NEIGHBOURHOOD, GEOMETRY AND INITIAL CONDITIONAL EFFECTS ON FAIRNESS AND AGENT LONGEVITY IN LARGE-SCALE SUGARSCAPE MODELS

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ABSTRACT

The agent-based SugarScape model of Epstein and Axtell has attracted interest and use in a number of sociological contexts. The model is usually configured on a regular square lattice of 51x51 cells. However, in this paper we explore how the model scales to much larger lattices and how it behaves with a range of different neighbourhood connectivity sizes and geometries including hexagonal, triangular, Moore and radial-proximity neighbourhoods. We compare the popular twin sugar peak initial model configuration with that of a flat sugar-field and use a number of metrics including the Gini coefficient of income distribution and other agent statistics to discuss the quantitative potential for this model on large scale systems. We focus on use of a single agent species type interacting solely with the spatial sugar-field. We report on how initial conditions dramatically affect the inequalities in agent opportunity and how this drives the model to longer agent population survival times.

KEY WORDS
agent-based modelling; SugarScape model; complex system; emergence; locality; geometry effects.

1 Introduction

Agent-based modelling (ABM) continues to be a powerful tool for exploring complex systems [8] in the sciences [26] as well as in sociological contexts [17, 18, 29]. Societies of agents [32] can be formulated in a number of simulation formats and can make use of techniques such as Markov chains [3]. We employ a spatial agent “animat” approach [12, 19], that provides a simulation environment for many simple individual animat agents to interact, and allows us to observe the macroscopic emergent patterns and other complex behaviours and capture them as quantifiable statistical metrics. This approach has been successful in modelling systems such as predator-prey ecologies [30].

The SugarScape family of models introduced in the social sciences by Epstein and Axtell [10, 11] has been widely explored to model income and financial consequences of a system of individuals or agents that interact on a spatial environment.

The SugarScape model has much in common with other automata models and also with game-theoretic spatial automata systems such as the Game of Life [16]; the Hawk-Dove model [21]; the Snowdrift model [15]; and cyclic automata [14,20] that can be used to study large scale emergent effects in complex environments.

In this present paper we explore relatively large spatial SugarScape systems of wealth-accumulating agents that are initialised in various starting patterns, with a fair and uniform wealth distribution. We observe how under appropriate conditions, considerable individual inequalities of agent wealth distribution emerge with benefits to the properties of the societal system as a whole. Figure 1 shows coloured agents superposed on an evolving sugar-field shaded on a gray-scale. Yellow agents have a lower band wealth and red agents have a higher band of accumulated sugar wealth.

Work has been reported in the research literature on SugarScape systems for modelling: medical systems [4, 22]; automotive applications [24]; and hazard management and terrorist attacks [7]. Newer variants of the SugarScape model incorporate social networks [31] and have been used for insurance applications [33] as well as tradi-
tional uses in wealth distribution [27], wealth adjustment [28] and tax model systems [1, 2].

Various simulation frameworks [25] have been used and reported for SugarScape simulations [5, 6, 13] with some recent work making use of specialist parallel computing platforms [23] such as graphical processing units [9]. For this present work we implemented our own simulation framework using the Java programming language and various modern constructs to optimise the performance and size of simulated system that was possible.

Our article is structured as follows. In Section 2 we describe the SugarScape model and how we simulate it using different neighbourhood sizes and lattice geometries. We also discuss some of the quantifiable metrics we use to track growing inequalities of wealth in the model system – such as the Gini coefficient. We present some selected results including screen-dump images, sugar-field surface plots and plotted metrics in Section 3. In Section 4 we discuss some of the implications of the model variations and offer some tentative conclusions and suggested areas for further study in Section 5.

2 Simulation Method

The SugarScape Algorithm used in this present work is based on a sugar-field of N sites with NA agents distributed randomly and uniformly across it spatially.

Algorithm 1 Single Species SugarScape Agent Model.

choose lattice size, shape, eg square 256²
choose neighbourhood N, eg Nearest, Moore, or Radial
for all runs do
initialise N sugar-field sites by pattern eg Flat/Peaks
initialise NA unique agent locations randomly
for all time steps, eg 2,000 do
for all agent i ∈ NA do
identify unoccupied neighbouring site with most sugar
pick one at random tie-break if more than one move to chosen new site
metabolise, using up “cost Cm of moving” sugar units from accumulated wealth
metabolise, using up “cost Cs of surviving” sugar units from accumulated wealth
consume, accumulating income I units of sugar from new site
end for
remove dead agents (with negative wealth)
record measurements, eg Gini coefficient
exit when no live agents remain
end for
normalise averaged measurements

We have simplified the model so that I is 2 units of sugar and the costs of moving CM and surviving CS are one unit of sugar each respectively.

The Java simulation system we developed allows us to plug in different lattice geometries including square, hexagonal and triangular. We can also plug in neighbourhoods including nearest-neighbour (4 neighbours on square, 3 on triangular and 6 on hexagonal); Moore neighbourhood (8 neighbours on square lattice) and larger neighbourhoods based upon a radial proximity distance. We experimented with radii of neighbourhoods from 1,2,3,4 and 5 giving neighbourhood set sizes of from 4 up to 80.

A convenient metric to quantify the wealth distribution or fairness of the agent system is the Gini coefficient G{wi} which is defined over the set of individual wealth values wi of the i = 1...NA agents as follows:

\[ G = \frac{N_A + 1}{N_A - 1} - \frac{2}{N_A(N_A - 1)} \sum_{i=1}^{N_A} R_i w_i \]  (1)

This is the Jasso-Deaton formulation and requires the rank Ri for each individual agent, sorted by wealth in descending order, so that the wealthiest agent has rank 1 and the poorest a rank of NA. The mean agent wealth is defined as \( \mu = \frac{1}{N_A} \sum_{i=1}^{N_A} w_i \) in the usual manner.

Other metrics that can be sampled include the mean, maximum and minimum wealth values for the agent population, and it is also useful to track the number of live agents remaining in the population and also simple maximum and mean quantities of sugar remaining at sites across the sugar-field.

We investigated two different sugar-field initialisation patterns. The first is the oft-studied twin mounds or peaks of sugar pattern. We generated this using the formula:

\[ F_2(x, y, t = 0) = F_2 \sin(2\pi x/L_x) \cdot \sin(2\pi y/L_y), s.t. F_2(x, y, t = 0) > 0 \]  (2)

This generates the twin peaked pattern shown in the figures and is useful as it clearly gives an unfair advantage to those agents who are near one of the two peaks and disadvantages agents located on the flat plains. We used a value of \( F_2 = 255 \).

A uniform flat field distribution was also investigated as an example that gives all agents equal statistical opportunities. In order to facilitate a reasonable comparison between the two different patterns, we set:

\[ F_0(x, y, t = 0) = F_0 \]  (3)

with a normalisation condition on \( F_0 \) so that:

\[ \frac{1}{N} \sum_{x,y,t=0} F_2(x, y) = \frac{1}{N} \sum_{x,y,t=0} F_0(x, y). \]  (4)

There is therefore an equal amount of total sugar in each system, and the same number of agents distributed randomly in each simulation initialisation. We used a ten percent formulation by spatial density of agents (\( N_A = \)
0.1N) which allows agents room to move. We restricted
the models so that only one agent can occupy a site at a time
but culled dead agents from the system so they no longer
blocked the movement of live agents.

3 Simulation Results

We investigated a number of different sugar-field
sizes including \(N = 64^2, 256^2, 1024^2, 2048^2\). These are
all feasible to run using our Java simulation code on desk-
top resources. In the case of large neighbourhood sizes the
very large systems slow down exponentially - largely due
to agent site traversal and memory cache missing effects.
For consistency we focus on the \(N = 256^2\) systems for
the results presented below. Each system is based on a av-
erage over at least ten independent runs. This is sufficient
to obtain suitably indicative error bars on the plots, calcu-
lated from standard deviations based on these independent
sample runs.

Figure 2 shows a progression of screen-dump snap-
shots of the model system as simulated on a \(N = 256^2\)
square lattice system. This is most convenient to il-
lustrate as it maps neatly onto a pixel by pixel screen-
dump. Other lattices such as triangular and hexagonal
are discussed below. These images are for time steps:
0, 16, 32, 64, 128, 256, 512, ... with the final shot taken at
step 534 just before all remaining agents starved as all the
sugar in the field was used up. We have coloured agents
by band of wealth so that yellow agents have a wealth of
between 0 and 16 units of sugar and those with 17 units or
above are coloured red. We discuss the wealth distributions
below, but these images make it clear that agents located
close enough to “find” the peaks or mounds of sugar are
likely to become super-rich compared to agents located on
the flat plains - who are likely to starve to death, or those
starting at the foot of the slopes which will eke out a sur-
vival but cannot compete with their richer brethren.

A different scenario emerges if we distribute the sugar
completely evenly so that agents all are given a fair or equal
opportunity to forage.

Figure 3 shows how agents move around the flattish
landscape more or less evenly consuming the sugar. Again
the agents are coloured by band of wealth. In this scenario
no agents become super rich, but they move around, graze
on sugar, gradually all consume it, and eventually die of
starvation.

These two scenarios provide our base for comparison
and so we can proceed to investigate how the different lat-
tices and geometries affect matters.

Figure 4 shows a selection of bulk metrics for the sys-
tem initialised with the twin sugar peaks. The metrics are
plotted against time on a log-log scale. The total sugar
available in the whole spatial field tails off relatively lin-
erly initially, but this spatial structure steepens as agents
use up the sugar in the heaps. The number of ants remains
relatively flat in this model after the initial cull from those
that starve in the flat plain regions due to a total lack of
available sugar there. The number of agents crashes when
the sugar runs out. The max and mean sugar availability
in each cell roughly follow the same trend as total sugar
in the system. The maximum wealth of any agent climbs
steadily until soon after the start of the population crash.
Figure 3. Progression of snapshots of coloured agents superposed on gray-scale sugar-field with initial flat configuration.

Figure 4. Metrics for the Twin Peak Initial Configuration on a $256^2$ lattice.

The last remaining agents are still relatively rich, but even the super-rich agents lose out before starvation as they frantically move around looking for sites with remaining sugar. Note that the mean agent wealth does not follow the same shape trend as maximum agent wealth - reflecting the uneven distribution of individual wealths among agents, that amplifies with time in this system.

Figure 5 shows bulk metrics for a system initialised with a flat even sugar distribution. The total sugar tails off more sharply but flattens out at the end of the curve indicating unforaged sugar is left in the system - unfound by the last remaining spatial agents before they starved. Likewise the number of agent/ants and the total agent wealth in the system tail off with a sharper crash than in the case of the twin-peaks initialised system.

Note also that the minimum sugar in the field is non-zero for much of the time in this model. The sugar-field is initially a flat uniform value but is eroded by wandering agents. Mean, minimum and maximum sugar all follow similarly shaped curves however.

Note that mean agent wealth is smoothly changing only over a limited scale except towards the time of the population crash. It fluctuates wildly at the end as there are some random patches of agents that survive up until the end, while others evaporate through starvation. This is spatially random, with a range of different patch sizes that give rise to these diverging fluctuations.

Figure 5. Metrics for the Flat Initial Configuration on a $256^2$ lattice.

Figure 6 shows the Gini coefficient evaluated for the twin peak initialised model system. It is shown on a lin-lin scale (top) and on a lin-log scale (below) to emphasise the early time stages more. Each of the coloured lines represents a different geometry/neighborhood. Broadly they group by size of neighborhood. So the triangular lattice which has 3 neighbours has a lower Gini coefficient at the mid time ranges than does the systems with a larger neighbourhood size.
A Gini coefficient of unity represents a fair distribution, but note that it falls rapidly as some agents attain more wealth than others. Generally the highly localised lattice and neighbourhood geometries worsen the unfairness. If the system allows large agent movements from the radial proximity neighbourhood scheme for example, then the Gini coefficient saturates out at close to 0.5 for much of the duration of the simulation. This changes and large fluctuations appear as the agent population starts to crash once the bulk of the sugar peaks have been used up. Looking at the lin-log plot we see that at the early stages the localised neighbourhoods retain an initial fairness of distribution of wealth. Once agents have started to move towards the peaks however this does not persist and the Gini coefficient falls accordingly. Generally the curves are quite smooth and the error bars indicate typical behaviours not dominated by random pathologies in the initial agent spatial distributions. The kinks and ringing/overshooting phenomena are consistent across the independently sampled runs and appear to be characteristic of the particular twin-peak initialisation pattern.

Figure 7 shows the Gini coefficient for the flat sugar-field initialised system. Again locality encourages initial fairness as the lin-log plot shows, but as time develops the system shows something of the opposite effects from the neighbourhood sizes and lattice geometries. Fairness is encouraged by agents that cannot move large distances for the flat initialisation configuration. In this model most agents stay relatively uniformly “middle class” with few exceptionally rich agents. In addition, agents die off more steadily without the initial 50% cull as the twin-peaks model exhibits. The flat system crashes with divergent fluctuations in agent wealth distributions and hence in the Gini coefficient around the time of the crash.

The different behaviours between the twin-peaked sugar mounds system and the flat sugar distribution system are partially explained by observing the changes in the sugar-field spatial pattern as it is consumed.

Figure 8 shows surface plots at times 0, 84, 182, 533 for the twin-peaked sugar-field. Agents erode the peaks from the top downwards as they seek sites with the highest sugar content. As agent pressure mounts the peak is eroded and the last snapshot is captured just after one of the peaks has been completely consumed, and the other has already
been reduced to stubble.

The twin-peaked uneven distributed system would typically end with the entire sugar-field consumed and any remaining agents finally starving. The flat system however would usually give rise to some unconsumed random patches of sugar remaining where agents were unable to find it before starving. Since the uniform distribution of agents in the twin peak system would usually mean approximately half the agent population (in the flat plains) died out quite early, it was found that on average this system would run to approximately 600 steps before all agents died. Whereas the flat plains system would give all agents an equal opportunity but would therefore typically last only around 300 steps before all sugar was consumed and all agents starved.

4 Discussion

In addition to tracking the Gini coefficient, a time plot of the number of live agents is also revealing about the effects of the different initialisation patterns and lattice geometries on the model system behaviours.

Figure 9 shows the number of agents in the model systems as time progresses. There are two groups of curves shown. The flat sugar distribution crashes quite sharply at around a time of 250-300 steps and the sharpness appears to be related to the neighbourhood size. The triangular system...
is least sharp and the radial proximity system with radius 5 is the sharpest.

The twin-peaked system has half the agents rapidly die out and then settles to a steady agent population before the crash. In this case however the systems with a higher number of neighbours crashes earlier and the systems with smaller neighbourhoods crash later. We hypothesise that since the sugar stubble from the eroded peaks is arranged in a relatively localised pattern, it profits agents not to try to move too far away from eroding sugar remnants. Once again the higher neighbourhood sized systems are sharper in their crash behaviour.

Figure 10 shows a single time snapshot of the wealth distribution \( \{ w_i \} \) at time \( t = 150 \) steps. This time was chosen as it represents a regime when both model systems still have a large and viable agent population. The frequency populations are on a log-base-2 scale but the wealth bins are linear. In the flat uniform initialised system only a few wealth values ever occur. Most agents have similar wealth values. In the twin peaked system the distribution is much wider and less even. Some agents achieve high wealth values but most have small wealth accumulations of sugar.

In this work we have focused on finite sugar resources and therefore on models with a definite time to completion time. The simulations end with all agents starved. Other reported work focuses on dynamic equilibria effects. In such systems sugar is replenished and an attempt is made to keep the agent population alive and viable. To attain this new sugar is injected into the system continually. Our work provides a definite set of base parameters and quantifiable observations upon which to properly calibrate and interpret a dynamic continual agent population system.

5 Conclusion

We have implemented a SugarScape simulation model capable of scaling to large agent populations and sugar-field sizes. Our model is also capable of supporting different lattice and neighbourhood geometries and we have investigated these in the context of two model initialisation configuration patterns. Despite our simplification of the SugarScape model and utilisation of only a few model parameters, the classic twin peaked sugar mound pattern initialisation reproduces some of the behaviours reported in the SugarScape literature - namely that of growing inequality of the agent wealth distribution.

We have shown how to quantify the agent wealth inequalities using the Gini coefficient and have also related this behaviour to other bulk systems metrics such as agent population and sugar-field statistics. Our experimental technique has minimised interpretation of statistical anomalies that would arise from single system runs - multiple runs being possible from a performance optimised code capable of running large systems sizes in reasonable execution times.

We have confirmed the qualitative observation that the SugarScape model can illustrate the contrast between fair systems with relative equality where individuals fare approximately equally well, and unfair systems where there is a wide disparity between individual wealths but for which the system as a whole does better - and lasts longer before agents all die out.

There is scope for the investigation of a number of other microscopic agent behaviours and a determination of their effects on the bulk system. Multiple agent species that interact with one another by breeding or predating on one another is one possibility. Agents as part of a system where they can trade commodities (sugar) or multiple commodities with one another opens up scope for use of the base model to study trading ensembles. Applying more sophisticated geometries and agent barriers or preferred routes would also allow investigation of more realistic scenarios with connecting routes, such as roads or airline networking.

In summary, the SugarScape model offers a rich set of modelling possibilities in sociological and financial modelling applications where a realistically large number of interacting agents may be able to better capture complex, emergent behaviour than could a more analytical approach or formulation. A scalable and high performance simulation code will be important to attain realistically large system sizes of individual agents.

References


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