COUPLED ACOUSTIC-STRUCTURAL-PIEZOELECTRIC MODELING OF SYNTHETIC JET
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ABSTRACT
Synthetic-jet actuators (SJA) have been employed in a variety of roles including aerodynamic boundary layer control. In their most widespread configurations, these zero net mass-flow devices are actuated by a piezoelectric, periodically-oscillating which alternatively admits and then expels a gas from an enclosed cavity through a nozzle. The cyclic fluid motion in and out of the cavity forms a synthetic outward bound jet. The aim of the current paper is to present the development and validation of a modeling procedure based on finite elements, which includes multiphysics simulation. The SJA model was based on fully-coupled acoustic, structural and piezoelectric modeling. The predicted results were favorably compared to experiment. While coupled fluid-structural piezoelectric models are available, the proposed method is a much more convenient quick-design tool for SJAs due to reduced computational times based on the acoustic fluid approximation.

KEY WORDS
Synthetic Jet, Acoustic, Piezoelectric, Finite Element

1. Introduction
Synthetic-jet actuators have been the topic of numerous research endeavors [1-5]. These endeavors were motivated by the potential applications of these devices in aerodynamic flow-separation mitigation, chemical mixing [2-4]. Synthetic Jet Actuators (SJAs) are mechanical actuators that create a synthetic jet. A synthetic jet is a flow directed from the actuator nozzle to the environment and which is characterized by zero-net mass flux. The zero-net mass flow is realized by having the actuator ingest and then exhaust a fluid through an intake-outlet nozzle. The outward-bound flow may form, due to external fluid viscosity, vortex pairs which propagate away from the nozzle. The ingest-exhaust process is repeated at a certain actuation frequency, which generates a steady mean flow away from the actuator nozzle. The steady flow generation, combined with the zero-net mass flux property, allows synthetic-jets to be used in aerodynamic flow control [2], mixing chambers [3] and gas turbine combustion chambers[4].

Several types of SJA actuation method exist [1-4]. Piston-and-crankshaft – based mechanical designs were one of the existing types of SJA. Another SJA type featured an electromagnetically-driven diaphragm. The most common, third SJA-type used a piezoelectric disk to actuate the diaphragm. It is the third actuator type that was studied in the current work. In particular, the actuator featured axial symmetry around a central axis (Figure 1). Other geometry configurations exist for SJAs, but the axisymmetric configuration is the most widespread.

Figure 1 - A synthetic jet actuator

The orifice, or nozzle, periodically and alternatively intakes and expels the fluid. The oscillating diaphragm was composed of a unimorph piezoelectric actuator construction: a flat, circular metal shell bonded to a smaller diameter, piezoceramic round disk. The piezoceramic round disk was polled in the thickness direction (3 direction) and tended to expand, if free, in the planar directions (1 direction). Since the piezoceramic disk was bonded to the metal disk, however, as the piezoceramic disk tended to expand radially it actually flexed the composite diaphragm vertically, due to its offset from the neutral axis of the composite diaphragm. The charge (voltage) to the piezoceramic disk was applied to the parallel faces of the piezoceramic disk. The cylindrical housing of the SJA was welded to the oscillating diaphragm and to a round circular plate with the inlet-outlet nozzle. The SJA considered was physically an enclosed cavity with a single inlet-outlet.
The axisymmetric SJA has one fundamental acoustic (Helmholtz) frequency. The current work has focused on developing and experimentally validating an analytical model, created within finite element software, of the third type of SJA; the analytical model would feature coupled structural-piezoelectric-acoustic physics interaction. The studies SJA would be operating with air as the working fluid. The small size, simplicity, and ruggedness of a piezoelectric design would allow its use in aerospace applications for aerodynamic control.

1.1 Piezoelectric Materials

The current work focuses on piezoelectrically-driven SJAs. Piezoelectric materials are characterized by their exhibition of the direct piezoelectric effect. The direct piezoelectric effect means that any mechanical stress applied to the material itself results in a developed charge within the material. Piezoelectric materials also exhibit the inverse piezoelectric effect, as an applied charge results in material deformation or induced strain. Several materials exhibit piezoelectric behavior. Examples of these materials are quartz, rochelle salt, lead-titanate-zirconate (or PZT piezoceramics), barium titanate, bone and polyvinylidene fluoride (piezopolymers). Only PZT-type piezoceramics were considered in this study.

Piezoelectricity can be described using coupling between mechanical and electrical field electric equations. The electrical equations are:

\[ D = \varepsilon E \]  

where \( D \) is the electric charge density vector, \( \varepsilon \) is the permittivity matrix, and \( E \) is the electric field strength vector. The mechanical portion of the equations, which relate stress and strain, is written in matrix form as:

\[ S = sT \]  

Where \( S \) is strain vector, \( s \) is compliance matrix, and \( T \) is stress vector.

Piezoelectric materials combine these two constitutive equations into coupled equations, called the piezoelectric constitutive law, written in the Strain-Charge form as (8)

\[ S = s_E \cdot T + d_T \cdot E \]  

\[ D = d \cdot T + \varepsilon_T \cdot E \]  

The matrix subscripts in the above equations describe the conditions under which the material property data was measured. The subscript \( E \) denotes a constant electric field, and the subscript \( T \) denotes a constant stress field. The superscript \( T \) indicates a transposed matrix.

Data for piezoelectric materials may be given in Strain-Charge form, or Stress-Charge form (used in input for ANSYS finite element code). The piezoelectric constitutive law and associated conversions are shown in Equations 5-9.

\[ T = c_E \cdot S - e' \cdot E \]  

\[ D = e \cdot S + \varepsilon_s \cdot E \]  

\[ c_E = s_E^{-1} \]  

\[ e = d \cdot s_E^{-1} \]  

\[ \varepsilon_s = \varepsilon_T - d \cdot s_E^{-1} \cdot d_T \]  

In the Stress-Charge form, \( T \) is the stress vector, \( c \) is the stiffness coefficients matrix, and \( e \) is the piezoelectric coupling coefficients matrix.

2. Model Development

This section will describe the finite element model (FEM) development, including mesh development, material properties, boundary conditions (loading and constraints), and solver algorithm considerations. The goal of the work was to develop a finite-element based model, employing fully coupled acoustic-structural piezoelectric coupling of an axisymmetric, piezoelectrically-driven SJA. The advantage of having a coupled acoustic-piezoelectric-structure model is that the formation and characteristics of the exterior pressure distribution in vicinity of SJA could be better understood a priori to experiment. The finite element model developed incorporated axisymmetric structural-piezoelectric and acoustic finite elements, along with discrete spring and damper elements at the oscillating diaphragm hinges. The finite element model replicated an SJA found in the work of Gallas [11].

2.1 Modeling

The FEM of an axisymmetric SJA was developed within ANSYS™ software [7], Figure 2, where:

- \( R_s \) = Radius of substrate membrane (and chamber radius)
- \( R_{\text{piezo}} \) = Radius of Piezoelectric membrane
- \( D \) = Chamber depth
- \( L \) = Outlet half-length
- \( r \) = Outlet radius

Data for piezoelectric materials may be given in Strain-Charge form, or Stress-Charge form (used in input for ANSYS finite element code). The piezoelectric constitutive law and associated conversions are shown in Equations 5-9.
needed to be at least four elements thick in order to prevent shear lock. The brass layer provided coincident nodes with the piezoelectric and fluid meshes. The complete model, including exterior fluid is shown in Figure 3; the SJA is shown at top. A typical meshed model of only the synthetic jet device only is shown in Figure 4. The exit section and transition areas in the acoustic fluid domain were characterized by high pressure gradients. As such, these sections were modeled with a refined mesh to capture pressure spatial distribution.

Figure 3 - Meshed geometry used for 2-D axisymmetric model

Figure 4 - Zoom of SJA-only mesh

In Figure 4, the upper layer is the piezoelectric layer; the middle layer is the substrate layer; the lower layer is the fluid enclosed within the cavity, including the outlet. The model is axisymmetric about the vertical center line on the left-hand side.

The metal substrate and piezoelectric layers were modeled with axisymmetric PLANE13 elements. PLANE13 is a coupled-physics, structural element that allowed, due to presence of a VOLT degree of freedom, modeling of both piezoelectric and brass materials. Each element had four nodes. For the acoustic fluid domain, FLUID29 element was used. FLUID29 elements were also used to model the fluid inside SJA cavity and exterior. This element type allowed coupled-field analysis between fluid and solid elements. One option for this element was to allow mesh deformation, which was required for this model. The infinite borders of the exterior fluid domain were modeled using line FLUID129 elements.

The substrate material (brass) was modeled as a linear isotropic material with properties as shown in Table 1. Note that metal portion of the diaphragm was also modeled using PLANE13 elements with piezoceramic option, since compatibility between these and the piezoceramic plate needed to be preserved. However, all piezoelectric specific material properties (i.e. relative permittivity) of brass were set to zero.

PZT5A was chosen as the material for the piezoelectric layer for initial study since it was also used by Gallas [11]. PZT5A was modeled as an anisotropic elastic material. Properties were arranged with poling defined in the Y direction (ANSYS required that arrangement of axes). The associated properties used in the material model were taken from Gallas [11] and are presented in Tables 2, 3, and 4.

Table 1 - Brass material properties

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Elastic Modulus EX</td>
<td>8.963E10 Pa</td>
</tr>
<tr>
<td>Poisson's Ratio PRXY</td>
<td>0.324</td>
</tr>
<tr>
<td>Density</td>
<td>8700 kg/m³</td>
</tr>
<tr>
<td>Relative Permittivity εᵣ (orthotropic)</td>
<td>0 (in X, Y, and Z directions)</td>
</tr>
</tbody>
</table>

Table 2 – PZT5A Anisotropic elastic matrix, stiffness form [c] (GPa)

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>XYZ</th>
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<tbody>
<tr>
<td>121</td>
<td>75.2</td>
<td>75.4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75.2</td>
<td>111</td>
<td>75.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75.4</td>
<td>75.2</td>
<td>121</td>
<td>0</td>
<td>0</td>
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Table 3 – PZT5 Piezoelectric stress matrix [ε] (C/m²)

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
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<tbody>
<tr>
<td>X</td>
<td>0</td>
<td>-5.4</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>0</td>
<td>15.8</td>
<td>0</td>
</tr>
<tr>
<td>Z</td>
<td>0</td>
<td>-5.4</td>
<td>0</td>
</tr>
<tr>
<td>XY</td>
<td>12.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>YZ</td>
<td>0</td>
<td>12.3</td>
<td>0</td>
</tr>
<tr>
<td>XZ</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

Table 4 – PZT5 Relative permittivity (F/m) and density (kg/m³)

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<tbody>
<tr>
<td>Relative Permittivity, X</td>
<td>1730</td>
</tr>
<tr>
<td>Relative Permittivity, Y</td>
<td>1700</td>
</tr>
<tr>
<td>Relative Permittivity, Z</td>
<td>1730</td>
</tr>
<tr>
<td>Density</td>
<td>7700</td>
</tr>
</tbody>
</table>
The properties of air, defined as the working fluid, were given in SI units through the FLDATA command [16]. Standard, built-in ANSYS models were used for density and viscosity. Axisymmetric boundary condition about the vertical symmetry axis included the piezoelectric and substrate nodes along the centerline to vertical (Y-direction) movement only. The edge of the oscillating diaphragm metal layer was connected with linear springs and dampers (COMBIN14 elements) to rigid nodes. A fluid-structure interaction condition was applied via the SF command [16] between the bottom surface of the brass substrate layer and the top surface of the fluid enclosure, which have coincident nodes due to the mesh defined earlier. An axisymmetric fluid boundary condition was defined along the actuator centerline. The enclosing fluid mesh was supposed to extend to infinity, which was modeled via FLUID129 acoustic line elements. The only applied load in the model was a sinusoidal voltage (25V amplitude), specified as discrete values at specific time steps. When this alternating voltage was applied to the piezoelectric layer, strain was induced in the piezoelectric layer and the composite piezoceramic-brass plate was displaced vertically. The displaced composite diaphragm plate mesh is shown below in Figure 5.

![Figure 5 - Displaced diaphragm mesh with physical constraints and voltage loads applied.](image)

Transient loads were applied and thus the results were obtained using a transient solver. The time step was 1/25th of the period corresponding to the voltage actuation frequency. This timestep provided a good balance between accuracy (judged by smooth data and stability) and processor time. For actual simulations, a timestep of 1/50th of the period may be more appropriate. ANSYS used sequential iterative coupling between physical environments. In this case, the mechanical physics environment was solved first, obtaining thus displacements. The acoustic model was then solved with the displacements introduced in the mechanical section, thus obtaining acoustic pressures. The solver iterated between the two physics environments until the predictions of the two physics environments converged. The model then moved to the next time step and the process repeated until a complete solution was obtained. The code assumed that the fluid was adiabatic.

Required spring constants to match the stiffness of the experimental data were $3 \times 10^{12}$ N/m for the X-direction spring and $3 \times 10^8$ N/m for the Y-direction spring. Note that these are both very stiff values. As a final improvement, damping was added as a material property for both brass and PZT-5A. Damping ratios for both materials was set at $3 \times 10^{-6}$.

3. Theoretical Results

The numerical results extracted the following parameters values from the model: first acoustic resonance, output sound pressure level at various distances from actuator centerline versus frequency. The first two parameters of interest were the first acoustic resonant frequency. It has been shown in Reference [1] that the actuator output pressure reached a local maximum when the piezoelectric diaphragm was actuated at that frequency.

The predicted fundamental acoustic resonance was also compared to that obtained from theory. The fundamental circular acoustic resonance of a cavity with a neck of area $S$ and length $L$ was found to be from Reference [9]:

$$R_s = 11.75 \text{ mm}$$
$$R_{\text{piezo}} = 10 \text{ mm}$$
$$D = 6 \text{ mm}$$
$$L = 1 \text{ mm}$$
$$r=0.3 \text{ mm}$$

![Figure 6 – SJA model showing spring elements and constraints imposed in ANSYS.](image)
\[ \omega_0 = c \sqrt{\frac{S}{LV}} \]  

The quantities used in calculation were: 
- \( c (343 \text{ m/s}) \), the speed of sound in standard-conditions air;  
- \( L' \) the effective length of the neck (outlet) for a baffled resonator;  
- \( a \) \((0.3 \times 10^{-3} \text{ m})\) the radius of the circular cross-sectional area of the neck;  
- \( S \) \((0.28 \times 10^{-6} \text{ m}^2)\) the area of the neck and \( V \) \((2.6 \times 10^4 \text{ m}^3)\) the volume of the vessel. The effective length of the flanged neck, \( L' \), of radius \( a \), had to account for the neck’s radiation mass loading and was calculated to be:

\[ L = L + 1.7 \times 10^{-3} + 1.7 \times 0.3 \times 10^{-3} = 1.51 \times 10^{-3} \text{m} \]  

Thus, the Helmholtz-theory predicted fundamental acoustic frequency for the SJA to be:

\[ f = \frac{\omega_0}{2\pi} = 460 \text{Hz} \]  

The finite element model predicted the first acoustic resonance to be 463 Hz, as will be seen in Section 5.

4. Experimental Methods

The SJA used for measurements is shown in Figure 7. The experimental setup used in determining the actuator’s output levels is shown in Figure 8. The actuator was placed in a small lathe’s chuck. The PCB Piezotronics 130D20 microphone (45 mV/Pa nominal sensitivity and 10 to 15000 Hz +/-2dB range) to was placed along the actuator’s centerline and the distance to the actuator output nozzle could be adjusted very precisely using the lathe’s positioning mechanism. The assembly was surrounded by sound-absorbing grey polymer foam. Measurements were taken of sound amplitude at different locations along the centerline corresponding to various actuation frequencies.

5. Discussion

The results of experiments corresponded well with numerical analysis. In Figure 9, the exit pressure profile along the actuator centerline is displayed along with the ANSYS predicted pressure profile. The pressure profile was obtained for a 450 Hz voltage-actuation frequency. The experimental pressure profile obtained immediately next to the outlet was lower in magnitude than the one obtained analytically. That was because the acoustic model assumes inviscid, incompressible flow, where in reality complex vortex flow occurs next to the outlet [5]. Thus the finite element acoustic-structural-piezoelectric model would be better at predicting pressure distribution at distance past the mixing portion of the synthetic jet (4-5 diameters away or 3-4 mm away from the outlet) which occurs next to the outlet [5].

Figure 10 presents a comparison of experimental vs. numerically predicted pressure profile at approximately 3 mm away from the outlet, on the centerline vs. frequency.
Figure 10 – Experimental vs. analytical pressure along centerline at 2.92 mm distance from outlet for various frequencies

The acoustic resonant peak pressure amplitude was numerically over predicted at resonance due to the same inviscid assumption of the acoustic finite element model. The location of the first acoustic natural frequency was predicted by the FEM to be 463 Hz.

6. Conclusion

A finite element based, coupled-field, structural-piezoelectric-acoustic model of an SJA has been developed and validated against experimental data. The model considers an axisymmetric SJA. The model used the voltage applied to the piezoelectric oscillating diaphragm, the geometry and material incorporated in SJA construction, and the driving frequency as input. The model predicted acoustic pressure distribution, as a result of SJA actuation, in the exterior fluid. The pressure distribution at discrete locations away from and along the SJA nozzle centerline were measured and compared to experimental data. Output pressure at discrete locations (along nozzle centerline) versus driving frequency and voltage were obtained and compared to experimental data. Good agreement was obtained between experimental data and corresponding analytical predictions. The model provides a good approximation of the fundamental acoustic frequency, as well as the centerline pressure distribution for the first acoustic Helmholtz resonance (since compressibility effects were included in model). One limitation of an acoustic-based finite element model would be that the model would not be able predict turbulent jet formation in surrounding fluid due to inherent inviscid-fluid assumptions that are associated with acoustic wave propagation equations.

References
