TRANSMISSION LINES MODEL WITH DIFFERENT BASIC STRUCTURES APPLIED TO TRANSIENT ELECTROMAGNETIC SIMULATIONS

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ABSTRACT

Models based on lumped elements are simple ones to represent systems composed by distributed parameters when models with lumped elements use a lot of these elements. However, such mentioned models usually present numerical problems because it is subject to the influence of numerical oscillations. When introducing new elements, such as damping resistors, in the mentioned models, the numerical oscillations can be minimized. The introduction of new elements increases the computational time for the analysis and simulations with the modified model. Thus, by applying \( \pi \) circuit cascades, simulations with abrupt changes in voltage or current are carried out using two different structures of \( \pi \) circuits with the purpose to achieve a similar accuracy and less computational time than those obtained with application damping resistor in every \( \pi \) circuit units.

KEY WORDS

Eigenvalues and eigenfuctions, linear systems, numerical analysis, power system transient simulation, state space methods, time-domain analysis, transmission line modeling.

1. Introduction

In analysis of electrical systems affected by electromagnetic transient phenomena, models in the time domain and frequency dependent are considered very efficient and accurate for this type of application [1]-[12]. But, it is searched the optimization for such models because there are still crucial points regarding the accuracy and efficiency [4]-[7]. Considering these points, simple modeling, such as applications constant transformation matrices or even models based on the lumped parameters [1]-[10], it is possible to minimize the influence of numerical oscillations in the obtained results introducing structural modifications in the circuits used to model electrical systems [4]-[10]. When considering the simplified representation of a transmission line by cascades of \( \pi \) circuits, the solution of this system can be performed by the trapezoidal integration [1], [8]. However, some results, particularly those related to rapid changes in voltage or current in system, are contaminated by numerical oscillations (Gibbs’ oscillations). The introduction of resistances in parallel with the elements in series of \( \pi \) circuits (elements that represent the longitudinal parameters of the line) can lead to minimization of the numerical oscillations [7]. The incorporation of an element in \( \pi \) circuit of each cascade used, however, increases the computational time for performing analyzes and simulations with this model [6].

Considering the alternate application of the damping resistor in a cascade of \( \pi \) circuits, tests are made checking for accuracy and computation time in comparison with the results obtained by the damping resistance application for all \( \pi \) circuits in the cascade. Such analyzes are based on step voltage inputs the respective voltage outputs. The comparisons are carried out based on voltage graphs as a function of the time and three-dimensional graphics that establish the relationship among the first voltage peaks with the number of \( \pi \) circuits and the damping resistance values during the first voltage reflection at the line end.

2. The Trapezoidal Rule

The trapezoidal rule is a numerical integration method based on transformation of differential equations to equivalent algebraic ones. The integral of a function is approximated by a first degree function, or the area of a trapezoid where the extreme points are approximated by points of the intersection between the function ant the first degree one. Improving the accuracy of the approximation, a big range of independent variable can be subdivided into smaller equal portions, called integration steps (Fig. 1).

![Figure 1. Trapezoidal rule.](image-url)
Applying the trapezoidal rule, it is obtained:
\[ \int_{x(k)}^{x(k+1)} f(x) \, dx \approx \frac{\Delta x}{2} \left[ f(x_{k+1}) + f(x_k) \right] \] (1)
Using (1), it is obtained:
\[ \int_{x(k)}^{x(k+1)} f(x) \, dx \approx y_{k+1} - y_k = \Delta y \] (2)
\[ y_{k+1} = y_k + \frac{\Delta x}{2} \left[ f(x_k) + f(x_{k+1}) \right] \]
The integration step is:
\[ \Delta x = x_{k+1} - x_k \] (3)

3. The Transmission Line Model

Analyzing wave propagation in transmission lines, these systems are decomposed into infinitesimal portions which can be represented by π circuits. There is a need to use a cascade with a large amount of π circuits and this cascade consists of three different types of π circuits: a circuit that represents the line power terminal, intermediate circuits and the circuit that represents the line end. In Fig. 2, the intermediate unit is shown [6].

Mesh voltages and node currents lead to:
\[ \frac{d v_k}{dt} = \frac{1}{L} \left( x_{k-1} - R \cdot i_k - v_k \right) \] (4)
\[ \frac{d i_k}{dt} = \frac{1}{C} (v_k - G \cdot i_k - i_{k+1}) \]

Each π circuit is described by two state variables (voltage and current). For a cascade with n π circuits, the following relationships are obtained:
\[ \dot{x} = [A]x + [B]u \quad B = \begin{bmatrix} \frac{1}{L} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{C} & -\frac{1}{C} \end{bmatrix} \] (5)
\[ x_n = [i_1 \quad v_1 \quad i_2 \quad v_2 \quad \cdots \quad i_n \quad v_n]^T \]
The A matrix is based on (4) and its structure is:
\[ A = \begin{bmatrix} \frac{-1}{L} & \frac{-1}{C} & 0 & \cdots & 0 \\ \frac{-1}{C} & \frac{-1}{C} & \cdots & \cdots & \cdots \\ 0 & \cdots & \frac{-1}{C} & \frac{-1}{C} & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & \frac{-1}{C} & \frac{-1}{C} \\ 0 & \cdots & 0 & 0 & \frac{-1}{C} \end{bmatrix} \] (6)
The damping resistor produces the modification shown in Fig. 3 and the new relationships of voltages and currents are in (7). The damping resistance is calculated by (8) and the proportional factor \( k_D \) is integer [7].

\[ \begin{cases} \dot{i}_k = v_{k+1} - R \cdot i_k - v_k \\ \dot{v}_k = \frac{1}{C} \left( i_k - (2G_D + G)v_k + G_D(v_{k-1} + v_{k+1}) - i_{k+1} \right) \end{cases} \] (7)
\[ R_D = k_D \frac{2L}{\Delta t} \leftrightarrow \quad \frac{1}{G_D} = \frac{1}{C} \quad k \text{ is even} \] (8)
The introduction of \( R_D \) leads to changes in the matrix structure shown in (9). The other elements remain the same as those in (6) [6], [7].
\[ A_{k} = \frac{1}{C} \begin{bmatrix} \frac{-2G_D + G}{C} & \cdots & \frac{-2G_D + G}{C} \\ \frac{G_D}{C} & \cdots & \frac{G_D}{C} \end{bmatrix}, \quad A_{k+2} = \frac{-2G_D + G}{C} \quad \text{integer} \] (9)
The R, L, G and C values are calculated by:
\[ R = R^\prime \frac{d}{n}, \quad L = L \frac{d}{n}, \quad G = G^\prime \frac{d}{n}, \quad C = C^\prime \frac{d}{n} \] (10)
\[ d \rightarrow \text{the line length} \]

4. The Numeric Routine Flowcharts

The flowcharts that illustrate the basic structure of numerical routines used to perform the simulations shown in this paper are shown in Figs. 4 and 5. Introducing the damping resistor in all π circuits, the flowchart is shown in Fig. 4 [6].

![Flowchart for the simulations with the damping resistance.](Image)

Considering the alternate application of the damping resistor in a cascade of π circuits, the flowchart in Fig 5 shows the basic steps of the numeric routine used for this type of simulations [13].
5. Obtained Results

Using a π circuit cascade consisting of alternating units with damping resistor and without this resistor, simulations were performed to analyze the influence of the mentioned resistor in decreasing numerical errors in the simulation of electromagnetic transients in transmission lines. Taking \( k_D = 15 \) and \( \Delta t = 60 \text{ ns} \), the number of π circuit was varied between 101 and 551 with a step 50. Odd numbers of π circuits were used to ensure that the first and last π circuit have the resistor \( R_D \). With 101 units, the decreasing in numerical oscillations was no significant for the carried out simulations. The voltage peak remains reaching the value of 2.5 pu. The limit of stability of the numerical method in this case is for the number of π circuits between 151 and 251 (Figs. 6 and 7).

![Figure 6: Output voltage for \( k_D = 15, \Delta t = 60 \text{ ns}, n = 151 \).](image)

![Figure 7: Output voltage for \( k_D = 15, \Delta t = 60 \text{ ns}, n = 251 \).](image)

Considering the number of π circuits of 151, the numerical oscillations are fast stabilized in each period between the wave reflections in the line terminals. In this case, the \( k_D \) factor was varied between 10 and 50 with step 5. It has been found that the numerical oscillations undergo a greater cushioning to between \( k_D \) values of 10 and 20. (Figs. 8 and 9). Given this range, the best option is \( k_D = 15 \) (Fig. 10) due to the concentration of the oscillations in a shorter time in each period of reflection of the wave. Subsequently, the number of π circuits used in the simulations was 201. Similar to \( n = 151 \), the results still show more controlled numerical oscillations (Fig. 11), but with \( k_D = 10 \) (Fig. 12), there is numerical instability. For \( k_D \) values above 20 (Fig. 13), a longer time for damping of oscillations is required in each period of reflection when compared with the results for \( k_D = 15 \). Using a large number of simulations, several values of the peak voltage during the first wave reflection were considered at the output terminal. The voltage peaks were obtained for a range of \( k_D \) from 10 to 50 and the number of π circuits from 101 to 401. It is The results are shown Fig. 14.

![Figure 8: Output voltage for \( k_D = 10, \Delta t = 60 \text{ ns}, n = 151 \).](image)

![Figure 9: Output voltage for \( k_D = 20, \Delta t = 60 \text{ ns}, n = 151 \).](image)

![Figure 10: Output voltage for \( k_D = 15, \Delta t = 60 \text{ ns}, n = 151 \).](image)

Regardless of whether or not to decrease the computational time required to transient simulations, it considers the alternating application damping resistance and leads to inconsistent and numerically unstable results. This is clearly observed when comparing the results shown in Fig. 14 with the results shown in Fig. 15. Fig. 15 shows results obtained previously with the damping resistor applied to all π circuits of cascade [13].

Figure 5. Flowchart for the alternate application of the damping resistor.
Simulating the propagation of electromagnetic transients, when transmission lines is modeled using cascades of \( \pi \) circuits, the obtained results can be highly influenced by Gibbs’ oscillations. Previous results obtained with the introduction of damping resistor in each \( \pi \) circuit of the cascade show significant reductions in numerical oscillations. This modification to the introduction of a new element in each \( \pi \) circuit causes an increase in the computational time required to carry out the digital simulations. With the objective of maintaining the minimized numerical oscillations and reduce computational time, it proposed a model based on two different \( \pi \) circuit structures: one without the damping resistor and the other with this resistor. So, the cascade of \( \pi \) circuits was composed by alternating these two types of \( \pi \) circuits. However, this proposed model is inadequate because the results are numerically unstable for certain damping resistor values and certain quantities of \( \pi \) circuits. Therefore, the use of damping resistor to all \( \pi \) circuits is still a better option than the model proposed in this work.

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### References


