A MULTI-LOOP CONTROL STRATEGY FOR ENHANCED NANOPOSIONING

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ABSTRACT
Vibration problems are inherent in most precision positioning systems. These systems are lightly damped and highly susceptible to mechanical resonance and nonlinearity, such as hysteresis and creep. Traditional approaches use a combination of damping and tracking controllers to deal with nonlinearity and resonance respectively. The damping controller utilised in this work to damp the mechanical resonance of the nanopositioning platform is the Integral Resonant Controller (IRC). This common approach is based on sole use of the integral controller (I) or proportional integral (PI) as a tracking controller to treat nonlinearity. This paper employs a multi-loop feedback scheme as a second-order integral tracking controller in conjunction with the damping controller (IRC). The proposed multi-loop feedback scheme delivers enhanced tracking performance with better compensation for hysteresis in comparison with single-loop tracking.

KEY WORDS
Nanopositioner stage, nonlinearity, resonance, feedback control.

1 Introduction
Nanopositioning systems are limited in their performance due to the presence of lightly damped mechanical resonance. The existence of nonlinearity, such as creep and hysteresis, is also a major obstacle. Hysteresis is an unwanted effect; when it is present, the performance of the control system is reduced. Therefore, to ensure better performance, it is necessary to compensate for hysteresis in nanopositioning applications, which further complicates the design [1]. The use of feedback in a single-loop scheme has commonly been the approach used to compensate for hysteresis. However, in the last decade it has been recognised that single-loop feedback has certain limitations in compensating for hysteresis. Single-loop feedback schemes are less sensitive to nonlinearity such as hysteresis; this can be related to the fact that single-loop feedback makes small corrections in order to track perfectly [2]. More robust dynamic performance can be achieved by using multiple feedback loop schemes as opposed to traditional single-loop schemes. Hysteresis can be problematic when there is a need to scan in a wider area as this requires high amplitude of the driven signal. Hysteresis can also obstruct the speed at which high frequency of the driven signal is used [3] [4][5]. The main challenge in controlling the piezoelectric nanopositioning stage is dealing with the non-smooth hysteresis [6]; the use of a multi-loop feedback scheme can generally cope with this.

The traditional approach to this problem uses the proportional integral (PI) or integral (I) as a tracking controller to treat nonlinearity such as creep and hysteresis, and damping controllers to treat the mechanical resonance of the platform [7]. This paper employs a second-order tracking controller in a multi-loop feedback scheme in conjunction with a damping controller. The proposed damping controller is the Integral Resonant Controller (IRC) [8].

The paper is organised as follows: section II describes the background theory and the hysteresis nonlinearity. The multi-loop feedback scheme and the inspiration for it are introduced in section III; results are also provided in this section to validate the enhanced performance of the scheme. Section IV proves the effectiveness and robustness of the proposed scheme and section V concludes the paper.

2 Background Theory
In this section, the open-loop transfer-function based model for one axis of the nanopositioner will be presented, along with a model of the measured hysteresis nonlinearity.

2.1 Nanopositioning Stage Model
In a nanopositioning system, the transfer function of the plant is identified using frequency response analysis. The transfer function is represented by a second-order model [3], as in the equation below:

\[ G(s) = \frac{\sigma^2}{s^2 + 2\zeta \omega_p s + \omega_p^2} \]

(1)

where \( \zeta \) is the damping ratio, \( \omega_p \) is the natural frequency and \( \sigma^2 \) is selected to manipulate the DC gain of the platform. This representation is only valid if the first dominant mode is considered. In the event that more than one mode
is considered, the overall transfer function for the piezoelectric platform can be represented by the summation of several second-order systems for each mode. For reasons of practicality, other modes can be truncated. Experimental results have been observed and the transfer function of the platform has been identified based solely on the first dominant mode, given by:

\[
\frac{4.746 \times 10^8}{s^2 + 910.1s + 2.927 \times 10^8}
\]  

(2)

It is worth noting that the mechanical resonance of the platform has been observed at 2700 Hz for the proposed fast nanopositioning platform. In order to adequately capture the dynamics of the system, a low pass filter with a 5 kHz cut-off frequency has been connected in series with the plant and the transfer function, as shown below:

\[
\frac{9.87 \times 10^8}{s^2 + 6.283s + 9.87 \times 10^8}
\]  

(3)

The addition of the filter has promoted the plant to fourth-order because to the two poles introduced by the filter. Figure 1 illustrates the complete open-loop model of one axis of the nanopositioner, which comprises the second-order plant model, the second-order low bass filter, and nonlinearities.

![Figure 1. Shows the overall modelling of the platform in the open-loop.](image)

### 2.2 Nonlinearities

Measurements have shown that at the higher scan rates typically used in nanopositioning applications, creep is a slow nonlinearity and therefore it can be neglected. Thus, only the hysteresis nonlinearity has been considered and modelled. In this work, the hysteresis has been modelled using the Bouc-Wen model [3]. The nanopositioning platform can be represented as a mass-spring damper and the relationship between the applied voltage and the displacement is nonlinear. The equation of motion for the piezoelectric platform can be described using the Bouc-Wen with nonlinear differential equations, as in the equations below [9][10]:

\[
\begin{align*}
mx + bx + kx &= k(du - h) \\
h &= \alpha du - \beta |a| h - \gamma u |h|
\end{align*}
\]

(4)

where \( h \) represents the non-linear relation between the lag force and the displacement. The parameters \( \alpha, \beta \) and \( \gamma \) have been identified to represent the hysteresis loop’s magnitude and shape. The applied voltage can be denoted \( u \), and \( x \) the displacement of the piezoelectric actuator; \( m, b, k \) and \( d \) represent the effective mass, damping coefficient, mechanical stiffness and effective piezoelectric coefficients respectively. The Bouc-Wen model has been selected as opposed to other models, due to its simplicity and ability to capture the major hysteresis cycles. The hysteresis has been realised in MATLAB using a nonlinear differential equation representation of the hysteresis cycles. The hysteresis has been realised in MATLAB using a nonlinear differential equation representation of the hysteresis cycles. The hysteresis has been realised in MATLAB using a nonlinear differential equation representation of the hysteresis cycles. The hysteresis has been realised in MATLAB using a nonlinear differential equation representation of the hysteresis cycles. The hysteresis has been realised in MATLAB using a nonlinear differential equation representation of the hysteresis cycles.

![Figure 2. a- Shows the integration of the Bouc-Wen model with the nanopositioning platform. b- The Bouc-Wen.](image)

Referring to figure 3, it is possible to compare the hysteresis generated from the experimental results and the modelled hysteresis. The parameters of the Bouc-Wen model have been tuned in such away as to mimic the hysteresis generated from the experimental results from the nanopositioning platform. The open-loop investigation of the of hysteresis modelling is illustrated in figure 3; a nonlinear relationship has been found between the control voltages applied to the piezoelectric and its generated displacements. The displacements generated are different for the forward and backward paths under the same voltage and the induced nonlinearity hysteresis noted displays 5 micrometres of lag in the displacement, as shown in figure 3. The
modelling of hysteresis in this paper is associated with the x-axis of the nanopositioning platform.

Figure 3. Measured and modelled hysteresis loops show that the hysteresis model accurately captures actual hysteresis.

3 Control Strategy and Motivation for the Multi-Loop Design

Controllability of the nanopositioning platform is challenging and difficult due to various control problems associated with the platform; hence a combination of different controllers is required. The initial approach for controlling the platform is to use a suitable damping controller for resonance, along with a well-designed tracking controller in a single-loop feedback system. In nanopositioning, the common tracking controllers are generally first-order and either proportional-integral (PI) or integral (I).

The control object in nanopositioning applications is there to keep the tracking error to a minimum. It is essential that applications requiring high linearity have feedback control, long-term position stability, repeatability and accuracy. Although the single-loop feedback control technique can improve the accuracy and dynamic response of piezoelectric actuators, feedback control law is limited in compensating for hysteresis. The first-order tracking controller does not track the input signal perfectly in the presence of hysteresis, which results in a distortion in the image in the raster scan. Hence, the first-order tracking controller does not compensate for hysteresis. A solution has been proposed to this whereby the order of the tracking controller is increased to a second-order controller. An attempt has also been made to employ the second-order controller in the single-loop feedback scheme with the transfer function given by:

\[ C_{track}(s) = \frac{k_t}{s^2} \]  

(5)

However, the root locus investigation has shown that the system has become unstable due to the locus on the right-hand side of the s-plane. Further, the bode plot analysis has shown a negative phase and gain margin for the proposed system. In order to overcome this problem, the multi-loop feedback scheme is proposed in this paper. The multi-loop feedback scheme is shown in figure 4.

![Multi-loop feedback scheme](image)

In figure 4, it can be seen that there are two controllers \( C_{track} \): the outer-loop feedback uses a first-order integral tracking with a transfer function of \( \frac{K_{T2}(s)}{s} \), and the inner-loop feedback also uses first-order integral tracking with a transfer function of \( \frac{K_{T1}(s)}{s} \). The damped system using the IRC controller has the following transfer function:

\[ G_{damped} = \frac{K_d * G}{1 - K_d * (G + d)} \]  

(6)

Multi-loop stability has been examined for the proposed plant by using the Routh-Hurwitz stability criterion. The characteristics equation of the multi-loop feedback scheme has been analysed in order to determine the range of \( K_{T1} \), \( K_{T2} \), \( k_d \) and \( d \) required for closed-loop stability.

The derivation of the overall transfer function for the multi-loop scheme is given by:

\[ \frac{Y(s)}{R(s)} = \frac{K_{T2} K_{T1} k_d G}{K_{T2} K_{T1} k_d G + 1 - dk_d - Gk_d + K_{T1} k_d G} \]  

(7)

Referring to equation 1, the transfer function of the second-order system can be minimised as follows:

\[ G(s) = \frac{\omega_p^2}{s^2 + \omega_p^2} \]  

(8)

This minimisation is based on the assumption that the DC gain of the system is equal to 1, which means that the value of \( \sigma^2 \) is equal to \( \omega_p^2 \). Further simplification can be explained: the damping ratio for the nanopositioning platform is very low and can be neglected for the purpose of mathematical analysis. It is worth noting that for the purpose of mathematical analysis, neither the filter nor hysteresis have been taken into consideration. The feed-through term can be determined mathematically as a negative value in the equation below [11]:

\[ d = \frac{-2 \sigma^2}{\omega_p^2} \]  

(9)
In this analysis, for the purpose of simplicity, the value of the feed-through term will be equal to -2, hence \( \sigma^2 \) is equal to \( \omega_p^2 \). The transfer functions for \( K_T(s) \), \( K_T(s) \) and \( K_d(s) \) are listed in equation (10):

\[
\begin{align*}
K_T(s) &= \frac{K_T}{s} \\
K_T(s) &= \frac{K_T}{s} \\
K_d(s) &= \frac{K_d}{s}
\end{align*}
\]

By substantiating all the above quantities in equation (7), the characteristic equation of the proposed feedback scheme can be determined and is given by:

\[
s^5 - dK_d s^4 + \omega_p^2 s^3 - (dK_d W_p^2 + K_d \sigma^2) s^2 +
K_T K_d \sigma^2 s + K_T K_T K_d \sigma^2 = 0
\]

(11)

Based on this assumption, equation (11) can be rewritten as follows:

\[
s^5 + 2K_d s^4 + \omega_p^2 s^3 + K_D W_p^2 s^2 + K_T K_d \sigma^2 s +
K_T K_T K_d \sigma^2 = 0
\]

(12)

It is important to note that the values of \( K_T \), \( K_T \) and \( K_d \) are positive. Due to the fact that the coefficients of \( s \) are positive, it can be suggested that the system is stable, however, there is a need to use the Routh-Hurwitz stability criterion to be absolutely sure of this. The value of the maximum damping gain \( K_d \) in the multi-loop feedback scheme has been determined and must obey the following quantity:

\[
K_d < \frac{\omega_p^2 + 2K_T K_T}{4K_T}
\]

(13)

Referring to equation (13), it has been found that decreasing the damping gain from its maximum value can achieve better tracking performance. Further, for a stable system in the multi-loop scheme, the values of the tracking gains must respond to the following quantities:

\[
K_T < \frac{2K_d}{3}
\]

(14)

\[
K_T < \frac{\omega_p^2}{4K_d + 2K_T}
\]

(15)

The characteristics equation can be characterised in terms of \( \sigma^2 \) and \( \omega_p^2 \) as shown in equation (16); this is because in this design, the DC gain is not 1.

\[
s^5 - dK_d s^4 + \omega_p^2 s^3 - (dK_d \omega_p^2 + K_d \sigma^2) s^2 +
K_T K_d \sigma^2 s + K_T K_T K_d \sigma^2 = 0
\]

(16)

In this work, the IRC damping controller has been used to damp the mechanical resonance of the nanopositioning platform. The value of \( K_d \), which has not been considered as the maximum damping, and is equal to 3000; the value of the feed-through term \( d \) is equal to -3.2426. This variation, as opposed to the previous value of -2 in the analysis, can be related to the fact that the DC gain is no longer equal to 1. The values of \( K_T \) and \( K_T \) have been selected as 8500 and 1000 respectively. The selection of the tracking gain has taken into consideration various criteria, such as simulation error (maximum linearity) and robust stability. It has been noted that reducing the damping gain from its maximum value results in better tracking performance, particularly at high frequencies. The value of the tracking gain in the outer-loop (\( K_T \)) is highly dependent on the damping gain \( K_d \) as can be seen in formula (14). Vibration problems can occur if there is improper selection of the \( K_T \) without reducing the damping gain. On the other hand, increasing the tracking gain (\( K_T \)) in the inner-loop results in decreasing the maximum error without causing any vibration. The Routh-Hurwitz table is shown below to highlight the fact that the proposed section of the gains is within the boundaries of stability. As can be seen from table 1, there are no right-hand side poles because there is no sign of change in the first column. The roots of the polynomial coefficients are given by:

\[
\begin{align*}
\omega_p^2 &\quad \omega_p^2 \\
3.162 &\quad 4.4390 \times 10^{10} \\
2.5618 &\quad 2.8890 \times 10^{20}
\end{align*}
\]

Table 1. Routh-Hurwitz Table

In order to test the performance of the proposed method, a 20 Hz triangle input with a 50 micrometre travel range has been applied to the plant, as is evident from figure 5(a). The open-loop and closed-loop tracking performances of a triangle wave have been plotted, as shown in figures 5(a), (b) and (d). The open-loop tracking performance for the triangle wave has deviated from a straight line as a result of hysteresis, as is depicted in figure 5(b). Excitation of the high-frequency components of the resonant mode has occurred as a vibration problem at the turnaround, as is clear in figure 5(c).
Figure 5. Shows the tracking performance of the platform for a triangle signal: a-The reference signal 20 (Hz). b-Hysteresis effect in terms of deviation from the straight line. c-Vibration distortion due to excitation of the resonant mode in open-loop. d-The time domain tracking in closed-loop.

The tracking error for the open-loop and closed-loop is plotted in figure 6.

Figure 6. Shows the tracking error of the platform for a triangle signal in the open-loop and closed-loop.

The proposed control strategy has shown improved tracking performance and there is a significant increase in the linear part associated with perfect tracking (as compared with the open-loop), as is clear from figure 6. The appearance of the linear part in the error plot suggests the multi-loop feedback scheme has provided noticeable compensation for hysteresis. Although error has been noted at the high frequency component of the turnaround, the turnaround itself is not taken into consideration in the raster scan, which avoids this area. The appearance of error at the turnaround can be related to the fact that the roots of the plant are coming closer to the imaginary axis.

Following examination of the system’s performance, a single-loop feedback scheme has been employed for fairness of comparison. The comparison has shown no linearity in the error plot in the case of a single-loop feedback controller, even for low frequencies. It can be concluded that the multi-loop feedback controller performs better and remains linear even with the existence of hysteresis, which makes the use of this scheme in nanopositioning applications preferred.

The raster scan is presented in figure 7 to verify the robustness of the multi-loop feedback scheme. The lateral movement has been generated by applying a triangle wave in the x-axis and ramp (staircase) in the y-direction. The vibration problem has been diminished in the closed-loop and an accurate image is generated.

Figure 7. Shows the raster scan of the nanopositioning platform.

Further analysis to test the robustness of the proposed controller has been undertaken, as will be clear in the following sections.

4 Controller Performance

Robustness refers to the capacity of the closed-loop system to be unresponsive to parameter variation and to attenuate external disturbances. Therefore, robustness against resonant frequency changes and disturbance rejection is discussed in the following part. Although robust stability has been obtained in the previous section, it is important to confer robustness analysis in nanopositioning applications.

4.1 Robustness to Resonant Frequency Variation

Nanopositioning platforms are highly susceptible to parameter variations such as resonant frequency [12][13]. A 5%, 10% and 15% reduction in the resonant frequency has been considered and the open-loop and closed-loop frequency response has been plotted for all of these, as is shown in figure 8.
There has been no evidence that a change of 15% influences the tracking error with a proper selection of the tracking gains. Improper selection of the tracking gains may not provide high quality resonant change rejection, whilst reducing the damping gain from its maximum value has no impact on the rejection of resonant changes.

Disturbance signal is an unusual and undesirable signal; it is present in order to quantify its effect. Attenuating different disturbances in order to obtain high precision nanopositioning systems is an important control issue. Neglecting the disturbance can also be detrimental to maintaining ultra-high precision mechatronics systems [14]. The closed-loop system in this paper has only one output signal and one input signal. However, in the event of a disturbance, there will be two input signals (D(s) as disturbance and R(s) as input signal) and the transfer function can be determined by taking into account both inputs. It is assumed that the D(s) is the input signal and R(s)=0 and Y(s) comprise the output signal: the transfer function can be determined as in equation 18.

\[
Y(s) = \frac{G(1 - K_d d)}{1 - K_d d + GK_d + GK_T1K_d + GK_T2K_T2K_d} \tag{18}
\]

With reference to the disturbance, the bode plot of the transfer function for the output signal has been plotted, as in figure 9. Figure 9 reveals that changing the resonant frequency does not have a significant influence on the disturbance rejection, showing that the proposed technique has offered reasonable disturbance rejection. At low frequencies the multi-loop has provided high disturbance rejection up to 1000 Hz, however, at the resonant frequency the controller does not exhibit significant disturbance rejection. The final value theorem has been applied and the steady state error goes to zero for the step input.

4.2 Disturbance Rejection Profile

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5 Conclusion

The main contribution of this paper is to develop a multi-loop feedback scheme that can compensate for nonlinear hysteresis at the precision positioning stage. The proposed multi-loop feedback scheme has overcome the inherently harmful nonlinearity hysteresis and enhanced the tracking performance of the nanopositioning piezoelectric actuator. The new control strategy has also reduced the effect of the mechanical resonance variation. In order to manage the output in a way that is closer to the desired pattern, the paper has introduced an innovative loop to be used as a new input. According to the simulation results, the developed and verified scheme provides better compensation for hysteresis; the results have shown satisfactory performance in terms of the induced error generated.

References


