SYSTEM ROBUSTNESS AND SENSITIVITY IN CASE OF PARAMETER UNCERTAINTIES, DISTURBANCES AND NOISE

Prof. Kamen M. Yanev
Department of Electrical Engineering, Faculty of Engineering and Technology,
University of Botswana, Botswana
yanevkm@yahoo.com

ABSTRACT
This paper suggests a method of design and application of a robust controller able to suppress within specific limits the effects of control system parameters uncertainties, disturbances and noise. It also contributes to the analysis of system’s robustness by looking at its sensitivity in terms of all disturbing factors. The results of the system sensitivity in the frequency domain display the unique property of the designed robust controller, being effective for each case of parameters uncertainties, disturbances and noise. The performance of the robust controller is also assessed in terms of the system’s transient response.

KEY WORDS
Parameter Uncertainties, Disturbance, Noise, Robust Control, D-Partitioning Analysis, Sensitivity Assessment.

1. Introduction
The reason for this research is to reveal the exclusive property of a two-stage robust controller with two degrees of freedom in suppressing the effects of system parameter uncertainties, as well as rejecting the effects of the disturbances and noise on the system’s operation. A wide literature review discloses that there is a deficiency in the analysis on this matter.

The designed robust controller enforces preferred system damping, stability and transient response [1], [2], [3]. System sensitivity analysis relevant to the effects of system parameters uncertainties, external disturbances and noise is applied in the frequency domain. The results of

the analysis of the original system are compared with the system sensitivity after the robust compensation, proving the efficiency of the proposed robust controller.

2. Design of a Robust Controller
Robust methods aim to achieve robust performance and stability in the presence of system variable parameters, disturbances and noise. A control system is robust if it has low sensitivities and preserves its stability and performance in spite of plant parameter variations or external disturbances and noise. The block diagram at Figure 1 shows a robust control system where disturbing factors are entering the system in three different locations.

The robust controller consists of a series stage \( G_S(s) \), connected in series with the original plant \( G_P(s) \) and involved in a unity feedback. A forward stage \( G_F(s) \) connected in series with the unity feedback loop. The controller operates with two degrees of freedom. In case of control systems of Type 0, an integrating stage \( G_I(s) \) is also added in series with \( G_S(s) \) to maintain a zero steady-state error.

The system parameters uncertainties may be caused by variety of ambient or internal reasons, but are assumed to appear within the plant. It is believed that the disturbances \( D(s) \) become visible at the system output. The noise \( N(s) \) is read on the sensor inputs. Each one of these different types of variable and disturbing factors has an additive component coming into view at the system output [4], [5].

Figure 1. Robust control system subjected to parameters uncertainties, disturbances and noise
2.1 Analysis of the Original System and Design of the Robust Controller Stages

System analysis and the robust controller design are illustrated for system consisting of an armature-controlled dc motor and a type-driving mechanism [6] (Figure 2). There are uncertainties of the system’s gain $K$ for the reason of ambient temperatures effects. Also, the system can experience problems in maintaining stability, due to variation of the type-driving mechanism time-constant $T$.

![Figure 2. Armature-controlled dc motor and a type-driving mechanism](image)

Initially, the transfer function of the robust controller is considered as $G_c(s) = 1$. The transfer function of DC motor can be represented as follows:

$$G_m(s) = \frac{\omega_n}{E_a} = \frac{K}{(1+0.5s)(1+0.8s)}$$

(1)

Where

- $J = 0.01 \text{ kg.m}^2$ is the load inertia
- $B = 0$ (negligible) is the load damping
- $R_a = 0.5 \Omega$ is the armature resistance
- $L_a = 0.4 \text{H}$ is the armature inductance
- $K_e = 0.1 \text{V/rad/sec}$ is the voltage constant of the motor
- $K$ (variable) is the DC motor gain constant

The type-driving mechanism is described by the following transfer function with a variable time-constant:

$$G_r(s) = \frac{\omega_n}{\omega_n} = \frac{1}{1+T_3s}$$

(2)

Considering equations (1) and (2), the open-loop transfer function of the system is presented in equation (3). Initially, it is suggested that the gain is set to $K = 10$, while $T$ is variable.

$$G_{op}(s) = \frac{10}{(1+T_3s)(1+T_3s)(1+T_3s)} = \frac{10}{0.4T_3s^3 + (1.3T_3 + 0.4)s^2 + (1.3T_3)s + 1}$$

(3)

The regions of stability reflecting the variation of $T_3$ are determined by implementing the D-partitioning analysis [3], [7]. The time-constant $T_3$ is presented as follows:

$$T_3 = \frac{0.4s^2 + 1.3s + 11}{0.4s^3 + 1.3s^2 + s}$$

(4)

By applying equation (4), the D-partitioning curve, shown in Figure 3, is plotted for the frequency within the range $-\infty \leq \omega \leq +\infty$ and is determined by the code:

```matlab
>> T3=tf([-0.4 -1.3 -11],[0.4 1.3 1 0])
Transfer function:
-0.4 s^2 - 1.3 s - 11
---------------------
0.4 s^3 + 1.3 s^2 + s
>> nyquist(T3)
```

The D-partitioning determines that only D1(0) and D2(0) are regions of stability, being always on the left-hand side of the D-partitioning curve within the range $-\infty \leq \omega \leq +\infty$.

![Figure 3. D-Partitioning in terms of $T_3$](image)

As seen from Figure 3, the closed loop system will be stable only if $0 < T_3 < 0.25 \text{ sec}$ and $T_3 > 1.5 \text{ sec}$, but will be unstable if $0.25 \text{ sec} < T_3 < 1.5 \text{ sec}$.

This is confirmed by the system step responses illustrated in Figure 4. If the time-constant is $T_3 = 0.1 \text{ sec}$, or $T_3 = 5 \text{ sec}$, the system is stable, while at $T_3 = 0.7 \text{ sec}$ the system is unstable. The system steady-state error is $e_{ss} \neq 0.$
To improve the system performance, the following design steps are considered for developing a robust controller:

**Step 1:** A unity feedback system of the $G_P(s)$ is having a closed-loop transfer function presented as:

$$G_{CL}(s) = \frac{10}{0.4T_3 s^3 + (1.3T_3 + 0.4)s^2 + (1.3 + T_3)s + 1 + K}$$

**Step 2:** Applying the ITAE criterion, the optimal value of the time-constant $T_3$ is determined by the code:

```matlab
>> T3=[20:0.01:35];
>> for n=1:length(T3)
    G_array(:,:,n)=tf([10],[0.4*T3(n) (1.3*T3(n)+0.4) (1.3+T3(n)) 11]);
End
>> [y,z]=damp(G_array);
>> [y,z]=damp(G_array);
>> plot(T3,z(1,:))
```

Figure 5. Time-constant $T_3 = 27.06$ sec corresponding to relative damping ratio $\zeta = 0.707$

It is seen from the plot in Figure 5 that by observing the ITAE criterion, the relative damping ratio becomes $\zeta = 0.707$ if the time-constant is tuned to $T_3 = 27.06$ sec.

**Step 3:** If the value of the time-constant $T_3 = 27.06$ sec is substituted in equation (5), the transfer function of the closed-loop system is modified to:

$$G_{CL}(s) = \frac{10}{10.824s^3 + 35.578s^2 + 28.36s + 11}$$

As a result the system’s desired closed-loop poles become $-0.466 \pm j0.466$. This outcome is determined from the code:

```matlab
>> GCL0=tf([10],[10.824 35.578 28.36 11])
>> damp(GCL0)
```

<table>
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<th>Eigenvalue</th>
<th>Damping</th>
<th>Freq. (rad/s)</th>
</tr>
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<td>7.07e-001</td>
<td>6.56e-001</td>
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<tr>
<td>-4.64e-001 - 4.64e-001isol</td>
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<tr>
<td>-2.36e+000</td>
<td>1.00e+000</td>
<td>2.36e+000</td>
</tr>
</tbody>
</table>

**Step 4:** After approximating of the poles values to $-0.5 \pm j0.5$, the series controller stage $G_S(s)$ zeros can be placed also at $-0.5 \pm j0.5$. Thus, the transfer function of $G_S(s)$ is presented as:

$$G_S(s) = \frac{(s + 0.5 + j0.5)(s + 0.5 - j0.5)}{0.5} = s^2 + s + 0.5$$

**Step 5:** An integrating stage $G_I(s)$ is connected in cascade with the series controller $G_S(s)$ and the plant $G_P(s)$ to eliminate the steady-state error $e_{ss}$ of the system:

$$G_{OL}(s) = G_I(s)G_S(s)G_P(s) = \frac{K(s^2 + s + 0.5)}{0.5s(1 + T_3s)(1 + 0.5s)(1 + 0.8s)}$$

**Step 6:** When $G_{OL}(s)$ is involved in a unity feedback system, its closed-loop transfer function is determined as:

$$G_{CL}(s) = \frac{10(s^2 + s + 0.5)}{0.5s(1 + T_3s)(1 + 0.5s)(1 + 0.8s) + 10(s^2 + s + 0.5)}$$

**Step 7:** It is seen from the equation (9) that the closed-loop zeros will attempt to cancel the closed loop poles of the system, being in their vicinity. This can be avoided if a forward controller $G_F(s)$ is added to the closed-loop system, as shown in Figure 1. The poles of $G_F(s)$ should cancel the zeros of the closed-loop transfer function $G_{CL}(s)$. Accordingly, the transfer function of $G_F(s)$ is:

$$G_{F0}(s) = \frac{0.5}{s^2 + s + 0.5}$$

**Step 8:** Finally, the transfer function of the total robust compensated system is represented by equation (10):
$$G_{T_0}(s) = G_{F_0}G_{CLOS}(s) = \frac{10}{0.5s(1+T_3)(1+0.5s)(1+0.8s) + K(s^2 + s + 0.5)} = (11)$$

$$G_T(s) = \frac{10}{0.2s^3 + (0.65T_3 + 0.2)s^2 + (0.65 + 0.5T_3)s^2 + + 0.5s + 10(s^2 + s + 0.5)}$$

2.2 Performance Assessment after Compensation

The D-partitioning in terms of the variable time-constant $T_3$ can be determined from the characteristic equation of (11) based on the total robust compensated system:

$$T_3 = -\frac{0.4s^3 + 21.3s^2 + 21s + 10}{0.4s^4 + 1.3s^3 + s^2}$$

(12)

The D-partitioning curve, revealed in Figure 6, is plotted within the range $-\infty \leq \omega \leq +\infty$ and is determined by the code:

$$>> T3 = tf([-0.4 -21.3 -21 10],[0.4 1.3 1 0 0])$$
$$>> nyquist(T3)$$

As seen from Figure 6, the D-partitioning determines that only D(0) is a region of stability, since its position is on the left-hand side of the D-partitioning curve for the frequency variation from $-\infty$ to $+\infty$. Analyzing the regions on the T-Plane, only the real values of $T_3$ are considered. The total robust compensated system will be stable for any value of $T_3 > 0$.

$$G_T(s)_{T_3 = 0.1} = \frac{10}{0.04s^4 + 0.53s^3 + 21.4s^2 + 21s + 10}$$

(13)

$$G_T(s)_{T_3 = 0.8} = \frac{10}{0.32s^4 + 1.44s^3 + 22.1s^2 + 21s + 10}$$

(14)

$$G_T(s)_{T_3 = 2} = \frac{10}{0.8s^4 + 3s^3 + 23.3s^2 + 21s + 10}$$

(15)

The step responses for the three different cases of the robust system are plotted in Figure 7 using the code:

$$>> GT01=tf([10],[0.04 0.53 21.4 21 10])$$
$$>> GT08=tf([10],[0.32 1.44 22.1 21 10])$$
$$>> GT2=tf([10],[0.8 3.3 23.3 21 10])$$
$$>> step(GT01,GT08,GT20)$$

Figure 7. Step responses with a robust controller ($T_3 = 0.1$ sec, $T_3 = 0.8$ sec, $T_3 = 2$ sec at $K = 10$)

Due to the applied robust controller, the control system becomes quite insensitive to variation of the time-constant $T_3$, as seen from Figure 7. The step responses for cases of $T_3 = 0.1$ sec, $T_3 = 0.8$ sec and $T_3 = 2$ sec coincide.

Since the discussed system is with two variable parameters, now the gain will be altered, applying: $K = 5$, $K = 10$, $K = 20$, while keeping the system’s time-constant at $T = 0.8$ sec. These values are substituted in equation (11). As a result, the following outcomes are obtained:

$$G_{T0}(s)_{K = 5} = \frac{5}{0.32s^4 + 1.44s^3 + 12.1s^2 + 11s + 5}$$

(16)

$$G_{T0}(s)_{K = 10} = \frac{10}{0.32s^4 + 1.44s^3 + 22.1s^2 + 21s + 10}$$

(17)

$$G_{T0}(s)_{K = 20} = \frac{20}{0.32s^4 + 1.44s^3 + 42.1s^2 + 41s + 20}$$

(18)

The step responses for the different cases are determined by the following code and plotted as shown in Figure 8:
The performance evaluation for an average case at step GTK5, GTK10, GTK20

\[
\begin{align*}
\text{GTK20} &= \text{tf}([10],[0.16 0.72 21.05 20.5 10]) \\
\text{GTK10} &= \text{tf}([5],[0.16 0.72 11.05 10.5 5]) \\
\text{GTK5} &= \text{tf}([2.5],[0.16 0.72 6.05 5.5 2.5])
\end{align*}
\]

for cases of assumed that there are no disturbances Bode plots in the frequency-domain. Initially, it is parameter uncertainties is achieved with the aid of the sensitivity analysis of a system with respect to its dominant poles is insensitive to variation of the gain \(K\). The step responses for cases of \(K = 10, K = 20\) are coinciding, while for \(K = 5\) differs insignificantly from them.

It is obvious from both cases shown in Figure 7 and Figure 8 that the compensated system is satisfying the ITAE criterion and is having a relative damping ratio close to \(\zeta = 0.707\). It is also seen that after the robust compensation the steady-state error becomes \(e_{ss} = 0\).

The performance evaluation for an average case at \(K = 10\) and \(T_j = 0.8\) sec is achieved by following code:

\[
\begin{align*}
\text{GT10} &= \text{tf}([10],[0.32 1.44 22.1 21 10]) \\
\text{damp} &\text{GT10}
\end{align*}
\]

Eigenvalue Damping Freq. (rad/s)
-4.91e-001 + 4.89e-001i 7.09e-001 6.93e-001
-4.91e-001 - 4.89e-001i 7.09e-001 6.93e-001
-1.76e+000 + 7.88e+000i 2.18e-001 8.07e+000
-1.76e+000 - 7.88e+000i 2.18e-001 8.07e+000

It is seen that the relative damping ratio enforced by the compensated system dominant poles is \(\zeta = 0.709\).

2.3 Sensitivity Assessment of the Robust Control System in Case of Parameter Uncertainties

The sensitivity analysis of a system with respect to its parameter uncertainties is achieved with the aid of the Bode plots in the frequency-domain. Initially, it is assumed that there are no disturbances \(D(s)\) and noise \(N(s)\) applied to the system. Also, the original control system with a unity feedback has a transfer function of the type:

\[
W(s) = \frac{G(s)}{1 + G(s)}
\]

If the variation of any of the parameters of the original open-loop system is represented by the plant transfer function \(G_P(s)\), the sensitivity of \(W(s)\) with respect to any variation of \(G_P(s)\) is determined as follows [7], [8], [9]:

\[
S^W_G(s) = \frac{dW(s)/W(s)}{dG_P(s)/G_P(s)} = \frac{G_P(s)^{-1}}{1 + G_P(s)^{-1}} \frac{1}{1 + G_P(s)}
\]

The best sensitivity value is regarded as \(S^W_G(s) = 0\).

In addition, a practical design criterion on sensitivity, modified in the frequency format, can be formulated using the following approach:

\[
\left|S^W_G(j\omega)\right| = \frac{1}{1 + G_P(j\omega)} \leq k
\]

where \(k\) is a positive real number [7], [8], [10].

Considering the system with the transfer function described by equation (3) and substituting it into (20) for \(T_j = 0.8\) sec and the three different successive cases \(K = 5, K = 10\) and \(K = 20\), the sensitivities of \(W(s)\) with respect to any variations of \(G_P(s)\) of the original system are:

\[
S^W_G\text{ Original } K = 5(s) = \frac{0.32s^3 + 1.44s^2 + 2.1s + 1}{0.32s^3 + 1.44s^2 + 2.1s + 6}
\]

\[
S^W_G\text{ Original } K = 10(s) = \frac{0.32s^3 + 1.44s^2 + 2.1s + 1}{0.32s^3 + 1.44s^2 + 2.1s + 11}
\]

\[
S^W_G\text{ Original } K = 20(s) = \frac{0.32s^3 + 1.44s^2 + 2.1s + 1}{0.32s^3 + 1.44s^2 + 2.1s + 21}
\]

Taking into account the total robust control system described by equation (11) and substituting it in (20), the sensitivities of the robust control system for \(T_j = 0.8\) sec and the cases \(K = 5, K = 10\) and \(K = 20\) are determined as:

\[
S^W_G\text{ Robust } K = 5(s) = \frac{0.32s^4 + 1.44s^3 + 12.1s^2 + 11s + 5}{0.32s^4 + 1.44s^3 + 12.1s^2 + 11s + 10}
\]

\[
S^W_G\text{ Robust } K = 10(s) = \frac{0.32s^4 + 1.44s^3 + 22.1s^2 + 21s + 10}{0.32s^4 + 1.44s^3 + 22.1s^2 + 21s + 20}
\]

\[
S^W_G\text{ Robust } K = 20(s) = \frac{0.32s^4 + 1.44s^3 + 42.1s^2 + 41s + 20}{0.32s^4 + 1.44s^3 + 42.1s^2 + 41s + 40}
\]

For the sensitivity comparison, the functions described by the equations (22), (23), (24) and (25), (26), (27) are plotted in the frequency domain in Figure 9, following the code:

\[
\begin{align*}
\text{GOriginalK5} &= \text{tf}([0.32 1.44 22.1 21 10]) \\
\text{GOriginalK10} &= \text{tf}([0.32 1.44 22.1 21 10]) \\
\text{GRobustK20} &= \text{tf}([0.32 1.44 22.1 21 10]) \\
\text{GRobustK5} &= \text{tf}([0.32 1.44 22.1 21 10])
\end{align*}
\]
>> GRobustK10=tf([0.32 1.44 22.1 21 10],[0.32 1.44 22.1 21 20])
>> GRobustK20=tf([0.32 1.44 42.1 41 20],[0.32 1.44 42.1 41 40])
>> bode(GOriginalK5,GOriginalK10,GOriginalK20,G0RobustK5,GRobustK10,GRobustK20)

Figure 9. Sensitivity of the original and the robust control system (Variable gain $K$)

As seen from Figure 9, the sensitivity of the original system at different gains is $S_G^W(s) \geq -21.6$dB in the range $2 \text{ rad/sec} < \omega < 10 \text{ rad/sec}$, reaching $S_G^W(s) = 21.6$dB for the case $K = 10$. The sensitivity of the robust control system is $S_G^W(s) \leq 0$dB or is of a negligible value. The results reveal that the robust system is with considerably lower sensitivity values compared with the original one.

If the same original system $G_P(s)$, described by equation (3) is substituted into (20) for $K = 10$, while the time-constant is varied as $T_3 = 0.1 \text{ sec}$ $T_3 = 0.8 \text{ sec}$ $T_3 = 2 \text{ sec}$, the sensitivities of $W(s)$ with respect to any variations of $G_P(s)$ of the original system are:

$$S_G^W \text{original } T=0.1(s) = \frac{0.04s^3 + 1.7s^2 + 1.4s + 1}{0.04s^3 + 1.7s^2 + 1.4s + 11}$$ (28)

$$S_G^W \text{original } T=0.8(s) = \frac{0.32s^3 + 1.44s^2 + 2.1s + 1}{0.32s^3 + 1.44s^2 + 2.1s + 11}$$ (29)

$$S_G^W \text{original } T=2(s) = \frac{0.8s^3 + 3s^2 + 3.3s + 1}{0.8s^3 + 3s^2 + 3.3s + 11}$$ (30)

Next, the transfer function of the total robust control system described by equation (11) is substituted in equation (20). The robust system sensitivities for $K = 10$ and $T_3 = 0.1 \text{ sec}$ $T_3 = 0.8 \text{ sec}$ $T_3 = 2 \text{ sec}$ are determined as:

$$S_G^W \text{ Robust } T=0.1(s) = \frac{0.02s^4 + 0.465s^3 + 10.7s^2 + 10.5s + 5}{0.02s^4 + 0.465s^3 + 10.7s^2 + 10.5s + 15}$$ (31)

$$S_G^W \text{ Robust } T=0.8(s) = \frac{0.16s^4 + 0.92s^3 + 11.05s^2 + 10.5s + 5}{0.16s^4 + 0.92s^3 + 11.05s^2 + 10.5s + 15}$$ (32)

$$S_G^W \text{ Robust } T=2(s) = \frac{0.4s^4 + 1.7s^3 + 11.65s^2 + 10.5s + 5}{0.4s^4 + 1.7s^3 + 11.65s^2 + 10.5s + 15}$$ (33)

To compare the functions described by the equations (28), (29), (30) and (31), (32), (33), they are plotted in the frequency domain as shown in Figure 10 based on the following code.

>> GOriginalT01=tf([0.04 1.7 1.4 1],[0.04 1.7 1.4 11])
>> GOriginalT08=tf([0.32 1.44 2.1 1],[0.32 1.44 2.1 11])
>> GOriginalT2=tf([0.08 3.3 1],[0.08 3.3 11])
>> GRobustT01=tf([0.02 0.465 10.7 10.5 5],[0.02 0.465 10.7 10.5 15])
>> GRobustT08=tf([0.16 0.92 11.05 10.5 5],[0.16 0.92 11.05 10.5 15])
>> GRobustT2=tf([0.4 1.7 11.65 10.5 5],[0.4 1.7 11.65 10.5 15])
>> bode(GOriginalT01,GOriginalT08,GOriginalT2,GRobustT01,GRobustT08,GRobustT2)

Figure 10 shows that the sensitivity of the original system is $S_G^W(s) \geq 0$dB in the range $0.2 \text{ rad/sec} < \omega < 10 \text{ rad/sec}$, reaching $S_G^W(s) = 24.2$dB for the case $T = 2 \text{ sec}$. The sensitivity of the robust control system at different gains is $S_G^W(s) \leq 0$dB or is of a negligible value.

2.4 Sensitivity Assessment of the Robust System in Case of External Disturbance and Noise

In many control system applications, the system must yield performance that is robust not only to parameter variations, but also to external disturbances and noise.

It is suggested that the series controller $G_S(s)$ is to be designed with the objective, the output signal $C(s)$ to be insensitive to the disturbance $D(s)$ or to the noise $N(s)$ over the frequency range in which the disturbance or the noise are dominant.
The forward controller $G_F(s)$ is to be designed to achieve the desired transfer function $W_{\text{Robust}}(s)$ between the input $R(s)$ and the output $C(s)$ of the system. The sensitivity of $W_{\text{Robust}}(s)$ with respect to any variations of $G_P(s)$ is:

$$S_{GP}^{\text{WR}}(s) = \frac{dW_{\text{Robust}}(s)/W_{\text{Robust}}(s)}{d[G_F(s)G_P(s)][G_F(s)G_P(s) + 1]} = \frac{1}{1 + G_P(s)G_F(s)}$$ \hspace{1cm} (34)

Taking into account the block diagram at Figure 1, the disturbance-to-output transfer function is presented as:

$$Q_{\text{Robust D}}(s) = \frac{C(s)}{D(s)} = \frac{G_F(s)G_P(s)}{1 + G_F(s)G_P(s)}$$ \hspace{1cm} (35)

The best disturbance rejection is at $Q_{\text{Robust D}}(s) = 0$.

It is seen that the result of equation (35) is similar to equation (21) if the series robust controller stage $G_S(s)$ is present. Therefore the disturbance-to-output transfer function and the sensitivity function are identical.

From the block diagram at Figure 1, the noise-to-output transfer function is:

$$Q_{\text{Robust N}}(s) = \frac{C(s)}{N(s)} = \frac{G_F(s)G_P(s)}{1 + G_F(s)G_P(s)}$$ \hspace{1cm} (36)

The ideal case of noise rejection is $Q_{\text{Robust N}}(s) = 0$.

The noise-to-output transfer function, described by equation (36) is identical to the input-to-output transfer function of the closed-loop section of the system, except for the negative sign.

Taking into account the discussed system and its related series controller and also considering equations (3) and (35), the disturbance-to-output transfer function can be represented for the cases before and after the application of the designed robust controller as follows:

$$Q_{\text{Original D}}(s) = \frac{C_{\text{Original}}(s)}{D(s)} = \frac{1}{1 + G_P(s)} = \frac{0.32s^3 + 1.44s^2 + 2.1s + 1}{0.32s^3 + 1.44s^2 + 2.1s + 11}$$ \hspace{1cm} (37)

$$Q_{\text{Robust D}}(s) = \frac{C_{\text{Robust}}(s)}{D(s)} = \frac{1}{1 + G_F(s)G_P(s)} = \frac{0.32s^3 + 1.44s^2 + 2.1s + 1}{0.32s^3 + 21.44s^2 + 22.1s + 11}$$ \hspace{1cm} (38)

The improvement of the disturbance rejection is analyzed by comparing the Bode magnitudes of the transfer functions $Q_{\text{Original D}}(s)$ and $Q_{\text{Robust D}}(s)$ in the frequency domain.

The functions (37) and (38) are plotted in the frequency domain with the aid of the following code and shown in Figure 11:
As seen from the magnitude Bode plot in Figure 12, the noise rejection of the original control system is $Q_{\text{Original}N0} \geq 0$ dB in the frequency range $0.9 \text{ rad/sec} < \omega < 3.6 \text{ rad/sec}$, reaching $Q_{\text{Original}N0} = 21.9$ dB at 2.71 rad/sec.

Figure 12. Noise rejection of the original and the robust control system

After applying the robust controller, the noise rejection is considerable. The amplitude Bode plot of the noise-to-output functions is $Q_{\text{Robust}N0} \leq 0$ in the complete frequency range.

3. Conclusion

The major achievement of this research is the proposed design strategy of a robust controller that is able to suppress the effects of all stressful factors and uncertainties to which a control the system is subjected.

The suggested design of a robust controller demonstrates that by implementing required dominant system poles, the controller enforces the requested relative damping ratio and therefore the desired system performance. For systems of Type 0, an additional integrating stage ensures a steady-state error equal to zero.

The application of the robust controller leads to system’s insensitivity to the variation of its parameters within specific limits. The D-partitioning analysis reveals a considerable improvement of the system’s margin of stability after the robust compensation. The analysis in the time-domain after the robust compensation demonstrates the system’s robustness and the insignificant difference in the system’s performance if varying different system parameters. It is seen also that the system’s steady-state error becomes zero after the robust compensation.

Since the design of the robust controller is based on the desired relative damping, its unique property is that it can function successfully for any of the system’s parameter variations or simultaneous variation of a number of parameters. This property is proven by the comparison of the system’s transient responses. It is confirmed that the system performance remains robust and insensitive in case of any simultaneous variations of the time-constant and the gain within specific limits.

The sensitivity analysis illustrates that the application of the robust controller significantly reduces the system sensitivity to parameters uncertainties and therefore improves the system’s robustness. The introduction of the robust controller also causes substantial suppression of the disturbance and noise, bringing their additive components to the system’s output signal near to zero.

The accomplishments in this research are supposed to lead to the advancement of the knowledge in the area of robust systems design and analysis. It is bringing new practical elements to the effectiveness of suppressing the effects of any parameters uncertainties and rejecting system’s disturbances and noise. The research in this field is of significance because of its considerable practical effect. It can be used for substantial improvement of the operation of industrial control systems with variable parameters due to different ambient conditions or are exposed to disturbance and noise.

References