system. The research motivation of switched systems originates from their great efficiency in modeling and control of numerous real-world systems such as robot control, chemical process, motor control, industry control systems and power systems, etc. [1-3].

The issue of Lyapunov stability of switched systems is a basic research topic at early time. Based on the core of classical Lyapunov stability theory and the feature of switched systems, researchers proposed common Lyapunov function approach and multiple Lyapunov functions method to analyze the stability of system [4-8]. We often use common Lyapunov function method to stabilize the system under arbitrary switching, but this method is more conservative. A common Lyapunov function may not exist for some switched systems, but we can use the multiple Lyapunov functions technique to find some properly switching signals to guarantee the switched system are still stable. Besides, average dwell-time approach which the switching law is time dependent have been proposed to analyze switched systems [9-10]. Usually Lyapunov asymptotical stability is enough for practical applications, however, large values of the state are not acceptable, for instance in the presence of saturations. In these cases, the concept of finite-time stability (FTS) can be adopted, which focuses on a bound of system trajectories over a fixed short time. It is worth to point that a finite-time stable system may not be Lyapunov asymptotical stable, and vice versa. Finite-time stability analysis method of switched systems is proposed by [11-12]. Furthermore, finite-time stabilization for discrete-time switched system and switched delay systems is studied in [13-14].

In actual control system, actuators may happen to failures owing to aging of the component and the severe operating condition which always makes system be of the poor stability and dynamic property. Fault-tolerant control is an effective method to ensure the system to run correctly. Some methods of designing fault-tolerant controllers have been given to make switched systems be finite-time stable [15].

However, in many engineering systems, owing to the delay of detecting the system mode or the respond error of the controller, the switching instant of the controller may lag behind or exceed those of the subsystem. Asynchronous switching phenomenon between the controllers and subsystems inevitably exists in the closed-loop systems. Therefore, the synchronous switching assumption may be unfeasible. In this case, the former designed method of fault-tolerant controller for switched systems is invalid. So a valued question to study is how to solve fault-tolerant control problem of the system with asynchronous switching.

We focus on the problem of robust fault-tolerant finite-time control for switched systems of actuator failures with asynchronous switching in this paper. The rest of the paper is organized as follows. Section II presents some necessary preliminaries and definitions and describes the problem investigated in the paper. In Section III the robust fault-
tolerant finite-time stabilization problem with asynchronous switching is studied. Using multiply Lyapunov function method and average dwell-time approach, an asynchronously switched controller is designed, which ensures that the closed-loop system is robustly finite-time stable with actuator failures. Finally, a numerical simulation shows the efficiency of the proposed method in Section IV. Section V concludes the paper.

2. Preliminaries and Problem Statement

The switched system with actuator failures and uncertainty is considered as follows:

\[
\begin{align*}
\dot{x}(t) &= \hat{A}_{\sigma(t)}x(t) + \hat{B}_{\sigma(t)}u'(t) \\
x(t_0) &= x_0
\end{align*}
\]

(1a)

(1b)

where \(x(t) \in \mathbb{R}^n\) is the system state. \(u'(t) \in \mathbb{R}^p\) is the control input of an actuator fault, \(x(t_0) = x_0\) is the initial state of the system. \(\sigma(t) : [t_0, \infty) \rightarrow \mathbb{N} = \{1, 2, \ldots, N\}\) is the switching signal which is a right continuous piecewise constant function of time, \(N\) is the number of subsystems or modes. \(\hat{A}_i \in \mathbb{R}^{nxn}, \hat{B}_i \in \mathbb{R}^{nxp}\) for \(i \in \mathbb{N}\) are unknown real constant matrices representing time-varying parameter uncertainties and are assumed to be of the form\(^{[16]}\):

\[
\hat{A}_i = A_i + H_i U_i(t) E_{ai}, \quad \hat{B}_i = B_i + H_i U_i(t) E_{bi}
\]

(2)

where \(A_i, B_i, H_i, E_{ai}, E_{bi}\) are known real constant matrices with proper dimensions and \(H_i, E_{ai}, E_{bi}\) denote the structure of the uncertainties. \(U_i(t)\) is unknown time-varying matrices which satisfies:

\[
U_i^T(t)U_i(t) \leq I, \quad \forall t
\]

(3)

The uncertainty structures \(H_i U_i(t) E_{ai}\) and \(H_i U_i(t) E_{bi}\) are said to be admissible if (2) and (3) hold. As often assumed in research of the switched system, we consider that there is no state jump at the switching instants, i.e. the state of system (1) is continuous at any time. Corresponding to the switching signal \(\sigma(t)\) of the system, we denote the switching sequence by

\[
\{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k)), \ldots\}, k \in \mathbb{Z}
\]

where \(t_0\) is the initial switching instant and \(t_k\) denotes the \(k\)th switching instant which means that the subsystem \(\sigma(t_k)\) is activated when \(t_k \leq t < t_{k+1}\).

Owing to asynchronous switching between controllers and subsystems, the switched of controller do not coincide with those of system mode. We use \(\sigma'(t)\) to denote the real switching signal of controller, then the corresponding switching sequence of controller can be written as

\[
\{(t_0 + A, \sigma(t_0)), (t_1 + A, \sigma(t_1)), \ldots, (t_k + A, \sigma(t_k)), \ldots\}, k \in \mathbb{Z}
\]

which means that the sub-controller \(\sigma(t_k)\) is running at \(t_k + A \leq t < t_{k+1}\).

Then the asynchronously switched controller will be the form as follows:

\[
u(t) = K_{\sigma'(t)}x(t)
\]

(4)

The real control input of actuator fault is

\[
u'(t) = R_{\sigma'}u(t)
\]

(5)

where \(R_i\) for \(i \in \mathbb{N}\) are actuator fault matrices and have the following form:

\[
R_i = \text{diag}\{r_{i1}, r_{i2}, \ldots, r_{ip}\}
\]

(6)

\(Q_i = \text{diag}\{q_{i1}, q_{i2}, \ldots, q_{ip}\}\)

(7)

\(S_i = \text{diag}\{s_{i1}, s_{i2}, \ldots, s_{ip}\}\)

(8)

By (6)–(8), we have

\[
R_i = R_{i0}(I + S_i), \quad |S_i| \leq Q_i \leq I
\]

(9)

where \(|S_i|\) represents the absolute value of diagonal elements in matrix \(S_i\), i.e. \(|S_i| = \text{diag}\{|s_{i1}|, |s_{i2}|, \ldots, |s_{ip}|\}\).

**Remark 1** When \(r_{ik} = 1\), it represents the \(k\)th actuator channel of the \(i\)th subsystem is normal. \(r_{ik} = 0\) means the full failure of the \(k\)th actuator channel in the \(i\)th subsystem. \(r_{ik} < 1\) and \(r_{ik} \neq 1\) covers a partial failure of the \(k\)th actuator channel for the \(i\)th subsystem.

We get switched system (10) without considering the uncertainty in the system matrices \(\hat{A}_i\) and control matrices \(\hat{B}_i\)

\[
\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u'(t)
\]

(10)

Next, we shall recall the definition of the average dwell-time and introduce the concept of finite-time stability for switched system of actuator fault with asynchronous switching.

**Definition 1**\(^{[17]}\) For any \(t_2 > t_1 \geq 0\), let \(N_0(t_1, t_2)\) denote the switching number of \(\sigma(t)\) over \((t_1, t_2)\). If

\[
N_0(t_1, t_2) \leq N_0 + \frac{t_2 - t_1}{\tau_a}
\]

(11)

holds for \(\tau_a > 0\) and an integer \(N_0 \geq 0\), then \(\tau_a\) is an average dwell time and \(N_0\) is called the chatter bound.

**Definition 2** The switched system (1) is finite time robust fault-tolerant stable with the asynchronous switching control mode with respect to \((c_1, c_2, T_f, \sigma(t), \sigma(t))\) with \(c_1 < c_2\) and the switching law \(\sigma(t)\), if there exists the controller \(u(t) = K_{\sigma'(t)}x(t)\) to make \(\|x(t)\| \leq c_2, \quad \forall t \in [t_0, T_f]\), whenever \(\|x_0\| \leq c_1\).
The following lemmas will play an important role in our development of main results.

**Lemma 1**\([18]\) For matrices \(M_i, M_j\) of appropriate dimensions, the following matrix inequality holds
\[ M_i\Phi(t)M_j + M_j^T\Phi'(t)M_i^T \leq \beta M_i VM_i^T + \beta^{-1}M_j^TVM_j \]
where \(\Phi(t)\) is a time-varying diagonal matrix, \(\beta\) is a positive constant and \(V\) is a known real-value matrix satisfying \([\Phi(t)]^T \leq V\).

The main issue in this paper is given as follows:

For switched system (1), find a robust fault-tolerant controller ensuring finite-time stability with respect to \((c_1, c_2, T_f, \sigma(t), \sigma'(t))\) with the asynchronous switching control mode.

### 3. Main Results

Let \(\sigma(t_k) = j\) and \(\sigma(t_{k-1}) = i\), then at the switching instant \(t_k\) the \(i\)th subsystem switches to the \(j\)th subsystem and the \(j\)th subsystem is activated. Due to asynchronous switching, based on above corresponding switching sequence of controller, the switching instant of the \(i\)th controller is \(t_k + A_k\), i.e. the controller of the \(j\)th subsystem affects at time \(t_k + A_k\). Therefore, there exists mismatched period at time interval \([t_k, t_k + A_k]\), \(A_k > 0\) (or \([t_k + A_k, t_k]\), \(A_k < 0\)). During this period, the controller \(K_i\) of the \(i\)th subsystem affects the \(j\)th subsystem (or the controller \(K_j\) affects the \(i\)th subsystem).

When \(A_k > 0\), the switching time of the controller is lag behind those of the system. Denote \(\Omega_i\) is the state region of the matched time region between the \(i\)th subsystem and the controller \(K_i\), and \(\Omega_j\) is the state region of the mismatched time between the \(j\)th subsystem and the controller \(K_j\), where \(i, j \in \{1, 2, \cdots, N\}, i \neq j\). Then we have

\[
\Omega_i = \{x(t) \in R^n \mid \sigma(t_k + A_k) = i, \sigma(t_k) = j, t \in [t_k, t_k + A_k], k = 1, 2, \cdots\}
\]

\[
\Omega_j = \{x(t) \in R^n \mid \sigma(t_k) = j, \sigma(t_k + A_k) = j, t \in [t_k, t_k + A_k], k = 0, 1, \cdots\}
\]

Fig. 1 illustrates the asynchronous switching mode between the controller and the subsystems. From Fig. 1, we can see that the controller \(K_i\) of the \(i\)th subsystem affects the \(i\)th subsystem in the matched period \([t_{k-1} + A_{k-1}, t_k]\) and affects the \(j\)th subsystem in the mismatched period \([t_k, t_k + A_k]\).

![Fig. 1. The time sequence of switched system and controller (asynchronous switching mode)](image)

**Remark 2** The restriction of the mismatched period between the controller and subsystems

\[
A_k \leq \inf_{k \geq 0}(t_{k+1} - t_k)
\]

ensures that there always exists a period \([t_{k-1} + A_{k-1}, t_k]\) such that the controller can affect the corresponding subsystem. This period is said to be matched in the follows.

Substituting (4) and (5) to system (10), we can obtain the closed-loop system

\[
\dot{x}(t) = (A_{\sigma(t)} + B_{\sigma(t)} R_{\sigma(t)}(K_{\sigma(t)}))x(t)
\]

**Lemma 2** For \(i, j \in \mathbb{N}\) and \(i \neq j\), suppose that there exist matrices \(P_i > 0, P_j > 0, K_i, R_i\) and constants \(\mu_i > 1, \mu_j > 1, \lambda^+ > 0, \lambda^- > 0\) such that

\[
P_i < \mu_i P_i, P_j < \mu_j P_j
\]

\[
(A_i + B_i R_i K_i)^T P_i + P_i (A_i + B_i R_i K_i) < \lambda^- P_i
\]

\[
(A_j + B_j R_j K_j)^T P_j + P_j (A_j + B_j R_j K_j) < \lambda^+ P_j
\]

If average dwell-time of the switching signal \(\sigma(t)\) of the system satisfies

\[
\tau_a > \frac{\ln(\mu_i \mu_j)(T_f - t_0)}{\inf_{i \neq j}(\kappa^T \cdot \lambda_{\min}(P_i) \cdot \lambda_{\max}(P_j))}
\]

where \(\kappa = \lambda_{\min}(P_i) \cdot \lambda_{\max}(P_j)\), then switched system (10) is finite-time stable with respect to \((c_1, c_2, T_f, \sigma(t), \sigma'(t))\) where \(T(t_0, T_f)\) represents the total matched control time and \(T^*(t_0, T_f)\) is the sum of the mismatched control time during the interval \([t_0, T_f]\).

**Proof** See Appendix.

**Remark 3** From (14) and (15), the closed-loop subsystem is not stable in finite-time interval owing to \(\lambda^+ > 0\) and \(\lambda^- > 0\). The whole system is finite-time stable, although asynchronously switched controller can not stabilize the
subsystem in the matched period and the mismatched period of finite-time interval. It is very different from reference [19] that requests the subsystem is exponentially stable in the matched period based on the theory of classical Lyapunov stability. Therefore, by Lemma 2, we can see that it is unnecessary to request that the closed-loop subsystem is stable during the matched period or mismatched period in the sense of finite-time stability.

In fact, (16) of Lemma 2 also shows that if switching law of the system can be pre-specified, i.e. $\tau_a$ is a known constant, the matched period $T^-(t_0, T_f)$ and the mismatched period $T^+(t_0, T_f)$ have the relation as follows

$$\lambda^* T^-(t_0, T_f) + \lambda^* T^+(t_0, T_f) < \ln\left(\frac{c_i^2}{\sqrt{c_i}}\right) \frac{\ln(\mu_1 \mu_2) (T_f - t_0)}{\tau_a}$$

**Remark 4** Notice that $R_i$ is not a known matrices and $P_iB_iR_iK_i$ is nonlinear, so in order to obtain the solution of $K_i$, we need to deal with (14) and (15) to be solvable forms.

The following theorem gives the design method of a fault-tolerant controller for the system (10) with asynchronous switching.

**Theorem 1** For $\forall i, j \in N$ and $i \neq j$, suppose that there exist matrices $X_i > 0$, $X_j > 0$ and constants $\alpha_i > 0$, $\zeta_j > 0$, $\mu_i > 1$, $\mu_2 > 1$, $\lambda^+ > 0$, $\lambda^- > 0$ such that

$$X_i < \mu_1X_i, X_j < \mu_2X_j$$

$$\begin{bmatrix}
\Lambda_i & Y_i^TR_i0^{j1/2} \\
* & -\alpha_i I
\end{bmatrix} < 0$$

$$\begin{bmatrix}
\Lambda_{ij} & (Y_iX_i^{-1}X_j)^TR_j0^{j1/2} \\
* & -\zeta_j I
\end{bmatrix} < 0$$

(17), (18) and (19) are not a set of linear matrix inequalities (LMI). In order to get feasible solutions, (18) can be firstly solved to obtain $\hat{X}_i$ and $\hat{Y}_i$. Then substituting $X_i$ and $Y_i$ into (17) and (19) leads to a set of LMIs. Scalars $\mu_1$, $\mu_2$ and $\lambda^+$ are tuning parameters which need to be given. By adjusting these parameters appropriately, the feasible solutions $X_i$, $Y_i$ and $X_j$ can be found to make (17) and (19) hold. If (17) and (19) have no feasible solution under the chosen parameters, $\mu_1$, $\mu_2$ or $\lambda^+$ can be set to be larger to expand the range of solutions. Following this guideline, the solution of (17) and (19) will be found. By this method, one can avoid to solve bilinear matrix in equalities (BMs) of (19) which will bring in some complex on computation.

Next, we will be in a position to address robust fault-tolerant controller design method of the system (1) with asynchronous switching and time-varying parameter uncertainties.

**Theorem 2** For $\forall i, j \in N$ and $i \neq j$, suppose that there exist matrices $X_i > 0$, $X_j > 0$ and scalars $\alpha_i > 0$, $\zeta_j > 0$, scalars $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0$, $\zeta_j > 0$, $\alpha_i > 0
\[ \varepsilon_i > 0, \quad \xi_j > 0, \quad \delta_i > 0, \quad \psi_j > 0, \quad \mu_1 > 1, \quad \mu_2 > 1, \]
\[ \lambda^+ > 0, \quad \lambda^- > 0 \] such that
\[ X_{ij} < \mu_i X_i, \quad X_i < \mu_2 X_{ij} \]
\[ \Pi_i \leq X_{ij} \]
\[ Y_i^T R_i \delta^2 \quad E_{\alpha_i} X_i + E_{\beta_i} R_i Y_i + \alpha_i E_{\theta_i} Q_i B_i^T \]
\[ \Pi_i \leq -\alpha_i I \]
\[ \Pi_j \leq (X_{ij}^T X_{ij})^T \quad \delta^2 \quad E_{\alpha_i} X_i + E_{\beta_i} R_i Y_i + \alpha_i E_{\theta_i} Q_i B_i^T \]
\[ \Pi_j \leq -\xi_i J \]
\[ \Pi_j \leq -\xi_j J \]

If average dwell-time of the switching signal \( \sigma(t) \) of the system satisfies
\[ \tau_a > \frac{\ln(\mu_1 \mu_2_2)(T_f - t_0)}{\ln(\frac{\mu_1 \mu_2_2}{\xi_i} \times \mu_2)} - \lambda \tau^+ (t_0, T_f) - \lambda \tau^- (t_0, T_f) \]
then switched system (10) is finite-time stable with respect to \( (\xi_i, \xi_j, T_f, \sigma(t), \sigma'(t)) \) under the feedback controller
\[ u(t) = K_{\sigma(t)} x(t) \]
where \( T(t_f, T_f) \) represents the total matched control time and \( T(t_0, T_f) \) is the sum of the mismatched control time during the interval \([t_0, T_f] \). \( \Pi_i = (A_{\alpha_i} X_i + B_{\alpha_i} R_i Y_i) + (A_{\beta_i} X_i + B_{\beta_i} R_i Y_i)^T + \alpha_i B_{\theta_i} Q_i B_i^T \]
\[ (\xi_i + \delta_i) H_i H_i^T - \lambda_i X_i \]
\[ \Pi_j = (A_{\alpha_j} X_j + B_{\alpha_j} R_j X_i) X_i + (A_{\beta_j} X_j + B_{\beta_j} R_j X_i)^T + \xi_j B_{\theta_j} Q_j B_j^T + (\xi_j + \delta_j) H_j H_j^T - \lambda_j X_j \]
\[ X_{ij} \leq \inf_{i \leq j} \{ \lambda_{\alpha_i} (X_i), \lambda_{\alpha_j} (X_j) \} \]
\[ \sup_{i \leq j} \{ \lambda_{\alpha_i} (X_i), \lambda_{\alpha_j} (X_j) \} \]
Proof: Substituting (4) and (5) into (1), the closed-loop system becomes \( \dot{x}(t) = (\hat{A}_{\alpha_i} + \hat{B}_{\alpha_i} R_{\alpha_i} \sigma(t), K_{\sigma(t)}) x(t) \).
Write
\[ \hat{T}_i = \left[ \begin{array}{ccc} \lambda_i & Y_i^T R_i & j^{1/2} \\ 0 & -\alpha_i I \end{array} \right] \]
where
\[ \hat{\lambda}_i = (\hat{A}_{\alpha_i} + \hat{B}_{\alpha_i} R_{\alpha_i}) + (\hat{A}_{\beta_i} + \hat{B}_{\beta_i} R_{\beta_i}) \lambda_i + \alpha_i \hat{B}_{\theta_i} Q_i B_i^T - \lambda_i X_i \]
By Schur Complement Lemma, \( \hat{T}_i < 0 \) is equivalent to
\[ \hat{\lambda}_i + \alpha_i^{-1} Y_i^T R_i Q_i R_i Y_i < 0 \]
(26)
Substitute (2) to (26), we have
\[ \Lambda_i + H_{\alpha_i} U_i (E_{\alpha_i} X_i + E_{\beta_i} R_i Y_i + \alpha_i E_{\theta_i} Q_i B_i^T) \]
\[ + \alpha_i H_{\theta_i} U_{\hat{i}} E_{\theta_i} Q_i E_i U_i^T H_i^T < 0 \]
By Lemma 1, (27) is equivalent to
\[ \Lambda_i + \alpha_i H_i U_i E_{\theta_i} Q_i E_i U_i^T H_i^T < 0 \]
Therefore, we can come up with the following
\[ \alpha_i H_i U_i E_{\theta_i} Q_i E_i U_i^T H_i^T < 0 \]
We can easily get \( \hat{T}_i < 0 \) from (24) and the following

4. Numerical Simulation
Consider system (1) with the following parameters:
\[ A_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_i = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_i = \begin{bmatrix} 0.2 & 0.5 \\ 0.3 & 0.1 \end{bmatrix}, \quad E_{\theta_i} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}, \quad H_i = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \]
The actuator fault model with parameters as follows:
\[ 0.1 \leq r_{11} \leq 1.3, \quad 0.1 \leq r_{12} \leq 1.2, \quad 0.2 \leq r_{21} \leq 1, \quad 0.1 \leq r_{22} \leq 1 \]
Then we have
\[ R_{\alpha_1} = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.65 \end{bmatrix}, \quad \lambda_1 = \begin{bmatrix} 6/7 & 0 \\ 0 & 11/13 \end{bmatrix}, \quad \lambda^- = 100, \lambda^+ = 10, \mu_1 = \mu_2 = 20, T_f = 0.007, t_0 = 0, \]
\[ \tau_a = 0.0041, \quad c_1 = 1, c_2 = 100 \] and by Theorem 2, we can obtain a set of feasible solutions
Then from Remark 3 we know that the matched period $T^-(t_0,T_f)$ and the mismatched period $T^+(t_0,T_f)$ satisfy the following relation
\[
100T^+(t_0,T_f) + 10T^-(t_0,T_f) < 0.1899
\]

Note that $T^+(t_0,T_f) + T^-(t_0,T_f) = 0.007$, then we have
\[T^+(t_0,T_f) < 0.0013\]
\[0.0057 < T^-(t_0,T_f) < 0.007\]

So we find the value range of $T^-(t_0,T_f)$ and $T^+(t_0,T_f)$ with given value of dwell time $\tau_a$. We choose $\Delta_k = 0.001$ and unknown time-varying matrices
\[
U_i(t) = \begin{bmatrix}
\sin t & 0 \\
0 & \sin t
\end{bmatrix}
\]

Subsystem 1 and subsystem 2 have the following fault matrices
\[
R_i = \begin{bmatrix}
0.6 & 0 \\
0 & 0.8
\end{bmatrix}, \quad R_2 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

The state response and the corresponding control law of the resulting closed-loop system are shown in Fig. 2 and Fig. 3 under the given condition.

Fig. 2 state response $x(t)$ of the system

Fig. 3 switching control law $u(t)$ of the system

From above figures, it is concluded that the robust fault-tolerant asynchronously switched controllers $K_1, K_2$ ensures that system (1) is finite-time stable with respect to $(1,100,0.007,\sigma(t),\sigma'(t))$. Therefore, the proposed method is effective to overcome actuator failures despite of asynchronous switching.

5. Conclusion

In this paper, we address the finite-time robust fault-tolerant control for switched system with asynchronous switching. Due to the asynchronous switching phenomenon widely existed in switched system and the importance of finite-time stability for switched system, a robust fault-tolerant asynchronously switched controller of switched system has been developed and the corresponding stabilizing switching law is derived by the average dwell time approach. All conditions are given in a set of matrix inequality format and depend on the switching mode of the system. A numerical simulation shows the effectiveness of the obtained results.

Acknowledgment

This work was supported by the Pre-Research Foundation of PLA University of Science and Technology under Grant no. 20110316.

References


and the actuator fault input is \( f^+ \) and \( f^- \), the Lyapunov-like function is
\[
V_t(x(t)) = x^T(t)P_t x(t)
\]
of feedback switched systems, \textit{Automatica}, 45(7), 2009, 1703-1707.


\section*{Appendix}

\textbf{Proof} In case of \( \Delta_k > 0 \) and \( x(t) \in \mathcal{Q}_1 \), for the \( i \) th subsystem, the corresponding controller is \( u(t) = K_i x(t) \) and the actuator fault input is \( u'(t) = R_i u(t) \). Then we can obtain the following closed-loop system
\[
\dot{x}(t) = (A_i + B_i R_i K_i) x(t)
\]
For system (28), we consider the Lyapunov-like function candidate as follows
\[
\dot{V}_i(t) = x^T(t) P_i x(t)
\]
By (14), we have
\[
\dot{V}_i(t) < \lambda^2 V_i(t)
\]
In case of \( \Delta_k > 0 \) and \( x(t) \in \mathcal{Q}_2 \), owing to asynchronous switching, for the \( j \) th subsystem, the sub-controller is still \( u(t) = K_j x(t) \) and the actuator fault input is \( u'(t) = R_j u(t) \). Therefore the closed-loop system is described as
\[
\dot{x}(t) = (A_j + B_j R_j K_j) x(t)
\]
For system (31), we choose the following Lyapunov-like function
\[
\dot{V}_j(t) = x^T(t) P_j x(t)
\]
By (15), it can be derived that
\[
\dot{V}_j(t) < \lambda^2 V_j(t)
\]
From the definition of \( \mathcal{Q}_1 \), \( \mathcal{Q}_2 \), the Lyapunov-like function (29) and (32) is transformed as follows
\[
\dot{V}_i(t) = x^T(t) P_i x(t) \quad t \in [t_k, t_{k+1}], \quad k = 1, 2, \ldots
\]
and
\[
\dot{V}_j(t) = x^T(t) P_j x(t) \quad t \in [t_k, t_{k+1}], \quad k = 0, 1, \ldots
\]
Let \( t_0 < t_1 < t_2 < \cdots < t_k = T_f \) is the switching time in \([t_0, T_f]\), we define piecewise Lyapunov-like function as follows:
\[
V(t) = \begin{cases}
\dot{x}^T(t) P_i x(t), & t \in [t_i, t_{i+1}], i = 0, 1, \ldots, k - 1 \\
\dot{x}^T(t) P_j x(t), & t \in [t_i, t_{i+1}], i = 0, 1, \ldots, k - 1
\end{cases}
\]
By (30) and (33), using the iterative method leads to
\[ V(t) < e^{\lambda(t_k+\Delta_k+\lambda \Delta_k)} V(t_k+1) \]
\[ < \mu \lambda e^{\lambda(t_k+\Delta_k)} V(t_k+1) \]
\[ < \mu \lambda e^{\lambda(t_k+\Delta_k+\lambda \Delta_k)} V(t_k) \]
\[ < \mu \lambda e^{\lambda(t_k+\Delta_k+\lambda \Delta_k)} e^{\lambda(t_{k-1}+\Delta_k)} V(t_{k-1}+\Delta_k) \]
\[ < \mu \lambda e^{\lambda(t_k+\Delta_k+\lambda \Delta_k)} e^{\lambda(t_{k-1}+\Delta_k)} e^{\lambda(t_{k-2}+\Delta_k)} V(t_{k-2}+\Delta_k) \]
\[ < \mu \lambda e^{\lambda(t_k+\Delta_k+\lambda \Delta_k)} e^{\lambda(t_{k-1}+\Delta_k)} e^{\lambda(t_{k-2}+\Delta_k)} e^{\lambda(t_{k-3}+\Delta_k)} V(t_{k-3}) \]
\[ \ldots \]
\[ < \mu \lambda e^{\lambda(t_k+\Delta_k+\lambda \Delta_k)} e^{\lambda(t_{k-1}+\Delta_k)} e^{\lambda(t_{k-2}+\Delta_k)} \cdots e^{\lambda(t_{k-N}+\Delta_k)} V(t_0) \]
\[ < \mu \lambda e^{\lambda(t_k+\Delta_k+\lambda \Delta_k)} e^{\lambda(t_{k-1}+\Delta_k)} e^{\lambda(t_{k-2}+\Delta_k)} \cdots e^{\lambda(t_{k-N}+\Delta_k)} V(t_0) \]

where \( T^+ (t_0, T_f) \) represents the total mismatched control time during the interval \([t_0, T_f]\). \( T^- (t_0, T_f) \) is the sum of the matched period in \([t_0, T_f]\). By (34), we can obtain

\[ V(t) \geq \inf \{ \lambda_{\min}(P_i), \lambda_{\min}(P_j) \} \|x(t)\|^2 \]  

(36)

Since

\[ V(t_0) \leq \sup \{ \lambda_{\max}(P_i), \lambda_{\max}(P_j) \} \|x(t_0)\|^2 \]  

(37)

and

\[ \|x(t_0)\| \leq c_1 \]  

(38)

we can deduce that

\[ V(t_0) \leq \sup \{ \lambda_{\max}(P_i), \lambda_{\max}(P_j) \} c_1^2 \]  

(39)

Altogether (35)-(39), it easily follows that

\[ \|x(t)\|^2 \leq \mu_{\max}^2(\mu_1 \mu_2)^{k(t_0, T_f)} e^{\lambda T^+ (t_0, T_f) + \lambda T^- (t_0, T_f)} \frac{c_1^2}{\kappa} \]  

(40)

where \( \kappa = \sup \{ \lambda_{\min}(P_i), \lambda_{\min}(P_j) \} \).

Note that \( k(t_0, T_f) = N_0 \), we get

\[ k(t_0, T_f) \leq N_0 + \frac{T_f - t_0}{\tau_a} \]  

(41)

Without loss of generality, let \( N_0 = 0 \), thus

\[ k(t_0, T_f) \leq \frac{T_f - t_0}{\tau_a} \]  

(42)

This together with (16) gives

\[ \mu_{\max}^2(\mu_1 \mu_2)^{k(t_0, T_f)} e^{\lambda T^+ (t_0, T_f) + \lambda T^- (t_0, T_f)} \frac{c_1^2}{\kappa} < c_2^2 \]  

(43)

Substituting (43) into (40) leads to

\[ \|x(t)\|^2 < c_2 \]  

(44)

Thus, the proof is completed. In case of \( \Delta_k < 0 \), the research approach is similar to above proof procedure, and we would get the same conclusions. It is omitted here.