ABSTRACT
This paper provides further advancement of the strategy for analysis and design of servo robust control systems with backlash nonlinearities. A case of a control system, consisting of linear and nonlinear sections is discussed, where the nonlinearity is a backlash element. Taking into consideration the Bilinear Tustin Transform and the Euler’s approximation, a robust controller design is achieved by a number of successive steps. By applying the Goldfarb stability criterion, the performance of the control system is examined before and after the application of the robust controller proving its robustness. Based on the difference equations of its stages, the robust controller is realized by two microcontrollers. This research is a further development of the author’s work on the analysis and design of linear, nonlinear and digital robust control systems. The suggested tool for analysis and design of servo nonlinear robust control systems is essential and beneficial for the further development of control theory in this area.

KEY WORDS
Servo System, Nonlinear, Interaction, Describing Function Analysis, Robust Control, Microcontrollers;

1. Introduction
The current research is suggesting a strategy for design of a servo robust control system with a backlash nonlinear element. The stability evaluation of the linear section of the system is a substantial precondition in the overall system assessment [1]. A stable linear section is an important requirement, to achieve the overall stability of the control system [2], [3]. The stability of the total system is assessed by the interaction between the linear and nonlinear stages of the control system. This is facilitated by the Goldfarb stability criterion, based on the Describing Function analysis. It examines the eventual existence of limit cycles, the system’s stability and its robustness [3], [4].

An optimal robust controller is implemented based on two-step compensation and operating in two degrees of freedom. Taking into account the Euler’s approximation, the robust controller stages are realized by implementing digital filters based on microcontrollers [5], [6].

2. Original Servo System
2.1 General Description
Backlash is one of the most important nonlinearities that limit the performance of speed and position control in industrial, robotics, automotive, automation and other applications. The control of systems with backlash has been the subject of study since the 1940s. Surprisingly few control innovations have been presented since the early path breaking papers that introduced the describing function analysis of systems with backlash.

Figure 1 represents a nonlinear servo system consisting of an amplifier, dc motor, a gear train mechanism and a load. Its lineal part that is preliminary reduced and approximated to a second order system with a transfer function \( G_P(s) \). Its nonlinear part is a backlash element, having a describing function \( N(M) \) [6].

The position of the output is fed back to the input, to generate the error signal. A control system of such type may create a limit cycle of specific amplitude and frequency, which in this case is undesirable effect and should be avoided.

The gear train consists of two gears. It is assumed that the inertia of the gears and load element is negligible compared with that of the motor. It is also assumed that there is no backlash between the motor shaft and the first gear. Backlash exists only between the first and the second gear. Also it is assumed that the gear ratio between the two gears is unity. The block diagram of the system can be modified as seen in Figure 2. In addition it includes a digital compensator that is to be designed.

Figure 1. Block diagram of a servo system with backlash

Figure 2. Modified model of the backlash control system with the digital compensator
The transfer function of the amplifier-motor combination is presented as follows:

\[ G_p(s) = \frac{K}{s(1+s)} = \frac{K}{s^2 + s} \]  \hspace{1cm} (1)

The plant’s gain K is the variable parameter due to some temperature effects on the system. The system is stable for any positive gain value or within the range 0 < K [7]. However, it will be seen that the interaction between the linear and the nonlinear sections of the system will cause problems of the system operation for high values of the gain K if the system is not compensated and made robust. The nonlinear section of the servo system plant is backlash nonlinearity [2], [7]. Its transfer characteristic and properties are shown in Figure 3.

![Figure 3. Characteristic and properties of the backlash](image)

The describing function of the backlash is presented as follows [2], [6], [7]:

\[ N(M) = \frac{1}{M} \sqrt{A^2 + B^2} \tan^{-1}\left(\frac{A}{B}\right) \]  \hspace{1cm} (2)

\[ A = \frac{2D}{\pi} \left(\frac{2D}{M} - 2\right) \]  \hspace{1cm} (4)

\[ B = \frac{M}{\pi} \left[\frac{\pi}{2} - \sin^{-1}\left(\frac{2D}{M} - 1\right) - \left(\frac{2D}{M} - 1\right)\cos^{-1}\left(\frac{2D}{M} - 1\right)\right] \]  \hspace{1cm} (5)

where \( M \) is the amplitude of the input variations.

2.2 The Concept of the Goldfarb Stability Criterion

The concept of the Goldfarb stability criterion also known as the Describing Function analysis can be explained with the aid of the block diagram of a basic nonlinear system. It consists of a nonlinear and a linear component as shown in Figure 4.

![Figure 4. Basic block diagram of a closed-loop nonlinear system](image)

According to the Describing Function analysis, the input signal to the nonlinear element is considered as:

\[ m(\omega t) = M \sin \omega t \]  \hspace{1cm} (6)

The Describing Function analysis assumes that the output of a nonlinear element is a periodic signal having the same fundamental frequency as that of the input where all harmonics and any dc component are neglected. Following this concept [2], [7], the transfer function of the closed-loop system shown in Figure 4 becomes:

\[ \frac{C(j\omega)}{R(j\omega)} = \frac{N(M, \omega)G_p(j\omega)}{1 + N(M, \omega)G_p(j\omega)} \]  \hspace{1cm} (7)

Accordingly, the characteristic equation of the closed-loop system is [8], [9], [10]:

\[ 1 + N(M, \omega)G_p(j\omega) = 0 \]  \hspace{1cm} (8)

from where

\[ G_p(j\omega) = -\frac{1}{N(M, \omega)} = Z(M, \omega) \]  \hspace{1cm} (9)

The Describing Function analysis can be considered as a linear approximation of a static nonlinearity limited to the first harmonic [2], [3], [11]. The harmonic balance equation (8) is similar to the characteristic polynomial function that leads to the Nyquist condition for closed-loop stability. Equation (9) that is derived as a result of equation (8) is considered as the basis of the Describing Function analysis.

If equation (9) is satisfied, then the system output will be experiencing a limit cycle. This corresponds to the case where the \( G_p(j\omega) \) locus passes through a critical point. While in the conventional frequency-response analysis of linear control systems, the critical point is (-1, j0), in the Describing Function analysis, the critical point is modified so that the entire \( Z(M) = -1/N(M, \omega) \) locus becomes a locus of critical points [2], [11]. Therefore, the relative location and intersection of the \( Z(M) = -1/N(M, \omega) \) locus and the \( G_p(j\omega) \) locus will provide the stability information [2], [3], [11], [12].

The stability of a nonlinear system is determined by plotting the \( Z(M) = -1/N(M, \omega) \) locus and the \( G_p(j\omega) \) locus on a common plane. There is an important precondition that \( G_p(j\omega) \) should correspond to a stable stand-alone system. While there are a number of rules for the application of the Describing Function analysis, here, only the rules related to its application to systems with backlash nonlinearity are considered:

If the \( Z(M) = -1/N(M, \omega) \) locus and the \( G_p(j\omega) \) locus intersect, the system output may exhibit a sustained oscillation, or a limit cycle. It is characterized by the magnitude \( M \) of the \( Z(M) = -1/N(M, \omega) \) locus and the value of the frequency \( \omega \) of the \( G_p(j\omega) \) locus at the intersection. A limit cycle may be stable or unstable.

If the \( G_p(j\omega) \) locus is not enclosing the points of \( Z(M) = -1/N(M, \omega) \) locus, corresponding to increment of \( M \), the closed-loop system is stable.
If the $G_P(j\omega)$ locus is enclosing the points of $Z(M) = -1/N(M, \omega)$ locus, corresponding to increment of $M$, the closed-loop system is unstable. For a lot of control systems having nonlinear elements, like on-off element with hysteresis, saturation, dead zone etc., stable limit cycle output oscillations is their natural performance. In case of control systems containing backlash nonlinearity, even a stable limit cycles are undesirable effect, since continuous oscillations in the gear box will cause its destruction [2],[11], [12].

### 2.3 Application of the Describing Function Analysis in Case of Backlash Nonlinearity

It is assumed that the parameters of the backlash nonlinearity are $K_1 = 1$ and $D = 1$ and are constant values. The function $Z(M)$ is plotted in accordance with equation (9). The results for different values of the amplitude $M$ are shown in Table 1.

<table>
<thead>
<tr>
<th>$M$</th>
<th>2</th>
<th>1.5</th>
<th>1.25</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(M)$</td>
<td>$0.6 \angle -36^\circ$</td>
<td>$0.4 \angle -45^\circ$</td>
<td>$0.2 \angle -55^\circ$</td>
<td>$0.1 \angle -65^\circ$</td>
</tr>
<tr>
<td>$Z(M)$</td>
<td>$-1.67 \angle +35^\circ$</td>
<td>$-2.5 \angle +45^\circ$</td>
<td>$-5 \angle +55^\circ$</td>
<td>$-10 \angle +65^\circ$</td>
</tr>
</tbody>
</table>

It was established that the linear section of the system $G_P(s)$ is stable for any value of the gain $K$. Under this condition, $G_P(j\omega)$ at different gains ($K = 4$, and $K = 5$), together with $Z(M) = -1/N(M)$ are plotted in the complex plane as shown in Fig. 5.

Taking into account the intention of designing a digital robust controller, the code for plotting $G_P1(j\omega)$ and $G_P2(j\omega)$ is introduced in the discrete time-domain by applying the bilinear transform, or the Tustin’s approximation [13] and is as follows:

```matlab
>> Gp1=tf([0 4],[1 1 0])
Transfer function:
  4
  ______
  s^2 + s
>> Gp2=tf([0 5],[1 1 0])
Transfer function:
  5
  ______
  s^2 + s
>> G1d=c2d(Gp1,0.1,'tustin')
Transfer function:
 0.009524 z^2 + 0.01905 z + 0.009524
--------------------------------------------------------
  z^2 - 1.905 z + 0.9048
Sampling time: 0.1
>> G2d=c2d(Gp2,0.1,'tustin')
Transfer function:
 0.0119 z^2 + 0.02381 z + 0.0119
--------------------------------------------------------
  z^2 - 1.905 z + 0.9048
Sampling time: 0.1
>> nyquist(G1d,G2d)
```

Figure 5. Application of the Describing Function Analysis for linear section gains: $K = 4$ and $K = 5$

As seen from Figure 5, there are two intersections between the loci of $Z(M)$ with each one of the loci $G_P1(s)$ and $G_P2(s)$. Applying the Goldfarb stability criterion reveals that the point on the $G_P1(s)$ loci ($M = 2$, $\omega = 1.6$) and the point on the $G_P2(s)$ loci ($M = 1.5$, $\omega = 1.2$) correspond to stable limit cycles.

The other two intersections between the $N(M)$ loci with each one of the loci $G_P1(s)$ and $G_P2(s)$, correspond to unstable limit cycles, since $G_P1(s)$ and $G_P2(s)$ enclose this part of $Z(M)$, related to increment of $M$. Although, in this case, the unstable limit cycles can still be described theoretically with their amplitude and frequency of oscillation, they cannot physically occur.

As already discussed, the stable limit cycles are undesirable effect for systems with backlash nonlinearity. Further, if the two loci are tangent, or almost tangent, a limit cycle or a slowly damped oscillation may occur, which is also undesirable effect for systems with backlash nonlinearity.

To avoid the limit-cycle behaviour, the gain of the amplifier must be decreased sufficiently so that the $G_P(s)$ locus is placed well apart the $Z(M)$ locus. Alternatively, the application of a robust controller will put apart the $G_P(s)$ and the $Z(M)$ loci, still keeping the desirable system gain and maintaining the system’s robustness.

### 3. Digital Robust Control Design for Case of Backlash Nonlinearity

A digital robust controller design employs a two-step compensation operating in two degrees of freedom. The design is based on the linear section prototype of the system and is considered as a general robust control design. Nevertheless, practice shows that it affects the
robustness of the total nonlinear control system within some quite reasonable bounds in respect to the original values of the variable parameters.

An optimal robust control is achieved by applying the ITAE criterion. Accordingly, a damping ratio of $\zeta = 0.707$ corresponds to optimal performance of a second order system. Comparison with other performance indices reveals that the ITAE provides the best selectivity in the sense that its minimum value is apparent and most easily detectable, as the system’s parameters are varied [8], [10], [14]. For higher order systems, a pair of dominant poles can represent the system dynamics and $\zeta$ can still be used as a relative damping ratio, to indicate the location of these poles. The relative damping is going to be considered as the required performance index (IP) of the system. To meet this IP, a robust controller consisting of a series and a forward stage is implemented [8], [10], [14], enforcing two dominant poles. The following design steps are considered:

**Step 1:** The closed-loop transfer function of the linear section of the servo system as a stand alone system is determined by applying a unity feedback to this section described by equation (1) and can be expressed as:

$$ G_{cl}(s) = \frac{K}{s^2 + s + K} \quad (10) $$

**Step 2:** The optimal gain $K$ of the continuous linear plant at a relative damping ratio $\zeta = 0.707$ can be determined by an interactive procedure with the plot of the $\zeta = f(K)$, shown in Figure 6. The following code is applied:

```matlab
>> K=[0:0.1:10];
>> for n=1:length(K)
    G_array(:,:,n)=tf([K(n)], [1 1 K(n)]);
end
>> [y,z]=damp(G_array);
>> plot(K,z(1,:))
```

Figure 6. Determination of the Optimal Gain K

It is seen from Figure 6 that the relative damping ratio is $\zeta = 0.7071 \approx 0.707$ if the gain is $K = 0.5$.

**Step 3:** By substituting $K = 0.5$ in equation (10) the transfer function of the closed-loop system becomes:

$$ G_{cl}(s) = \frac{0.5}{s^2 + s + 0.5} \quad (11) $$

The optimal system is assessed by the code:

```matlab
>> GCLO =tf([0 0.5], [1 1 0.5])
>> damp(GCLO)
```

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Damping</th>
<th>Freq. (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.00e-001 + 5.00e-001i</td>
<td>7.07e-001</td>
<td>7.07e-001</td>
</tr>
<tr>
<td>-5.00e-001 - 5.00e-001i</td>
<td>7.07e-001</td>
<td>7.07e-001</td>
</tr>
</tbody>
</table>

It is seen that at damping of $\zeta = 0.707$ the desired closed-loop poles are $-0.5 \pm j0.5$.

A comparison is done between the linear system performance and the system’s performance in case of its operation in the discrete-time domain.

If the system is digitalized, taking into account the condition of the Euler’s approximation [15], the sampling period $T_s$ is to be at least 10 times smaller than the minimum time constant $T_{\text{min}}$ of plant $G_p(s)$. In this case, the plant’s minimum time constant is $T_{\text{min}} = 1$ sec. Then the sampling period is chosen to satisfy the condition $T_s < 0.1T_{\text{min}}$ and for the case of the discussed system it is established as $T_s = 0.1$ sec.

By applying the bilinear transform or the Tustin’s approximation [4], [13], [15], the optimal open-loop and closed-loop transfer functions of the system are determined in the discrete-time domain:

```matlab
>> GdCLO = c2d(GCLO,0.1,'tustin')
```

**Step 4:** Then the two series controller zeros can be placed at $-0.5 \pm j0.5$ and its transfer function becomes:

$$ G_s(s) = \frac{(s + 0.5 + j0.5)(s + 0.5 - j0.5)}{0.5} \quad (12) $$

**Step 5:** The series robust controller $G_s(s)$ is connected in series to the continuous linear section of the plant considered as a stand alone system $G_{cl}(s)$. As a result, their open loop transfer function is:
Step 6: Further, the unity feedback closed-loop transfer function is determined as:

\[ G_{CL}(s) = G_S(s)G_{CL,O}(s) = \frac{K(s^2 + s + 0.5)}{0.5(s^2 + s)} \] (13)

Step 7: From equation (14), it is seen that the closed-loop zeros will attempt to cancel the closed loop poles of the system being in their proximity. This can be avoided if a forward controller \( G_F(s) \) is added to the closed-loop system. The poles of \( G_F(s) \) have the objective to cancel the zeros of the closed-loop transfer function \( G_{CL}(s) \). As a result the transfer function of the forward controller can be designed as:

\[ G_F(s) = \frac{0.5}{s^2 + s + 0.5} \] (15)

Step 8: As a result, the transfer function of the linear total compensated robust control system becomes:

\[ G_T(s) = G_F(s)G_{CL}(s) = \frac{0.5K}{0.5(s^2 + s) + K(s^2 + s + 0.5)} \] (16)

4. Robustness Assessment of the Servo Control System

The linear section of the robust compensated system is assessed for two gain values \( K = 5 \) and \( K = 50 \), substituted in equation (16):

\[ G_{T1,K=5}(s) = \frac{2.5}{5.5s^2 + 5.5s + 2.5} \] (17)
\[ G_{T2,K=50}(s) = \frac{25}{50.5s^2 + 50.5s + 25} \] (18)

The step responses in the discrete-time domain for the two different gain values of the linear section are shown in Figure 7 and are obtained by the code:

```matlab
>> GT1=tf([0 2.5],[5.5 5.5 2.5])
>> GT2=tf([0 25],[51 51 25])
>> H1 = c2d(GT1,0.1,'tustin')
Transfer function:
0.002198 z + 0.002126
--------
z^2 - 1.901 z + 0.9048
Sampling time: 0.1
>> H2 = c2d(GT2,0.1,'tustin')
Transfer function:
0.002237 z + 0.002293
--------
z^2 - 1.9 z + 0.9048
Sampling time: 0.1
>> step(H1,H2)
```

It is seen that the transfer functions of equations (17) and (18) are transformed in the discrete-time domain as:

\[ H_{T1,K=5}(z) = \frac{0.002198 z + 0.002126}{z^2 - 1.901 z + 0.9048} \] (19)
\[ H_{T2,K=50}(z) = \frac{0.002234 z + 0.002315}{z^2 - 1.9 z + 0.9048} \] (20)

The step responses of the linear section of the robust compensated system in the discrete-time domain for the two different gain values are presented in Figure 7.

4. Describing Function Analysis of the Robust Servo Control System

The system performance after robust compensation is assessed with the aid of the Goldpharb stability criterion, taking into account equation (16) representing the transfer function of the total robust compensated linear section prototype.

As a precondition, both loci \( G_{DT1} \) and \( G_{DT2} \) at the two different gains \( (K = 4, \text{ and } K = 10) \), correspond to stable linear prototype sections, seen from Figure 8. In this case even a larger gain of \( K = 10 \) is suggested to emphasise the achieved robustness of the system.

The function \( Z(M) \) is plotted by applying equation (2) and the data for different amplitudes \( M \) are shown in Table 1.
Both $G_{dT}$ loci and $Z(M) = -I/N(M)$ loci are plotted in the complex plane following the code:

```matlab
>> GT1=tf([0 2],[4.5 4.5 2])
Transfer function:
2
-------------
4.5 s^2 + 4.5 s + 2
>> GdT1 = c2d(GT1,0.1,'tustin')
Transfer function:
0.001057 z^2 + 0.002114 z + 0.001057
-------------
z^2 - 1.901 z + 0.9049
Sampling time: 0.1
>> GT2=tf([0 5],[10.5 10.5 5])
Transfer function:
5
-------------
10.5 s^2 + 10.5 s + 5
>> GdT2 = c2d(GT2,0.1,'tustin')
Transfer function:
0.001133 z^2 + 0.002265 z + 0.001133
-------------
z^2 - 1.9 z + 0.9049
Sampling time: 0.1
>> nyquist(GdT1,GdT2)
```

Figure 8. Describing Function Analysis after Robust Compensation at Different Linear Section Gains $K = 4$ and $K = 10$

As already discussed, limit cycles should be completely avoided in systems with a backlash type of nonlinearity. As seen from Figure 8, this main objective is achieved after the application of the robust compensation. The locus $Z(M) = -I/N(M)$ became quite apart from the two loci $G_{dT1}$ and $G_{dT2}$, in this way preventing any undesirable limit cycles to occur during the system operation.

Further, from the comparison between Figure 5 and Figure 8, it is seen that the linear prototype section of the system becomes quite insensitive to uncertainties. After robust compensation, the two loci $G_{dT1}$ and $G_{dT2}$, almost coincide.

Although, the system is examined only for variation of the linear section prototype gain $K$, very similar results are achieved in case of a time-constant $T$ variation [9], [15], [16].

Experiments prove that for the case of a system with a backlash type of nonlinearity, the parameters of the nonlinearity (the slope $K_1$ and the dead zone $D$) may change insignificantly and are no challenge for the system performance. Hence, there is no need for assessment of the system sensitivity in terms of variation of these parameters.

5. Design of a Digital Robust controller Based on Microcontrollers

The design of the series and the forward digital controller stages founded on their analogue prototypes is realized again by applying the Tustin bilinear transform.

Taking into account equation (12), the transfer function of the modified series robust controller stage can be presented as:

$$G_{So}(s) = \left( s + 0.5 + j0.5 \right) \left( s + 0.5 - j0.5 \right) = 0.5 s (s + 1000)^2 = \frac{0.5 s}{s^2 + s + 0.5} \frac{s^2 + s + 0.5}{0.5 s^2 + 1000 s + 500000 s} \quad (17)$$

Two additional remote poles are added, in case of a requirement of the physical realisation of the stage. These poles will have negligible effect of the final system’s performance.

The digital equivalent of the series robust controller stage is realized by the code:

```matlab
>> Gs0=tf([1 1 0.5],[0.5 1000 500000])
Transfer function:
s^2 + s + 0.5
-------------
0.5 s^2 + 1000 s + 500000
>> Gs0d = c2d(Gs0,0.1,'tustin')
Transfer function:
0.0008083 z^2 - 0.001536 z + 0.0007314
-------------
z^2 - 1.922 z + 0.9231
Sampling time: 0.1
```

Following the code result, the transfer function of the series robust controller stage in the discrete-time domain is as follows:

$$G_{Sod}(z) = \frac{0.0008083 z^2 - 0.001536 z + 0.0007314}{z^2 + 1.922 z + 0.9231} \quad (18)$$
To enable the implementation of a microcontroller, the transfer function of the series digital robust control stage is represented by the following difference equation [16], [17]:

\[ y(kT) = 0.0008083x(kT) - 0.00153x[(k - 1)T] + \\
+ 0.0007314x[(k - 2)T] + \\
-1.922y[(k - 1)T] - 0.9231y[(k - 2)T] \]  \hspace{1cm} (19)

From equation (15), the modified transfer function of the forward robust stage can be presented as:

\[ G_{f0}(s) = \frac{0.05s + 0.5}{s^2 + s + 0.5} \]  \hspace{1cm} (20)

Here, an additional zero is introduced, to improve the speed and rapid performance of the system.

The digital equivalent of the forward robust controller stage is realized as follows:

\[
\begin{align*}
\text{>> Gf0=tf([0.05 0.5],[1 1 0,5])} \\
\text{Transfer function:} \\
0.05 s + 0.5 \\
\text{-------------} \\
\text{s^3 + s^2 + 5} \\
\text{>> Gf0d = c2d(Gf0,0.1,'tustin')} \\
\text{Transfer function:} \\
0.0001785 z^3 + 0.0002974 z^2 + 5.949e-005 z - 5.949e-005 \\
\text{-------------} \\
z^3 - 2.901 z^2 + 2.81 z - 0.9036 \\
\text{Sampling time: 0.1}
\end{align*}
\]

From the code the transfer function of the forward robust control stage, in the discrete-time domain is:

\[ \frac{G_{f0}(z)}{E(z)} = \frac{1.785 \times 10^{-4} z^3 + 2.974 \times 10^{-4} z^2 + 5.949 \times 10^{-5} z - 5.949 \times 10^{-5}}{z^3 - 2.901 z^2 + 2.81 z - 0.9036} \]  \hspace{1cm} (21)

To facilitate the implementation of a forward microcontroller, the transfer function of the forward robust control stage is expressed, by the following difference equation [16], [17]:

\[ m(kT) = 2.901m[(k - 1)T] - 2.81m[(k - 2)T] + \\
+ 0.9036m[(k - 3)T] + \\
+ 1.785 \times 10^{-4} e(kT) + 2.974 \times 10^{-4} e[(k - 1)T] + \\
+ 5.949 \times 10^{-5} e[(k - 2)T] - \\
- 5.949 \times 10^{-5} e[(k - 3)T] \]  \hspace{1cm} (22)

The design of two robust control stages is upgraded to digital filters based on microcontrollers, as shown in the block diagram of Figure 9.

The two microcontrollers are usually combined in a single chip of a multiple-input multiple-output MIMO microcontroller. Some of the latest produced microcontrollers can offer up to 200 MHz operating frequency. The discussed control system operates at sampling frequency easily managed by such a microcontroller.

6. Conclusion

Contribution of this research is the application of the analysis and design of a digital robust control for a nonlinear servo control system. Considering the Euler's approximation, the results of the research reveal that the analysis can be applied to the continuous plant of the system and further converted into the discrete time-domain. The stability of the linear section of the system is an important precondition for further application of the Describing Function analysis.

The system is assessed in terms of the interaction between the linear and the nonlinear sections with the aid of the Goldfarb stability criterion, based on the Describing Function analysis. The outcome proves that after implementation of the robust controller, regardless of the variation of the linear prototype section gain, the undesirable limit cycles can be completely avoided. A major advantage of the Describing Function analysis is the graphical display and the simplicity of its application for systems with complex dynamics in their linear parts.

An optimal robust controller is achieved by applying forward-series compensation with two degrees of freedom. The controller enforces the desired system performance. Analysis in the discrete-time domain before and after the application of the robust controller proves that the system becomes quite insensitive to its parameter variations.

To achieve digital robust control, the transfer functions of the two controller stages are presented by their difference equations. Microcontrollers are incorporated into the control system as forward and series robust control stages and can be programmed to solve their related difference equations. This strategy is the preferable due to the advantages of digital systems.

Even if the Describing Function analysis is considered as an approximate method, the accuracy of the analysis is very practical and useful. The Describing Function analysis is even more accurate for higher-order systems since they have better low-pass filter characteristics.
References


