OFFSHORE WIND FARM NETWORK CONNECTIVITY ANALYSIS AND OPTIMISATION USING GRAPH ALGORITHMS AND METRICS

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ABSTRACT

Offshore wind farms are becoming a very important contributor of electricity to national grid systems, including that of the United Kingdom. Wind farms vary considerably in their geographic size and hence capacity. A number of operational and practical factors must be taken into account in designing an optimal spatial layout to make best use of resources including the connectivity cabling of turbine platforms. We consider the cabling layout and some possible wiring patterns based on a minimum spanning tree (MST) approach using graph theory algorithms with weighted edges. We investigate patterns for some actual wind farms currently in operation or under construction off the UK coast. We also explore the scalability of such algorithms for simulated spatial patterns of wind turbines using MST wiring solutions applied to various regular and random turbine layouts.

KEY WORDS
electrical network; offshore wind farm; turbine; minimal spanning tree; weighted graph.

1 Introduction

Generating electricity from renewable power sources is an ongoing challenge globally. Wind farming using a collection of wind turbine generator towers has become a popular and economically feasible solution in many regions [8,40]. Locating wind farms offshore rather than on-land is seen as potentially more hazardous and expensive in the short term, albeit with the potential for greater wind conversion efficiency and wind availability. Social factors such as the desirability to hide the turbines away from living areas on land are also important in some countries such as the UK. It is also feasible to scale up an offshore farm to contain thousands of turbine towers, whereas this is not practical for land-bound farms.

Graph and network analysis techniques [26] are finding uses in many practical applications and studies of complex systems [37]. Computational science and numerical simulation techniques allow a study of varied and large network model systems [32]. In this article we investigate the use of graph analysis methods in studying the cable connectivity layouts of typical offshore wind farms. Usually an offshore farm can be laid out in a much simpler and regular pattern of turbine towers than an on-land farm. Typically the sea-bed over a chosen area is regular enough that a regular pattern is possible, whereas on-land there are inevitably many obstacles including terrain variations and buildings and land use conflicts.

Wind power [2,4] generally makes use of a farm approach comprising multiple wind turbines [18,41] and in that sense is a specific subset of the multi-source power generation problem [3,30]. Such distributed systems have a complex set of vulnerabilities [6,9] including electrical ones as well as structural ones [7,25] and reliability issues [28,35] also comprising graph structural issues as well as electrical ones [38,39] and black out analyses [19] and protection strategies [24].

There are a number of criteria that go into optimising any electrical power networks including wind farms [20,23]. We do not explore the electrical engineering aspects in this present work but rather focus on the graph network [22,29] and spatial aspects. We also do not explore the complex design space of operating conditions, resource allocation and wind farm life-cycles. Although there is scope for further work on use of graph algorithms to study the reliability
and failure modes [1, 14] of such networks [5, 15], we also do not cover that in this present paper.

Planning for offshore wind farms remains a complex problem [11, 36]. While agents [31] and other optimisation techniques can be used for some aspects we focus on graph and network aspects in this present paper.

We investigate a number of UK offshore wind farms and the total cable length used in connecting individual turbines up to aggregation points. We compare these with the minimum spanning tree (MST) connectivity solutions [27] that can be calculated using a graph algorithm [10, 21, 33, 34] based on planar spatial vertex locations. A typical such solution is illustrated in Figure 1 which shows a MST cabling layout for a simulated wind turbine layout, with 100 individual turbines located randomly.

We also consider some of the regular and random spatial arrangements for offshore wind turbines and how this affects the cost of connecting them up. We compare total edge length for various placement strategies and find they are all describable using a power-law relationship in terms of the number of vertices or turbine towers in the farm cluster.

Our article is structured as follows: In Section 2 we give some background to the offshore wind farm layout problem and the connecting cable aspect in particular. We describe the graph algorithmic approach to computing a minimum spanning tree cable connectivity solution in Section 3. We give a selection of computed solutions for various wind farms in Section 4 as well as presenting scalability results for various wind turbine placement strategies. We offer some discussion in Section 5, including comments on the variations between actual and possible connectivity solutions. Finally we give some tentative conclusions and areas for further study in Section 6.

2 Wind Farms

Wind farms are becoming very prevalent in the UK and internationally. While generally it is cheaper and less hazardous to build wind farms on-shore, there are potentially higher wind speeds and capacities offshore and larger turbine towers with larger blades can also lead to greater efficiencies. Wind farms located offshore are also perceived by the public as less obtrusive, although care is likely needed in the future to avoid large scale offshore farms becoming an obstacle to navigation. This will be particularly problematic as offshore farms are scaled up in the future to sizes comprising thousands of turbine towers.

Figure 2 shows a typical wind turbine tower. For various technical reasons including blade weight and stability, three-blade towers are the most common current design, and blade radii of up to 100 metres are becoming possible. These sizes are set to increase to 150m and beyond. Planned and operational offshore wind farms are shown in Figure 3, with green for actual sites and red patches for planned (at the time of writing) sites. At present, UK farms typically comprise clusters of 30-100 individual wind turbine towers, laid out in a pattern appropriate to the sea-bed and common wind conditions. Turbines need to be placed a minimal distance apart to avoid turbulence effects in the wake of one tower, affecting another tower that is downwind.

Table 1 summarises some typical properties of some selected UK offshore wind farms. Figure 3 shows the location of some of the planned and actual sites and in this present article we focus on a selected few near to Hull and the Humber or with interesting properties.

Data are taken from online information provided at the web site: http://www.4coffshore.com/offshorewind/ although we emphasise that our analysis and calculations in this present paper are only indicative.

Figure 2. A typical wind turbine tower.

Figure 3. Map showing planned and actual offshore wind farms around the UK. Circled codes refer to the six farms discussed.
Table 1. Properties of some selected actual and planned Specific UK offshore Wind Farms, with reference to the Map in Figure /reffig:windfarm-map-uk.

of statistical properties and in particular, any errors due to transcription of data into a form suited for the calculations discussed are entirely our own responsibility.

There are various criteria which determine the spatial layout and spacing of individual turbine towers. Power and wind utilisation effects appear to give rise to empirical ratios for minimal spacings relating to the turbine blade diameter $D$ and spacings of 3 to 4 times $D$ appear to be commonly used [8]. Layout geometries vary considerably - as will be seen in the farm map plans illustrated in Section 4, but the connectivity cable layout appears to be much less constrained, and a review of the literature suggests there has been less work done in optimising this aspect of offshore wind farms. Cable connectivity is the main focus of this present article and we investigate the use of graph algorithms such as Prim’s algorithm for constructing a minimum spanning tree of all the turbines, so that they are connected to an aggregating point which can have a cable connecting the farm to an on-shore collection point and hence to the national power grid.

3 Spanning Tree Network Generation

A minimum spanning tree (MST) [21, 33] is a tree-form of a graph or network that spans or connects all the vertices (or nodes) and minimises the total weight of all the edges. In an offshore wind farm the vertex nodes are the turbine towers and the edges are the connecting cables. Locating the towers in an appropriate pattern has been discussed in Section 2, and we focus here on the cable layout pattern, for a given spatial layout of vertices or turbine towers.

Topologically we can view the farm as a graph $G$ of vertices $V$ connected together by edges $E$. A great deal of research has been reported on the construction of MSTs and there are a number of algorithmic variations, especially those optimised for very large graph sizes with a large number of vertices $N_v$ or edges $N_e$. We use a variant of the algorithm discovered by Dijkstra [10] and Prim [34] and widely known as Prim’s algorithm. For completeness we summarise this in listed in Algorithm 1.

For a practical application we ascribe spatial coordinates to each vertex and hence we can compute the Eulerian spatial distance been all possible edges in the graph.

Algorithm 1 Prim’s algorithm for constructing an MST of Wind Farm turbine towers.

<table>
<thead>
<tr>
<th>Named Farm</th>
<th>Cables to Shore</th>
<th>Turbine Towers (planned)</th>
<th>Map Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrow</td>
<td>7</td>
<td>272 (408)</td>
<td>0</td>
</tr>
<tr>
<td>Watermost Rough</td>
<td>1</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>Humber Gateway</td>
<td>1</td>
<td>74</td>
<td>2</td>
</tr>
<tr>
<td>Dudgeon</td>
<td>1</td>
<td>(68)</td>
<td>3</td>
</tr>
<tr>
<td>Sheringham Shoal</td>
<td>2</td>
<td>90</td>
<td>5</td>
</tr>
<tr>
<td>Lincs</td>
<td>6</td>
<td>124</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 4. Barrow wind farm complex - showing multiple separate turbine clusters and to-shore cable routes.

We use a variation of Prim’s Algorithm for computing a minimal spanning tree (MST) that has been implemented using adjacency lists and which scales adequately for the size of systems we discuss in this paper. We employ specialist graph software tools [13] to render the resulting tree graphs constructed.

In addition to the rendered graph we can compute various properties, the most interesting for our present analysis is the total weighted edge length of the constructed trees. This is a quantitative metric that gives a good approximation to the length of cable that must be laid and the associated seabed trench length dug.

4 Results

We analyse the wind farm layouts listed in Table 1 and give comments on the connectivity patterns used and in particular how they compare to the MST connectivity tree computed. We also compute total edge weight - or cable length both for the existing connectivity layout - where it is known - and that of our simulated MST connectivity pattern.

We also discuss some simulated turbine layouts and the connectivity patterns that our MST algorithm generates for them.

Figure 4 shows a typical wind farm layout for the large installation that is planned for offshore of Barrow in north west England. The system comprises several clusters of turbine towers, and we were able to obtain connectivity data only some of them. This is the largest installation we con-
considered and it is unusual in having several cable connections to shore - each from a different aggregation point. We speculate this may be because of the contributions of different contractors/operating companies or due to construction at different times and likely a complex operational plan.

Figure 5 shows a simpler installation off the east coast at Dudgeon. The planned installation has a single aggregation point around the centre of the farm, although our computed connection shows a connection pattern that is shorter overall.

Figure 6 shows the Sheringham Shoal installation layout which has two separate clusters of turbine towers and two associated to-shore connection cable routes.

Of particular interest to us are the Humber Gateway installation for which the layout is illustrated in Figure 7, and also the Lincolnshire (Lincs) installation as shown in Figure 8. The Humber Gateway farm has a single connection route to shore and our computed MST connection is significantly shorter in overall length. The Lincs Farm complex has a more sophisticated cluster structure and multiple to-shore cable routes.

The “Watermost-Rough” Wind Farm has an interesting tree connectivity layout and is shown in Figure 9. This is a relatively small farm and our computed MST connectivity pat-
tern is statistically similar but still shorter than the one used. It is also interesting to consider a range of theoretical turbine layout patterns and to explore the cabling connections that the MST algorithm uncovers for these. We can then compare the patterns and properties for these theoretic layouts to the practical schemes for particular UK farm locations and installations.

Figure 10 shows simulated layout patterns for regular lattice patterns as well as a random one. The edge weights have been computed using precise Eulerian distances from the spatial layout and the MST algorithm has chosen the spanning trees as shown. Note that in the case of the regular lattices there are a lot of degenerate duplicate trees with precisely the same length. Other criteria such as balance of the tree structure or cable laying practicalities could be applied to choose amongst the degenerate cases.

In the case of the random network, the trees are typically quite deep, and while optimising cable length this may not necessarily be a practically desirable solution as it introduces great risk consequences to single points of failure.

5 Discussion

Figure 11 shows a plot of the total weighted edge length $L_{\text{tot}}$ (or spatial cable length) versus the number of vertices $N_v$ (or number of individual wind turbines) plotted on a log-log scale. The slope of a least-squares fit therefore determines the exponent $\nu$ in a power-law relationship:

$$L_{\text{tot}} = A \cdot N_v^\nu$$

(1)

Where $A = e^c$ and $c$ is the fitted intercept. In fact in this context $A$ is of less direct interest and determines only the absolute length scale. In the work reported here we have converted all length scales into a unit system consistent with our plots. In the log-log scale this does not affect the fitted slopes and hence power law exponents.

The scalability analysis can be summarised by the slopes of the straight lines fitted to the log-log plots shown in Table 2.

<table>
<thead>
<tr>
<th>Layout</th>
<th>Slope (Power Law)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>1.13 ± 0.01</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>1.06 ± 0.02</td>
</tr>
<tr>
<td>Named Specific</td>
<td>1.00 ± 0.0001</td>
</tr>
<tr>
<td>Triangular</td>
<td>0.65 ± 0.15</td>
</tr>
<tr>
<td>Random</td>
<td>0.5 ± 0.005</td>
</tr>
</tbody>
</table>

Table 2. Power Law Scalability of total cable length versus number of turbine towers for various possible Wind Farm Layouts

The implication from the good straight line fits is that the relationship $L_{\text{tot}} \approx N_v^\nu$ well describes the data and that larger wind farm turbine connectivity layouts would likely scale up their cable costs in this manner.

In all cases except the triangular layout there is a single best fitted linear trend. We speculate that the even-odd parity of the generated patterns for this layout have contributed to two separate lines, both of which have broadly similar slopes but different intercepts. As discussed, the slope is the more useful attribute in our scalability analysis.

It is interesting to note that even the total edge length of the MST solution to the random turbine layout scales as a power law - albeit on a quite different scale than that for the regular layouts. We cannot imagine a scenario where a random turbine layout would be chosen operationally, although as individual devices failed and were deemed too expensive to replace, one might end up with a semi-random collection of operational turbine towers. In a sense the random case represents an extreme bound on the layout design that might arise from a complex sea-bed configuration and a very large collection of turbines laid out in a number of individually more regular farm clusters.

The specific named farms scale in a surprisingly precise manner - suggesting a particular algorithm or approach - unknown to the author - was used. Generally speaking the hexagonal/triangular turbine layouts represent a staggered
grid and may well be best for wind capture and minimising blade shadowing and wake effects. It appears there may be some room for improvement in the scalability of the actual farm connectivity approach used. It should be emphasised however this is only an opinion based on what is a very theoretical analysis, made without access to detailed operational constraints and other optimisation criteria that were likely taken into account for the designs of cable connections for the real farms studied. Nevertheless it may be worthwhile reappraising the connections and particular preferred location of aggregation points in these systems.

Table 3 summarises an interesting observation concerning the planned connectivity layouts from some of the installations described compared with the Minimum spanning tree solutions that we have computed. It is not possible to make simple comparisons with installations that have multiple clusters and hence multiple cable aggregation points, but for those farms that do, we observe a consistently shorter cable layout using our calculated connectivity pattern from that apparently being implemented. The percentage improvement is significant and ranges conservatively from 15 to 23 percent.

The ease of maintaining a farm with a common cabling system is likely to be an important design consideration, although it appears that some operational farms are already having new turbines added to them incrementally, with the potential to use newer emerging technologies or standards. Cables routes may also be constrained by seabed properties which may affect the ability to dig appropriate trenches for burying the cables in. We have only considered topologies for electrical feeds and not for control and monitoring and other communications, assuming such infrastructure is incorporated into the main trenched cable bundle.

While there are likely a range of operational reasons that make the optimal calculation more complicated, there would still seem to be considerable likely scope for further optimisation in presently calculated cabling connectivity patterns for UK offshore wind farms. There is scope for encapsulating the methods and approaches described here in an appropriate domain-specific language solution [16], making them more easily deployable in practical design use cases.

Connectivity reliability and fault tolerance requirements would also likely affect the chosen solution and there is scope for incorporating such criteria into a more sophisticated analysis, optimising over more than simple total edge length. Some networks with loops might be favoured since this would give fail-over reliability in the case of for example a boat anchor damaging a single cable. Loops in power systems carry their own complications however and
additional control systems would likely be necessary - not always simple to deploy in the context of a submerged cable network.

6 Conclusion
We have studied a range of offshore wind farm layouts and in particular the cable connection patterns used to wire up collections of wind turbine platform towers. We encoded a set of layouts from some real UK offshore farms and compared the total edge length or cable length used with that of theoretical simulated layouts using a minimum spanning tree algorithmic approach. generally we find the MST solutions arrive at shorter cable lengths, although we recognize there are a range of other complex optimisation criteria involved including preferred location of aggregation points and offshore conditions.

The scalability of various turbine layout patterns along with the total edge length of their minimum spanning tree solutions has been our main focus. We find that there is a very good fit to a power law with total edge length scaling to a power of the number of turbine towers with an exponent of between 0.65 to 1.13 for regular layout patterns. A random layout represents a lower bound with an exponent found of 0.5.

We believe there is considerable scope for further use of graph analysis methods for the problem of optimising offshore wind farm layouts and connectivity patterns, which likely constitute complex systems in their own right. In particular there are other graph metrics such as tree depth, path-length statistics and weighted diameter which may also give useful insights into simulated connectivity solutions.

Finally, the methods we have employed here would extend to space-filling network problems in 3 dimensions as well as the planar 2-dimensional wind farm systems we have discussed. There is scope for deploying this approach in 3D manufacturing where wiring layouts in a dense system need to be optimised.

Acknowledgment
It is a pleasure to thank D. Wells for his helpful expert comments on the Offshore Wind Farm industry.

Table 3. Properties of the Cable Layout Patterns for Various Specific UK Wind Farm Layouts

<table>
<thead>
<tr>
<th>Named Farm</th>
<th>Total Edge-Length Listed from Map</th>
<th>Total Edge-Length Prim Generated</th>
<th>Percentage Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watermost Rough</td>
<td>4.62</td>
<td>3.92</td>
<td>15 %</td>
</tr>
<tr>
<td>Humber Gateway</td>
<td>6.81</td>
<td>5.26</td>
<td>22 %</td>
</tr>
<tr>
<td>Dudgeon</td>
<td>4.35</td>
<td>3.31</td>
<td>23 %</td>
</tr>
<tr>
<td>Barrow</td>
<td>N/A</td>
<td>13.68</td>
<td>-</td>
</tr>
<tr>
<td>Sherringham Shoal</td>
<td>N/A</td>
<td>6.52</td>
<td>-</td>
</tr>
<tr>
<td>Lincs</td>
<td>N/A</td>
<td>9.14</td>
<td>-</td>
</tr>
</tbody>
</table>

References
[12] Freeman, L.C., Borgatti, S.P., White, D.R.: Centrality in val-

175


