NEIGHBOURHOOD SIZE DEPENDENCE OF COOPERATOR CLUSTER EMERGENCE IN THE SPATIAL VOLUNTEERS’ DILEMMA

Ken A. Hawick
Department of Computer Science, University of Hull, Cottingham Road, Hull HU6 7RX, UK.
Email: k.a.hawick@hull.ac.uk, Tel: +44 01482 465181 Fax: +44 01482 466666

ABSTRACT
Public good scenarios, where shared resources must be managed to best effect, are important applications of agent-based modelling techniques and continue to be a challenge. Models of small numbers of agents and associated theoretical analyses can be insightful, but often large scale simulated systems are needed to capture the spatial and emergent complexities of realistic public goods scenarios. We develop the Volunteers’ Dilemma as a spatial agent-based model and explore how it is affected by changes in the temptation for individual agents to defect in groups of differing size. Our game playing group size is controlled by the lattice and neighbourhood geometry used, which we are able to make a parameter of the model through customised software. We explore spatial component cluster statistics for cooperation and defection and discuss their utility in understanding phase transitional behaviour in the model.

KEY WORDS
Spatial game theory; volunteers’ dilemma; agent-based model; complex system; emergence.

1 Introduction
Game theory has proved a valuable tool for understanding the complex phenomena that occur in sociological [3] and interactive systems. Spatial games played with individual agents that are constrained to interact with their local neighbours on, for example, a lattice can be especially revealing as they allow individual spatial regions to manifest different behaviours and to interact - much as real animals or people do in sociological settings [22]. Such spatial models go beyond theoretical analyses and can capture fluctuations and other details not possible in a mean-field approach.

Game theory and game models arise in a number of contexts including physical [11] and sociological systems [9]. Game theory [27, 30] makes a number of predictions concerning the behaviour of agents interacting in a spatial system [25]. Cooperation [4, 20, 26] is an important trait in agent-based systems and the study of the emergence [29] or suppression of cooperation in a complex system of many independent agents is an important area. Spatial emergence can be modelled using a game theoretic formulation [30] such as the iterated spatial prisoners’ dilemma [27, 28].

Figure 1. Growth of region of (red) defection on a hexagonal mesh, showing the largest cluster of defectors outlined in black against a background of blue cooperators/volunteers.

This scenario is based on the well-known two-player prisoners’ dilemma [4]. The Prisoners’ Dilemma is well known and has had much related work on it reported in the literature. An interesting variant is the Volunteers’ Dilemma, which has a more asymmetric character and models public goods situations [2, 8, 23] such as human volunteers or for example prairie dog groups where one has to “volunteer” to be a lookout - at some cost to the individual [36], but at huge public good to the group.

Figure 1 shows our implementation of the spatial Volunteers’ Dilemma on a hexagonal mesh with red defector agents encroaching upon a background of blue cooperators, with the largest cluster of defectors outlined in black.

Other related game-theoretic models include the Snow-drift model and the Hawk-Dove model [18]. The Snow-drift game model [10] is another interesting theoretical construct that can be used to model the microscopic individual agent behaviour in a spatial agent-based system [34] where many individual agents interact, each with a localised subset of the total number of agents. We describe this subset as their localised neighbourhood. The Prison-
Hawk-Dove and other spatial game-theoretic models. The Dilemma can be studied as a spatial agent-based model in Section 2. We present a selection of model studies of the Volunteers’ Dilemma including characterisations such as the optimal size of the social interaction group [1]; the asymmetry between cooperation and defection [6]; the cost of punishment [31]; social traps [33]; an inversion of cooperation population [38]; and spatial fluctuations [40,41].

The main focus and contribution of this present article is to incorporate a spatial structure with systemically parameterised neighbour structures so that the Volunteers’ Dilemma can be studied as a spatial agent-based model in a manner similar to the Prisoners’ Dilemma, Snow-drift, Hawk-Dove and other spatial game-theoretic models. Our article is structured as follows: We describe our formulation of the volunteers’ Dilemma as a spatial agent-based model in Section 2. We present a selection of model snapshots illustrating the systemic behaviours as defection temptation is varied and for different lattice and neighbourhood geometries and connectivities in Section 3. In Section 4 we discuss some of the implications of components and cluster formation and identify a split between differing lattices as well as some critical neighbourhood sizes separating different behavioural regimes. We offer some conclusions and suggested areas for further study in Section 5.
Algorithm 1: Spatial Volunteers’ Model algorithm.

- identify temptation $T$ and hence payoff matrix
- identify geometry, dimension, neighbourhood size $|N|$ for 20 independent jobs do
  - initialize $N$ agents as 50/50 volunteer/defector mix
  - for time $t \leftarrow 1$ to $t_{\text{steps}}$ do
    - for agent $i \leftarrow 1$ to $N_{\text{agents}}$ do
      - identify $|N|$ agents in spatial neighborhood of agent $i$
      - play game against group of $|N|$ neighbours
    - end for
    - compute total payoffs of all agents from all plays
    - for agent $i \leftarrow 1$ to $N_{\text{agents}}$ do
      - identify a random neighbour $n$
      - consider copying its strategy from $n$
      - note new state for agent $i$ at step $t + 1$
    - end for
    - commit new state $t + 1$ for all agents
    - gather statistical metrics for time-step $t + 1$
  - end for
- normalise statistical averages
- end for

Games. Once the payoffs have been computed a mechanism is introduced so that the agent copies the strategy of a neighbouring agent that has the highest payoff.

Typically agent systems can be updated synchronously (all at once) or asynchronously, by stepping through the system in a random order and updating in situ. For the work reported in this paper we focus on simple synchronous updates where every agent is updated exactly once at each time-step.

Various authors have identified mechanisms for the agent strategy propagation. A common approach adopted [10] is to use a weighting function with a small noise term $\kappa$ so that at each time-step the focal agent adopts the strategy of a randomly chosen neighbour with probability:

$$p = \frac{1}{1 + \exp(\delta z / \kappa)}$$ (5)

where $\delta z$ is the difference between the total payoff amounts earned by the focal agent and its neighbour.

Setting $\kappa = 0.1$ provides a small but sufficient noise term to prevent agent species from accidentally becoming completely extinct during the simulation, and is the approach that has been adopted by many researchers using similar models. Several authors have argued that a small noise term does in fact present a more realistic scenario for comparisons with experimental data than would a completely noiseless system. We have found that the system equilibrium values are not sensitive to the value of $\kappa$ although the simulation time to reach equilibrium is. Too small a noise value means a species can become extinct due to finite size effects.

Algorithm 1 therefore summarises how the simulation is implemented. Our code was implemented in Java and is configurable to support both graphical interactive parameter mode as well as a speed-optimised batch mode to gather statistical averages.

3 Selected Simulation Results

We present a range of experimental snapshots and metrics characterising the spatial and temporal behaviour of the spatial Volunteers’ Dilemma model.

Figure 2 shows some snapshots of the spatial Volunteers’ Dilemma model in action. Each image shows a reasonably well mixed and equilibrated configuration at temptation parameter values of: $T = 0.01, 0.020, 0.25, 0.50$. The final image is not well equilibrated however and shows the sparsening pattern of blue cooperators rapidly slowly vanishing against a field of red defectors at around $T = 0.75$. This emphasizes the difference between the spatial Volunteers’ Dilemma and the spatial Prisoners’ Dilemma - in the latter, the transition is around $T = 1.5$ and the spatial patterns are much more structured in the Prisoners’ Dilemma.

Generally at very low temptation parameter values cooperators subsist and continue to survive in the system but at quite low spatial densities and generally in quite small clusters. The nature of the Volunteers’ Dilemma is that just one cooperator is needed in each neighbourhood and it appears that only one occasional neighbour copies the cooperator strategy to make groups of small clusters of 1, 2 or 3 cooperators.

It is instructive to study the whole system by plotting the fraction of cooperators and defectors at different temptation parameter values.

Figure 3 shows the time evolution of the fraction of cooperators in the whole model system for a nearest neighbour connectivity on a square and periodic lattice. These results have been averaged over 15 or 20 runs each with independently seeded initial agent configurations of a uniform mix of cooperators and defectors. Each curve is for a different value of the temptation parameter $T$.

Figure 4 shows the corresponding trends for the fraction of defectors in the whole system. We observe fluctuations superposed on the curves arising from spatial regions or clusters of activity. In all cases the system reaches a dynamic equilibrium within a few hundred time-steps of the simulation. We would therefore appear to be justified in taking a representative average over the final 1000 steps of a 2000 step simulation. Aside from fluctuations, this gives a trend against temptation parameter $T$ which we can plot for each of the geometric and neighbourhood sizes investigated.

Figure 5 shows the final or dynamic equilibrium mean values of the fraction of cooperators present in the model system in the long term, plotted against the temptation parameter. Each curve in subsequent plots is for a different geometry or neighbourhood. A triangular lattice with three neighbours is plotted in red, a nearest neighbour of 4 neighbours is plotted in green; a hexagonal lattice with 6 neighbours is shown in blue; a Moore neighbourhood on a square lattice is in orange; and radial proximity connectiv-
Figure 2. Snapshots of the model for temptation values $T = 0.01, 0.02, 0.25, 0.50, 0.75$.

Figure 3. Time evolution of the fraction of cooperators (volunteer agents) in the system for a nearest neighbour square lattice configuration on a $128^2$ system.

Figure 4. Time evolution of the fraction of defectors (non-volunteering agents) in the system for a nearest-neighbour (N1) square lattice configuration on a $128^2$ system.

Ities with 20 or 48 neighbourhood sizes are shown in pink and black respectively. It is useful to see how the geometry/lattice arrangement - partially characterised by neighbourhood size, affects the fractional agent populations but also the cluster component size distributions.

Firstly Figure 5 shows the fraction of cooperators on a log-log scale. This is useful as it emphasizes what is essentially a power-law relationship at the lower part of the temptation parameter scale. To a reasonable approximation it appears that:

$$f_C \approx T^{1/3}$$

The numerically fitted exponent is $0.35 \pm 0.05$ and it appears to be a universal value independent of the number of neighbours and geometry for $T < 0.2$. 
Figure 5. Fraction of cooperator volunteers as a function of temptation $T$ on log-log scale.

Figure 6. Fraction of defecting agents as function of temptation parameter $T$.

Figure 7. Mean equilibrium number of cooperator volunteer clusters in the system as a function of temptation $T$.

Figure 8. Mean equilibrium number of defector clusters as a function of temptation $T$.

Figure 6 shows the corresponding plot for the fraction of defectors present in the system. This is plotted on a linear scale and the low $T$ shoulder regime is of similar shape for all the neighbourhoods/geometries tested. However at higher temptation values there are kinks and other features in the curves that suggest changes in behaviour.

These bulk metrics do not differentiate the behaviours on the various geometries and lattices very quantitatively or indeed precisely. Work in the literature for agent-based models suggest that examining cluster component characteristics can give more insights into what is taking place in the models at the different regimes.

Figure 7 shows the number of separate cooperator components in the system in the long term, for different temptation values and with the different coloured curves for each lattice/ neighbourhood arrangement. We observe some quite subtle and complex variations for the various regimes. The number of cooperator clusters starts low at low temptation since all cooperators are effectively in one giant connected super-cluster. As temptation to defect rises, the super-cluster is eroded and defectors start to dominate and at some point the curves stop rising, overshoot as it were, and settle downwards as cooperators start to disappear from the system.

Figure 8 shows the corresponding curves for the number of connected clusters of defectors in the system. This emphasizes the asymmetry between cooperators and defectors in the Volunteers’ Dilemma model. The number of defector clusters rises most dramatically for the lattice with the smallest number of neighbours - the triangular and nearest-neighbour square lattices - with 3 and 4 neighbours respectively. These two low neighbourhood sizes lead to an inability for defector agents to form very large connected
clusters except at quite high values of temptation.

We can also examine the size of the largest “super-cluster” of cooperators or defectors as temptation and lattice are varied.

Figure 9 shows the size of the largest cluster of cooperators where we have plotted it against the log of the temptation parameter to expose the details at low $T$. At low values of $T$ the cluster is relatively large for the lattices with smaller neighbourhood sizes before falling with increasing $T$ and with a cutoff point $T_c$ that is inversely related to neighbourhood size.

In contrast the size of the largest defector cluster as shown in Figure 10 rises to a plateau value for the higher connected lattices quite rapidly with rising $T$ and is markedly slower for the triangular and nearest neighbour square lattices.

4 Discussion

We have seen how the different neighbourhood sizes are a rough guide to the changes in behaviour for bulk and cluster statistics. A compound metric is the number of cooperators within the neighbourhood of a cooperator, appropriately normalised by lattice size and current number of cooperators.

Figure 11 shows this property plotted on a log-linear scale against temptation to defect. It shows how the lattice/neighbourhood arrangements clearly fall into two groups. The triangular, nearest neighbour square and hexagonal (3,4,6 neighbour) systems start high and fall, whereas the Moore, and radial proximity systems (8, 20, 48 neighbours) start low and rise with temptation. All appear to converge to a similar value at very high $T$. This plot suggests some sort of critical connectivity size of around 7 neighbours that divides the two regimes.

We note also that the radial proximity systems of radius 3 (20 neighbours) and 4 (48 neighbours) effectively behave the same in terms of the metrics we have presented. We might infer that there is a second critical number of neighbour above which it is “saturated” and we obtain the same behaviours. There is scope for experimenting with other arbitrarily chosen neighbour groupings of sizes between 8 and 20 to determine if this is indeed the case.

In our simplified model cooperation or volunteering comes at an individual cost. There is scope to follow the suggestions of other researchers and incorporate measurable rewards for cooperation [5] as it is believed that even small rewards destabilizes total defection across the whole system.

Our model is synchronous and instantaneous but there is also scope to defer volunteering [39] as it will be advantageous to individuals to delay cooperation or volunteering until the last possible moment in the hope that another
agent will volunteer.

We have focused on fixed, albeit quite large agent populations. There is scope to explore the effect of a dynamically changing agent population size as this will likely also affect the dynamics as it has been reported for the spatial Prisoners’ Dilemma [7].

5 Conclusion

We have shown how the Volunteers’ Dilemma can be implemented spatially with neighbourhood size as a parameter of the simulation. Our model involves individual agents playing the Volunteers’ Dilemma game with a spatial peer group of a size that is governed by the choice of lattice and neighbour proximity. We varied the temptation to defect for various systems and through appropriate averaging over independent runs identified some metrics that give insight into the bulk and regional structure of emergent agent behaviour.

We see that unlike other spatial game-theoretic games such as the Prisoners’ Dilemma, the Volunteers’ Dilemma exhibits a much more bimodal cluster structure. Clusters typically span the entire system as super-connected regions or are just very small dimers or trimers. We have identified parameter regions in the temptation but also in the neighbourhood size - or size of the social group playing the game - that appear to be separated into different behaviours. The number of defector and cooperator clusters are useful metrics, but the mean number of cooperators in the vicinity of a cooperator is also a useful divider or classifier showing the distinction between low connectivity lattices and high connectivity systems.

We restricted our model to the case where all defectors defect all the time, but it is possible to extend the model further by allowing the possibility of defectors to be stochastic. They defect with some probability. Our work reported in this article is the foundation for planned further study exploring the role of that probabilistic tendency to actually defect in spatial models.

Game theoretic models such as the Volunteers’ Dilemma give insights into the sharing and management of public goods and resources. Understanding these complex phenomena continues to be a challenge and practicable simulation of large scale agent-based models to carry out “what-if” analyses of realistic public good scenarios is very likely to continue to be an important application area in sociological and political science.

References

[17] Hawick, K.A., Scogings, C.J.: Cycles, transients, and complexity in the game of death spatial automa-


