A NOVEL CONTROL SYSTEM DESIGN FRAMEWORK AND ITS APPLICATION TO ROBUST STEEL ROLLING

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ABSTRACT
A control system design framework based on characteristic transfer functions is presented and applied to robust control of a tandem cold rolling mill. Compared to conventional design scheme such as Q-parameter approach, the proposed one enables numerical optimization with a prescribed order controller. Moreover, multi-objective control problems can be dealt with step-by-step in order of precedence. Namely, various practical control problems can be flexibly considered based on the scheme.

We, then, applied this scheme to thickness and tension control for a tandem cold mill rolling of steel. It is required high product quality in the tandem cold mill to compete globally. Several robust control strategies have been applied to design a controller for a rolling mill [10]-[14]. Among them, Inverse Linear Quadratic (ILQ) theory is successfully installed at real works [13],[14]. Simulation results described that the proposed scheme offers a potential for improvement compared to conventional ILQ design.

KEY WORDS
Control system design, robust control, rolling mill, transfer function, numerical optimization.

1. Introduction
Control system design is formulated as finding a controller to meet control requirements for a given plant. The control requirements are often represented by transfer functions of a closed loop system. Thus, describing all realizable closed loop maps is fundamental issue to execute the superior design.

The Q-parameter approach [1]-[4] based on so-called Youla (or Q)-parameterization [5] is one of the standard solutions of the problem. One can describe any realizable closed loop transfer function with the parameter $Q$ that is arbitrary proper stable transfer matrix. In the Q-parameter approach, to design a controller results in deciding the function $Q$ to realize desirable closed loop maps.

In this paper, we present alternative unified design approach [6],[7] based on characteristic transfer function matrix [8],[9]. As contrasted with functional optimization with respect to the $Q$, the proposed design scheme deals with non-convex optimization and enables numerical optimization for a prescribed order controller, since the design freedom appears as real variables in the coefficients of the characteristic transfer functions. Moreover, multi-objective control problems can be dealt with step-by-step in order of precedence by assigning each objective to a part of the variables. Namely, various

practical control problems can be flexibly considered based on the scheme.

In this chapter, firstly, we show the general control scheme. Then, we introduced theorems on realizable closed loop transfer functions referred to as Dual Model [6],[7]. Finally, control system design based on the dual model is developed.

2. Control System Design based on Dual Model

General control systems have the structure of Fig.1.

![Fig.1 General control system configuration](image)

Where, signal $r$, $u$, $y$, $y^*$ and $q$ mean reference inputs, control inputs, controlled outputs, feedback signals and disturbances, respectively. Signals $v$ and $v^*$ are conceptual signals added $z$ and $y^*$ respectively.

In Fig.1, we obtain the following one-to-one relationship between controllers and closed-loop system.


\[ \begin{bmatrix} W_{x_a} \ W_{x_b} \end{bmatrix} = (I - C_{a_x} \cdot P_{x_b})^{-1} \cdot [C_{a_x} \ C_{a_y}] \]  
(1)

\[ \begin{bmatrix} C_{a_x} \ C_{a_y} \end{bmatrix} = (I + W_{x_a} \cdot P_{x_b})^{-1} \cdot [W_{x_a} \ W_{x_b}] \]  
(2)

Where, \( W_{ab} \), \( P_{ab} \), \( C_{ab} \) denote transfer functions of a closed loop system, a plant and a controller, from signal \( a \) to signal \( b \), respectively.

Since the controller fixes all closed loop properties, it is noted that every property of the control system can be represented by the pair matrix \( [W_{x_a}, W_{x_b}] \), called Dual Model. Any other closed-loop transfer matrix can be expressed by the dual model. For instance, closed loop transfer matrix between disturbance and the controlled variables: \( W_{x_d} \) can be written as

\[ W_{x_d} = P_{x_d} + P_{x_d} \cdot W_{x_u} \cdot P_{x_u} \]  
(3)

Consequently, a scheme of control system design can be formulated with the dual model as,

Find \([C_{a_x} \ C_{a_y}] \ s.t. [W_{x_a} \ W_{x_b}] \in W_d \subset W_f \) for given \( P \)  
(4)
where, \( W_d \) stands for desired dual model with respect to considering control specifications, and \( W_f \) stands for feasible dual models.

When we execute the formula (4), describing universal set of \( W_f \) is necessary to obtain the best solution. In the next section, we focus on the feasible dual models and introduce regarding theorems.

### 2.2 Realizable Dual Model

There exist several forms of transfer function descriptions for multivariable linear time invariant systems. We treat transfer function matrix with characteristic polynomial as its common denominator named Characteristic Transfer function Matrices (CTM). A necessary and sufficient condition of CTM is shown as the following theorem.

**Theorem 1: CTM conditions** [8], [9]

Suppose \( N(s) \) be a polynomial matrix and let \( d(s) \) be a polynomial. Then, \( G(s) = \frac{N(s)}{d(s)} \) is a Characteristic Transfer Function Matrix if and only if \( G(s) \) satisfies the following conditions.

(i) Every element in \( G(s) \) is a proper real rational function.

(ii) Every \( k \)-th minor of \( N(s) \) contains \( d(s)^{k-1} \) or being zero. Where, \( k = 2, \ldots, p \). \( p = \text{rank } N(s) \).  

We, then, consider the central subject: realizable dual model based on CTM expression.

**Theorem 2: DM conditions** [7]

When, general control system configuration shown in Fig.1 is considered, a characteristic transfer-function matrices: \( \begin{bmatrix} L_1(s), L_2(s) \end{bmatrix} \), where \( L_1(s) \) and \( L_2(s) \) have compatible dimensions with the corresponding input signals \( r, v^* \) and output signals \( u \), can be a dual model for a given plant if and only if the following conditions are satisfied.

(i) \( L_1(s) \) is CTM of the plant between \( u \) and \( v^* \), and 
\[ \text{det}\left(\frac{L_1(\infty)}{d(\infty)}\right) \neq 0 \] .

(ii) \( \begin{bmatrix} L_1(s), L_2(s), L_3(s) \end{bmatrix} \) satisfies CTM conditions (Theorem 1) and its **System Zeros** includes all **System Poles** of the plant.

Remark

**System Zeros** are defined as the roots of zero polynomial derived from Rosenbrock’s system matrix, and **System Poles** are eigenvalues of the system.

### 2.3 Dual Model Matching Design Procedure

Control system design strategy based on the dual model is called Dual Model Matching (DMM) [5],[6]. The DMM design procedure is to execute the formula (4) starting from the end, and summarized as follows.

Step 1. Derive a model of a given plant: \( P \) as CTM form.

Step 2. Derive the feasible dual model: \( W_f \) with the specified order.

Step 3. Decide the dual model by fixing free parameters to satisfy control specifications.

Step 4. Reduce the dual model to the controller by substituting the fixed dual model into Eq. (2).

Step 2 is executed by using Theorem 2 with symbolic formula manipulations. We can exploit computer capabilities and application software such as Mathematica™ or Maple™. Step 3 and 4 depends on individual design problems. Analytic solution can be obtained for specific problems such as exact model matching, pole and zero assignment, decoupling, zeroing, and obtaining the robust steady state property. Numerical approach can be applied to optimization or inequality constraints arise from physical limitations as well as from desired objectives.

As compared to so-called Q-parameter approach, proposed scheme has the following features:
1. Order of the controller can be specified.
2. Step-by-step design can be executed in order of precedence.
3. Not only analytical but also numerical approach can be applied.

These features come from that the proposed framework is characterized by characteristic transfer matrices and the freedom appears on coefficients of polynomials instead of function $Q \in RH^\infty$. Furthermore, the proposed framework does not distinguish parameters in plants and those in controllers. Then, simultaneous design of structural and controller parameters can be executed naturally. Also, to shift conditions of controllers to over-all systems with relations (1) and (2), we obtain the proposed framework does not distinguish parameters in plants and those in controllers.

3. Robust Control of a Rolling Mill

3.1 Overview of Plant Description

Typical five-stand continuous tandem cold rolling mill is illustrated as shown in Fig. 2 [14].

![Fig. 2 Structure of a typical tandem cold rolling mill](image)

The target plant is the first stand of the mill, since the gage accuracy is greatly influenced by control accuracy at the first stand. The plant has two controlled variables; mill entry tension: $\Delta T_e$ and the exit gage of the first stand: $\Delta h_1$. These controlled variables and tension bridle roll (BR) speed: $\Delta V_\beta$ as well are feedback signals to the controller. On the other hand, the plant has two control inputs; the motor drive torque of the BR motor: $\Delta \tau_o$ and the roll position reference of the first stand: $\Delta s_1$.

To maintain high product quality in strip processing lines is challenging problem, since thickness and tension control interfere with each other in severe and variable rolling environment. Several advanced robust control theories have been applied to solve this problem. Among them, inverse linear quadratic (ILQ) theory has advantages to obtain both prescribed tracking property and quadratic evaluation optimization, and is successfully installed at real works [13],[14].

In the following, the results are described in accordance with design procedure mentioned in subsection 2.3.

3.2 Design Procedure

Step 1. Derive a model of a given plant: $P$ as CTM form

The CTM of the plant is derived as the following.

$$P_{\nu\nu}(s) = \begin{bmatrix} 4.84 \times 10^{-4} s + 0.0822 & 120.7 \\ -9.05 \times 10^{-1} & s + 5.32 \times 10^{6} \end{bmatrix} \begin{bmatrix} 0.227 \\ 0.794(s + 0.737 \pm 20.56i) \end{bmatrix}$$

where, input and output signals are

$$u = [\Delta \tau_o, \Delta s_1]^T, \ y* = [\Delta V_\beta, \Delta T_e, \Delta h_1]^T.$$

Step 2. Derive the feasible dual model: $W_\gamma$ with the specified order.

Step 3. Describe a desired dual model: $W_\nu$ according to control specifications.

Here, above two steps are considered simultaneously. The order of the controller and tracking property are equated with those of ILQ design, which are expressed by the next CTM.

$$W_\gamma(s) = \begin{bmatrix} 100(s + 20) \\ 0 \end{bmatrix} \begin{bmatrix} 1.34 \times 10^8 s^2 \\ (s + 20)(s + 0.18)(s + 10.6 \pm 20.6i) \end{bmatrix}$$

where, the reference inputs and controlled variables are both strip gage and strip tension.

Then, one of the dual model: $W_\nu$ is fixed with the above $W_\gamma$. Another feasible dual model $W_{\nu_0}$ is derived with the $W_\nu$ through Theorem 2. Due to lack of space, the result is omitted, however, it should be noted that the $W_{\nu_0}$ contains eight free parameters in numerator matrix.

The remaining control requirements are to obtain robust strip tension properties described by the performance index $J$ as,

$$J = \int_0^T \left( L^T \begin{bmatrix} W_{\nu\nu}(s) & 1/s \end{bmatrix} \right)^2 dt \ s.t. \ H W_{\nu\nu}(s) \leq 0.04 \quad (5)$$

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where, \( W_{q1}(s) \) is CTM between \( q \): output disturbance added to tension and \( y1 \): tension. \( W_{q1}(s) \) is CTM between the \( q \) and \( u1 \): motor drive torque of the BR motor: \( \Delta \tau_u \). The former CTM regards cancelling tension disturbances, and the latter regards robust stability under additive perturbations.

The feasible \( W_{q1}(s) \) is derived using Eq.(3) with the \( W_{r1} \) mentioned above as

\[
W_{q1}(s) = \frac{s^4 + k_{v1}^2 s^2 + k_{w1}^2 s^2 + k_{r1}^2 s}{(s+20)(s+0.18)(s+10.6+20.6i)}
\]

(6)

where, \([k_{v1}, k_{w1}, k_{r1}]\) are free parameters.

The feasible \( W_{q1}(s) \) is obtained by the next relationship, and also contains the same free parameters as in \( W_{q1}(s) \)

\[
W_{q1}(s) = \frac{P_{r1}(s)}{s} W_{q1}(s)
\]

(7)

Step 4. Decide the dual model by fixing free parameters to satisfy control specifications.

To minimize the index \( J \), numerical optimization technique can be applied. Here, we adopt Genetic Algorithms to solve this problem and obtained the next result.

\[
[k_{v1}, k_{w1}, k_{r1}] = [17.3, 399, -9.53]
\]

Step 5. Reduce the dual model to the controller

Five more underused parameters remains in \( W_{r1}, \) and other control specifications can be treated. In this design example, remaining parameters is set randomly and the design is finished.

Lastly, CTM of the unique controller \([C_r(s), C_{r1}(s)]\) is given as shown in the follows.

\[
C_r(s) = \begin{bmatrix}
-0.01 (s - 4.87) & 21.6 (s + 15755)(s + 0.014) \\
-2.34 \times 10^{-3} & 27.7 (s + 22.7)
\end{bmatrix}
\]

\[
(s - 15.3)(s + 7.20)
\]

\[
C_{r1}(s) = \begin{bmatrix}
-4.81 \times 10^9 (s + 124.9)(s + 2.06) & -0.074 (s + 10.66)(s + 0.7) & -0.021 (s + 12.1)(s + 0.40) \\
-4.37 (s + 20 + 7.38) & 2.70 \times 10^9 (s - 4.64 + 10.66) & -0.12 (s + 44.9)(s + 10.2)
\end{bmatrix}
\]

\[
(s + 15.3)(s + 7.20)
\]

3.3 Simulation Results

Simulation provides a comparison between the proposed DMM design and ILQ design. The \( \| \|_{\infty} \) of DMM and ILQ control system is 0.040 and 0.045 respectively. Fig.3 shows tension responses on step tension disturbance of both control systems. The responses show that DMM controller quickly and smoothly eliminates the influence of the disturbance compared to ILQ controller. In Fig.4, we see that DMM improves frequency response in especially around 20 (rad/s).

4. Conclusion

A control system design framework based on dual model matching theory has been presented. Various practical control problems can be flexibly considered based on the scheme. A gage and tension control for a tandem cold mill rolling of steel has been applied to the proposed design framework. Simulation results show that the proposed scheme offers a potential for improvement compared to conventional Invers Linear Quadratic design installed to actual works.

Acknowledgement

The authors are grateful to Mr. K. Kashima and Mr. S. Ootagoshi for assistance in several calculations.

References


