AN APPROACH FOR PATHOLOGICAL CONSTRAINTS IDENTIFICATION AND FEASIBILITY RECOVERY OF REACTIVE OPTIMIZATION USING INTERIOR POINT METHOD

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ABSTRACT
In this paper, the reason of reactive optimizing failure with large-scale power networks is discussed. A practical relaxed optimal power flow model (RelaxOPF) is presented to restore the solvability of reactive optimization solved by interior point method. In this model, pathological constraints which caused failure of OPF can be identified during optimizing; the problem is restored by minimum relaxation introduced into those pathological constraints. Numerical results of IEEE testing system and actual power networks show that the proposed method is feasible and practical.

KEY WORDS
optimal power flow; reactive optimization; pathological constraints; constraint relaxation

1. Introduction
The Interior Point Method has been widely used since first proposed by Karmarkar in 1984 [1], it’s now playing an important role in solving optimization problems with large-scale power networks [2–6]. For actual power system with thousands of buses, various constraints are involved for an OPF model such as reactive optimization. OPF may fall infeasible due to several reasons, such as overly strict restrictions of bus voltages, power transmission limits and so on. Then, system planning and dispatching staffs can merely check the calculation data relying on experiences and manually adjust constraints or network data. This conventional method is inefficient and often useless. Therefore it’s urgent to find a new approach to automatically identify pathological constraints which caused failure of OPF and restore feasibility of the problem with an approximate solution.

So far there are three kinds of methods aiming at the infeasibility of power flow and OPF. The first method [7] uses the eigenvalue, critical buses are identified by zero eigenvalue and corresponding eigenvector. The second method [8] considers the least square solutions of Newton method to solve the power balance equations, the insolvability degree was evaluated by the shortest Euclid distance of infeasible points to feasible boundary. These two kinds of methods only concentrate on the infeasibility of power flow. The third method is based on optimization algorithm. In [9], interior point method is applied to solve the load cutting off model and least square model considering various constraints. In [10], slack variables are introduced for upper and lower bounds of every constraint in OPF model and objective function is appended with penalty terms. There will be 2*N additional variables with voltage constraints for an N-buses reactive OPF problem, while most of which are unnecessary.

In this paper, a relaxed OPF model based on primal-dual interior point method is proposed. Pathological constraints which cause insolvability of the model can be detected during iteration by analyzing dual variables, after that relaxation is imported only to those critical constraints. The optimization will be restored with minimum relaxation of pathological constraints and an approximate solution will be obtained.

2. Pathological Constraints Identification

2.1 Solving reactive OPF by primal-dual interior point method
The reactive OPF problem could be presented by a universal nonlinear optimization model as below:

\[
\begin{align*}
\text{obj.} & \quad \min f(x) \\
\text{s.t.} & \quad h(x) = 0 \\
& \quad g_m \leq g(x) \leq g_M
\end{align*}
\]

(1)

The brief procedure solving OPF problem with interior point method is as below [11–12]:

Firstly the inequality constraints are transformed to equality ones by slack variables:

\[
\begin{align*}
g(x) + u &= g_M \\
g(x) - l &= g_m
\end{align*}
\]

(2)

Then the Kuhn-Tucker conditions of Lagrange Function are obtained:

\[
\begin{align*}
L_1 &= \nabla_x f(x) - \nabla_y h(x) y - \nabla_z g(x)(z + w) = 0 \\
L_2 &= h(x) = 0 \\
L_3 &= g(x) - l - g_m = 0 \\
L_4 &= g(x) + u - g_M = 0 \\
L_5 &= z - \mu L^T E = 0 \quad \Rightarrow LZE - \mu E = 0 \\
L_6 &= w - \mu U^{-1} E = 0 \quad \Rightarrow UWE + \mu E = 0
\end{align*}
\]

(3)
Thus the Kuhn-Tucker conditions are solved by Newton-Raphson method, the correction equations for primal and dual variables are:

\[
\begin{bmatrix}
H' \\
V^\top h(x) \\
0
\end{bmatrix}
\begin{bmatrix}
Ax \\
Ay \\
0
\end{bmatrix} =
\begin{bmatrix}
L_x \\
L_y \\
0
\end{bmatrix}
\]  
(4)

Where:

\[
H = -[\nabla^2 f(x) - \nabla^2 h(x) y - \nabla^2 g(x)(z + w)]
\]

\[
H' = H - \nabla g(x)[L' + \nabla^2 W] \nabla g(x)
\]

\[
L_i = L_i + \nabla g_i(x)[L_i' + ZL] + U^1(i, j - WL_i)
\]

Correction equations of slack variables \(l, u\) and Lagrange multipliers \(z, w\) are shown below:

\[
\begin{align*}
Al = \nabla g(x)^T \Delta x + (g(x) - l - u)
\end{align*}
\]

\[
\begin{align*}
Au = \nabla g(x)^T \Delta x - (g(x) + u - g_u)
\end{align*}
\]

\[
\begin{align*}
Az = -L^1 \nabla g(x)^T \Delta x - L^1((Lz - \mu e + z)(g(x) - l - g_u))
\end{align*}
\]

\[
\begin{align*}
Aw = -U^1 \nabla g(x)^T \Delta x - U^1((Uw + \mu e) - W)(g(x) + u - g_u))
\end{align*}
\]

And the iteration step sizes for primal and dual variables are:

\[
\begin{align*}
\text{step}_p &= 0.9995 \min \left\{ \frac{-g_i}{A_i} : A_i \leq 0, \frac{-g_i}{A_i} : A_i < 0, i \right\}
\end{align*}
\]

\[
\begin{align*}
\text{step}_d &= 0.9995 \min \left\{ \frac{-g_i}{A_i} : A_i < 0, \frac{-g_i}{A_i} : A_i > 0, i \right\}
\end{align*}
\]

The corrections above repeat until the convergent requirements are met or the maximum number of iterations is reached.

2.2 Method of detecting pathological constraints

It is obvious that every primal and dual variable should satisfy the constraints list below under normal condition.

\[
l, u, z, w > 0, \quad w < 0, \quad \gamma_i \neq 0 \quad (i = 1, 2, \cdots r)
\]  
(8)

In a convergent case, the correction step size of slack variable and dual variable should satisfy the conditions below:

\[
\Delta l_i < 0, \Delta u_i < 0, \Delta z_i < 0, \Delta w_i > 0
\]  
(9)

Equation (8) and (9) would drive the slack variables and dual variables towards zero and the iteration gradually converges.

In particular cases, the OPF model contains some inequality constraints with overly strict bound. These pathological constraints can never be satisfied during iteration, thus the corresponding slack variable \(l_i\) or \(u_i\) would turn negative:

\[
l_i < 0 \quad \text{or} \quad u_i < 0
\]  
(10)

According to the correction equations of \(z\) and \(w\) in Eq. (5), the value of \(\Delta z\) and \(\Delta w\) will accordingly turn opposite as below:

\[
\Delta z_i > 0, \Delta w_i < 0
\]  
(11)

Equation (11) will drive \(z_i\) or \(w_i\) back away from zero instead of towards zero and finally cause divergence of the OPF model. Therefore we can identify the pathological constraints by analyzing value of the dual variables during the iteration.

In this paper we propose a criterion to identify the pathological constraints for an OPF model:

\[
|z_i| > 10^6, |w_i| > 10^6 \quad (i = 1, 2, \cdots r)
\]  
(12)

When the dual variable \(z_i\) or \(w_i\) meets the criterion in Eq. (12), it can be concluded that the constraint responding \(z_i\) or \(w_i\) is pathological and needs adjustment.

3. Restoration of reactive OPF solvability

3.1 The RelaxOPF model

For a typical OPF problem of power system, the constraints can be briefly sorted into two parts. The first kind of constraints is not allowed to break, such as reactive limit of generators, tap limit of transformers. Yet the second kind of constraint such as bus voltage limit is allowed to be adjusted in a certain extent. The RelaxOPF model proposed in this paper tend to adjust the second kind of constraints in order to recover feasibility of reactive OPF problem and obtain an approximate solution.

For the pathological constraints of original model in Eq. (1), firstly we consider a rough expanding of the interval and a new model is obtained as below:

\[
\begin{align*}
\text{obj.} \quad & \min f(x) \\
\text{s.t.} \quad & h(x) = 0 \\
& g_m \leq g_i(x) \leq g_M \\
& g_m \leq g_i(x) \leq g_M
\end{align*}
\]

In Eq. (13) \(g_m(x_i)\) represents critical constraints that cause insolvability of OPF and the second kind of constraints. \(g_d(x_i)\) represents the first kind of constraints. \(g_m\) and \(g_M\) represent the original bounds while \(g_m'\) and \(g_M'\) represent the expanded lower and upper bound for those pathological constraints.

\[
\begin{align*}
g_m' &= g_m(1 + \varphi_{up}) \\
g_M' &= g_M(1 - \varphi_{ad})
\end{align*}
\]

Where \(\varphi_{up}\) is the relaxation factor of upper bound, \(\varphi_{ad}\) is of lower bound. The relaxation factor may be a single number or a vector. If \(\varphi\) is a constant, all of the constraints adopt the same expansion: if \(\varphi\) is a vector, critical constraints can adopt different coefficients respectively. Equation (13) and (14) is combined to expect a feasible solution with new interval of constraints.

In the other hand, the interval of pathological constraints cannot be expanded without any limitation, otherwise the result would be meaningless. Additional penalty term is introduced into objective function \(f(x)\) in order to consider the influence of constraint relaxing.

\[
\min f(x) \Rightarrow \min f(x) + M' \Psi(x)
\]

Where \(M\) could be a large positive number or a constant vector. In this paper we use one single number between 10^3 and 10^5 for simplicity.

3.2 Expression of penalty term

The additional penalty term \(\Psi(x)\) should be doubly differentiable. In this paper we design a typical quadratic
expression for bus voltage limits of reactive OPF problem as below:

\[
\Psi(V) = \begin{cases} 
M(V_i - V_{\text{max}})^2 & V_i > V_{\text{max}} \\
0 & V_{\text{min}} \leq V_i \leq V_{\text{max}} \\
M(V_i - V_{\text{min}})^2 & V_i < V_{\text{min}} 
\end{cases}
\]  \quad (16)

It’s obvious that the additional term in (16) presents a barrier property. The penalty function behaves zero when bus voltage is normally within \([V_{\text{min}}, V_{\text{max}}]\), however it grows rapidly when bus voltage \(V_i\) jumps out of the range. Eq. (16) prevents unboundedly loose of voltage limits and will lead to minimum relaxation of \(V_i\).

The diagram of RelaxOPF model is shown as Fig. 1.

4. Case Study

In this section, we present numeric results of IEEE testing system and actual power system by the proposed model and compare them with results of original model.

4.1 IEEE 39-buses system

A reactive OPF problem of IEEE39-buses system is designed for experiment. The relaxed model proposed in this paper is applied and different type of penalty terms is discussed.

4.1.1 Case of detecting pathological constraints

Reactive OPF problem of IEEE 39-buses system converges with bus voltage limit of 0.97~1.03p.u, yet it falls divergent when bus voltage limit is reduced to 0.98~1.02p.u. Curve of dual variables \(z_i, w_i\) corresponding with voltage limits during iteration is shown as Fig.2~Fig.4. Logarithmic ordinate is used for convenience.

Tab.1 Comparison with original OPF model and RelaxOPF model on case of IEEE 39-bus system

<table>
<thead>
<tr>
<th>Voltage range(p.u)</th>
<th>Original model</th>
<th>RelaxOPF Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9~1.1</td>
<td>Converge</td>
<td>Converge</td>
</tr>
<tr>
<td>0.95~1.05</td>
<td>Converge</td>
<td>Converge</td>
</tr>
<tr>
<td>0.97~1.03</td>
<td>Converge</td>
<td>Converge</td>
</tr>
<tr>
<td>0.98~1.02</td>
<td>Diverge</td>
<td></td>
</tr>
</tbody>
</table>

In order to examine the effect of RelaxOPF model, we set different bus voltage limits and compare results of relaxed model with original model. The penalty coefficient \(M\) is set to 1000.

In Fig.3 and Fig. 4, the dual variable \(w_{19}\) is related to voltage upper limit of bus19 and \(z_{20}, z_{33}\) is related to lower limit of bus20, bus33. Their value grow rapidly with the iteration, from which we can infer that the voltage upper limit of bus19 and lower limit of bus20, bus33 are too strict and should be relaxed.

4.1.2 Result of constraints relaxation

In order to examine the effect of RelaxOPF model, we set different bus voltage limits and compare results of relaxed model with original model. The penalty coefficient \(M\) is set to 1000.

In Fig.3 and Fig. 4, the dual variable \(w_{19}\) is related to voltage upper limit of bus19 and \(z_{20}, z_{33}\) is related to lower limit of bus20, bus33. Their value grow rapidly with the iteration, from which we can infer that the voltage upper limit of bus19 and lower limit of bus20, bus33 are too strict and should be relaxed.

In order to examine the effect of RelaxOPF model, we set different bus voltage limits and compare results of relaxed model with original model. The penalty coefficient \(M\) is set to 1000.
The original OPF model and RelaxOPF model converge with the same solution with bus voltage range of 0.97–1.03 p.u; however when the constraint is reduced to 0.98–1.02 p.u, the original model diverges while RelaxOPF model is still convergent with constraint relaxation for bus19, bus20 and bus33. It is quite clear that the relaxed voltage of bus19, bus20 and bus33 is still within the range of 0.97–1.03 p.u, which match the conclusion that original model converges with bus voltage constraints of 0.97–1.03 p.u.

4.1.3 Contrast of different additional penalty terms

Besides different value of penalty coefficient $M$, it’s also rational to use different form of penalty term to improve effect of RelaxOPF model. A new cubic form is presented below:

$$
\Psi'(V_i) = \begin{cases} 
M(V_i - V_{\text{max}})^3 & V_i > V_{\text{max}} \\
0 & V_{\text{min}} < V_i < V_{\text{max}} \\
-M(V_i - V_{\text{min}})^3 & V_i < V_{\text{min}} 
\end{cases}
$$

(17)

We compare the effect of cubic term in Eq. (17) with quadratic term in Eq. (16). As the original model diverge with bus voltage range of 0.98–1.02 p.u, results of RelaxOPF model with different penalty term are as below:

<table>
<thead>
<tr>
<th>0.98–1.02</th>
<th>Cubic</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>Voltage(p.u)</td>
<td>Bus</td>
</tr>
<tr>
<td>1</td>
<td>1.021098</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>1.023993</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
<td>1.030356</td>
<td>33</td>
</tr>
<tr>
<td>20</td>
<td>0.973147</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>1.025328</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1.027000</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>1.026483</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>1.025326</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>1.025885</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.978892</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>1.023488</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0.976155</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>1.022011</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>1.024342</td>
<td></td>
</tr>
</tbody>
</table>

For the same case that original model diverges, RelaxOPF model with cubic penalty term requires voltage limit relaxations of totally 14 buses, while the quadratic only needs 3 buses. Beyond that the relaxed voltage of bus19 using cubic term is higher than 1.03 p.u, which conflicts with the conclusion that the original model converge with limit of 0.97–1.03 p.u. The results of Tab.2 prove that quadratic term performs better than cubic term. That’s because a quadratic function has higher first derivative and second derivative rather than a cubic function within the interval of [0, 1]. The quadratic penalty term has better barriers effect than other forms.

4.2 Case of actual large-scale system

In this section a reactive OPF case of actual regional power grid is chosen. It contains 2414 buses with highest voltage degree of 500kV and lowest of 10kV, 387 generators, 2734 transmission lines, 2441 transformers and 343 shunt capacitors.

The original reactive OPF model diverge with bus voltage limit of 0.95–1.05 p.u. We use RelaxOPF model to solve the problem, the penalty coefficient $M$ is also set to 1000.

The relaxed model successfully converges within 21 iterations. Value of barrier parameter $\mu$ is shown as Fig. 5.

![Fig.5 Curve of $lg(\mu)$ during iteration](image)

Buses of which voltage limit has been relaxed are list as below:

<table>
<thead>
<tr>
<th>Bus Name</th>
<th>Lower bound relaxation(p.u)</th>
<th>Upper bound relaxation(p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(V&lt;0.95p.u)</td>
<td>(V&gt;1.05p.u)</td>
</tr>
<tr>
<td>THP/20kV.06BS</td>
<td>0.037056</td>
<td>-</td>
</tr>
<tr>
<td>THP/20kV.01BS</td>
<td>0.037056</td>
<td>-</td>
</tr>
<tr>
<td>THP/20kV.03BS</td>
<td>0.037056</td>
<td>-</td>
</tr>
<tr>
<td>THP/20kV.02BS</td>
<td>0.037056</td>
<td>-</td>
</tr>
<tr>
<td>THP/20kV.04BS</td>
<td>0.037056</td>
<td>-</td>
</tr>
<tr>
<td>THP/20kV.05BS</td>
<td>0.037056</td>
<td>-</td>
</tr>
<tr>
<td>JD/35kV.IBS</td>
<td>0.009346</td>
<td>-</td>
</tr>
<tr>
<td>MZ/35kV.IIBS</td>
<td>-</td>
<td>0.021489</td>
</tr>
<tr>
<td>SHB/35kV.IIBS</td>
<td>0.014117</td>
<td>-</td>
</tr>
<tr>
<td>YG/35kV.IBS</td>
<td>-</td>
<td>0.018061</td>
</tr>
<tr>
<td>CC/10kV.01IBS</td>
<td>-</td>
<td>0.008493</td>
</tr>
<tr>
<td>FC/15kV.7IBS</td>
<td>0.036348</td>
<td>-</td>
</tr>
<tr>
<td>WAN/220kV.IIBS</td>
<td>0.014144</td>
<td>-</td>
</tr>
<tr>
<td>XC/20kV.IBS</td>
<td>-</td>
<td>0.010191</td>
</tr>
<tr>
<td>QK/220kV.IBS</td>
<td>0.013433</td>
<td>-</td>
</tr>
<tr>
<td>CHC/35kV.IIBS</td>
<td>-</td>
<td>0.010507</td>
</tr>
<tr>
<td>DT/35kV.IBS</td>
<td>0.013257</td>
<td>-</td>
</tr>
<tr>
<td>DT/35kV.IIBS</td>
<td>0.011591</td>
<td>-</td>
</tr>
<tr>
<td>FY/10kV.IBS</td>
<td>-</td>
<td>0.035423</td>
</tr>
<tr>
<td>ND/500kV.IBS</td>
<td>-</td>
<td>0.027428</td>
</tr>
<tr>
<td>ND/25kV.3BS</td>
<td>0.029695</td>
<td>-</td>
</tr>
<tr>
<td>ND/25kV.4BS</td>
<td>0.029893</td>
<td>-</td>
</tr>
</tbody>
</table>

The results of Tab. 3 show that the total number of bus whose voltage constraint requires relaxing is only 22, that’s less than one percent of total buses. Most of the buses listed in Tab.3 belong to generator terminal bus or low voltage bus of transformer. The follow-up network data inspecting proves that there are errors with reactive
power parameter of related generators and topology of transformers. After correcting the parameter errors, the total number of buses that still require relaxation is reduced to 8.

5. Conclusion

The optimizing problem of actual power system usually performs as a large-scale nonlinear model. Iteration may diverge due to overly strict constraints, which is unpractical to manually check. In this paper, a relaxed OPF model is proposed to recover feasibility of OPF calculations. Pathological constraints that lead to insolvability of original model can be promptly identified during iteration, proper relaxation is introduced to those critical constraints and the original problem can be restored with an approximate solution.

Results of numeric experiments prove that the RelaxOPF model proposed in this paper has characteristics as follows:

1. When the original OPF problem is solvable, RelaxOPF model can converge to the same solution without any relaxation of constraints.
2. If the original problem is unsolvable, RelaxOPF model will restore the original problem with an approximate solution. In the meantime pathological constraints of the original problem will be automatically identified and relaxed, depending on which system planning and operating staffs can accordingly check the network data.
3. As the pathological constraints account only for a small part of all the constraints, RelaxOPF model does not markedly increase the computation work. It would be beneficial for OPF problems of large-scale power system.

References