ABSTRACT
The main purpose of this paper is to present the natural frequencies and mode shapes of a small unmanned aerial vehicle (sUAV), designed and test-flown at the Florida Institute of Technology obtained via experimental modal analysis (EMA). The EMA was performed in order to understand the UAV's modal characteristics as a first step aimed towards understanding the aeroelastic characteristics of the aircraft. The Variable Aspect Ratio (VAR) unmanned aerial vehicle (UAV) studied employs a telescoping wing which, by adaptively-changing its span can evolve optimally in all flight regimes. Data acquisition and analysis were performed with ModalVIEW. The modal tests employed hammer excitation and accelerometer response measurements. In particular, the study focused on the difference between modal characteristics of two different VAR wing configurations: fully retracted vs. fully extended wing.

KEY WORDS
Experimental Modal Analysis, UAV

1. Introduction
Unmanned aerial vehicles (UAV) or systems (UAS) are used in many civilian and military applications [1]. Often, these UAS are required to perform missions that require short-field take-off and landing, rapid cruise airspeed and maneuvering at fast airspeeds. These performance characteristics often are best performed with wing configurations that have different aspect ratios [1].

The aspect ratio ($AR$) [1], one of the most important parameters in aircraft design and is a function of the wing span $b$ and wing area $S$ (Equation 1). A change in the aspect ratio leads to a change in the performance of the aircraft (i.e. lift). A high aspect ratio aspect ratio would allow an aircraft to perform better in low airspeed conditions, usually during landing or takeoff, while a high aspect ratio is desirable during cruise. One such configuration, the telescoping wing, has been adopted by Raytheon Missile Systems on cruise missiles.

$$AR = \frac{b^2}{S}$$ (1)

Morphing was defined as the ability of a structure to change shape and wing morphing was identified as a method to allow optimal performance in various flight conditions without the need of employing complicated secondary control surfaces such as slats or flaps [2]. The Wright Brothers employed morphing wings as early as 1903 with their Wright Flyer. While most 20th century aircraft were built with fixed wing configurations, the current decade has brought new research into the concept of an active, morphing, aeroelastic wing. The Boeing X-53, for example, employed an Active Aeroelastic Wing (AAW) [3] whose aeroelastic flexibility allowed the optimization of aircraft performance in all flight regimes. The principle behind the AAW was to build a wing that could be used itself as a control surface. Several morphing wing designs [4] are currently proposed and implemented; perhaps the simplest one is the telescoping wing concept.

The telescoping wing is capable of modifying its span during flight and thus increase its aspect ratio. The extension direction is perpendicular to the chord and the telescoping wing is composed of a fixed center section and two extendable outer sections, one for the port and one for the starboard wings. An overlapping spar system is responsible for extending the wing.

A morphing wing aircraft was designed, manufactured and successfully flight tested at the Florida Institute of Technology (Figure 1). The mission of the aircraft was to perform low level photography missions, as well as to be a technology demonstrator for the telescoping
wing concept and an educational tool for the senior design class. The plane is shown in Flight in Figure 2.

Figure 1 VAR UAV and Data Acquisition System

Figure 2 VAR UAV in Flight

One research goal was to investigate the aeroelastic [5] characteristics of the wing (i.e. flutter speed), which could not be obtained without a preliminary modal analysis needed to identify the torsional and bending modes of the wing and horizontal stabilizer. Modal characteristics of wings are obtained using experimental modal analysis (EMA) [6]. It was thus needed to find the modal frequencies for the fully extended wing configuration first, as it would be expected that this configuration would be more prone to flutter. Two studies were conducted: aircraft on ground and aircraft suspended (in flight). The study presented in the current document investigated and compared the modal characteristics of the VAR UAV with extended wing in the two situations.

2. Experimental Modal Analysis

The methods and procedures used to conduct the series of modal analyses are presented in this section [6].

2.1 Brief Overview of Experimental Modal Analysis

A mode shape or a normal mode of a vibrating system is a pattern of motion in which all parts of the system move sinusoidally with the same frequency (natural frequency associated with the mode) and with a fixed phase relation. An object has, in theory, an infinite number of mode shapes and natural frequencies. If a structure is excited by a periodic force whose frequency matches any of the natural frequencies of the structure to which it is applied, a resonance condition occurs. At resonance, the structure is able to admit an unlimited amount of mechanical energy, which is why (in the absence of damping), the structure undergoes increasingly larger deformations and ultimately failure.

2.2 Experimental Modal Analysis Software

ModalVIEW™ software was developed with NI LabVIEW™, an open environment designed to make interfacing with any measurement hardware fast and simple. Developed by ABSignal, ModalVIEW was designed to acquire multichannel sound or vibration signals from a running machine or to obtain the static or dynamic loading of a structure by using data acquisition hardware. ModalVIEW allows the user to perform an experimental modal analysis. After acquiring a set of acceleration, velocity or displacement time histories of certain points on a structure generated by a time-varying excitation signal (force, acceleration or velocity, etc.), and the software can animate the response of a structure and show the structure's vibration at the measurement points. It also helps extract and visualize useful modal parameter information from acquired time-and frequency-domain experimental data. This software uses the Fast Fourier Transform (FFT) algorithm [7] which computes a sampled version of the frequency spectrum from sampled time signal. The discretized and finite-length spectrum is called Discrete Fourier Transform (DFT). The software produces the frequency response functions from the spectra of the response and excitation produced at each point on the structure under test. The software then performs a curve-fitting algorithm to extract the natural frequencies, damping ratios and mode shapes from the FRF. Theoretically, the FRF describes the input/output relationship between two points on a structure as in the frequency domain. In other words, the FRF is a measure of how much displacement, velocity, or acceleration response a structure has at an output point, per unit of excitation force at an input point. The Frequency Response Function (FRF) $H(j\omega)$ is generally computed by

$$H(j\omega) = \frac{X(j\omega)}{F(j\omega)}$$

where $X$ is the autopower spectrum of the response (in this case displacement) and $F$ is the autopower spectrum of the excitation, with $\omega$ being the circular frequency and $j$ the imaginary number. The spectra $X$ and $F$ are a result of the Fourier transform applied to the dynamic equations of
motion of the structure, which performs a transition of the equation from the time domain ($t$ is time) to the frequency domain. The equations of motion are:

$$M\ddot{x}(t) + [K]x(t) + [C]\dot{x}(t) = F(t)$$  \hspace{1cm} (3)

where $[M]$, $[K]$, $[C]$ are the mass, stiffness and damping matrices, respectively; $\{\ddot{x}\}$, $\{\dot{x}\}$, $\{x\}$ are the time-dependent, acceleration, velocity and displacement, respectively; $\{F\}$ is the force vector.

2.3 Methods

The methods used to perform the EMA was the roving response-fixed impact method. The response was measured by triaxial accelerometers, PCB Piezotronics Model Y356A16 with 100 mV/g nominal sensitivity for each of the three axes (1g=9.8 m/s²). The excitation was provided by an instrumented (force gauge) PCB Piezotronics 080C01 impact hammer with a nominal 12.3 mV/N sensitivity (Figure 3).

Impact testing is relatively fast, convenient and low cost; in this experiment one single driving impact point was chosen. This particular point was situated be situated at a location that would allow excitation of the main modes of interest, which must be away from a vibrational node. One wing, in its most simple approximation, could be modeled as a cantilevered beam. The highest vibration displacement on a single degree of freedom, cantilevered beam is located at the end of the beam: the wingtip would have the highest vibrational displacement as the wing is similar to a cantilevered beam in a first approximation. In the modal analysis experiment the middle of the wingtip had been chosen as the impact point for each measurement. Accelerometers situated at various points on the structure recorded the accelerations caused by the hammer impact. FRFs are computed one at a time after each impact. For each measured point, seven impacts (averages) are performed for accuracy and good coherence. Modal parameters are defined by curve fitting the resulting set of FRFs. The impact hammer was connected to the first channel in the NI DAQ 3174 data acquisition card. The accelerometers (two were used simultaneously) were connected to the rest of the available channels. In total, 7 data acquisition channels were used.

2.3.1 Base Model

From a testing point of view, a real structure can be sampled spatially at as many DOFs as needed. For practical reasons, the UAV has been represented with a finite number points; at each point the accelerations along three DOFs (X, Y and Z) were made. The accelerometer measurements were made at each one of the DOF. The finite set of DOFs is called the base model. The locations of the DOF were measured and input into the software. The retracted and the extended wing configuration were represented via a separate base model. The base model was used to visualize the mode shapes of interest and a result had to have the correct spatial resolution. The model consisted initially of 36 points (suspended configuration) and then was modified to include 72 measurement points (Figure 4) for the on-ground configuration due to spatial resolution needs dictated by the “harder” boundary condition and ensuing coupling.

2.3.2 Measurements

The measurement sessions were performed for the two configurations: aircraft resting on the ground and aircraft suspended by the wings [8]; the same acquisition parameters were used for both. After the hammer impact [9] and response measurement, ModalVIEW automatically generated a FRF for each channel from the response that was measured. The acquisition were performed using the following parameters:

- Sampling rate: 1652 Hz
- Resolution: 1 Hz
- Block size / number of points: 1652

The hammer impact technique needed to impart energy into exciting the modes of interest. The response at each DOF was acquired 7 times (corresponding to 7 impacts) and linearly averaged. Each impact had to be carefully made in order to avoid double-hammering and overloading. The coherence, a function that indicated whether the response of the structure was the result of the hammer impact, was for all FRF’s close to a value of 1 (except at the resonance peaks), which indicated that the measurements were performed correctly and repeatedly and that no extraneous noise influenced the measurements.
The choice of the sampling rates came from the Nyquist-Shannon sampling, which states that the sampling frequency needs to be at least twice the highest frequency of interest. The lower modes usually have the highest participation factors and are usually the ones of interest. Without prior knowledge of the modes of interest, a suitably high frequency sampling rate was chosen. Figure 5 shows the aircraft right before the modal analysis.

3. Results

The results obtained are presented. Figure 6 presents a sample of the FRF obtained. The FRF phase changed from $+180^\circ$ to $-180^\circ$ at each natural frequency. The magnitude FRF plot displayed local peaks at each natural frequency. From the plots, it was evident that careful curve-fitting of the FRF and mode shape extraction had to be made over portions of the entire frequency range. Figure 7 shows a sample FRF plot curve-fitted using the mode shapes found using a stabilization chart. The stabilization chart employed by the code to determine the mode shapes used the Least-Square Complex Function (LSCF) [5] and a model order using 40 terms. The mode was positively identified when the natural frequency, mode shape and the corresponding damping were determined; on the stabilization chart a stable mode was denoted by a green “s” symbol. The natural frequency and mode shape list is shown on the right side of Figure 7.

The mode shapes extracted were checked using the Modal Assurance Criterion (MAC). The Modal Assurance Criterion is a widespread tool for the quantitative comparison of two modal vectors. This statistical indicator is used to measure the differences between the modes shape, in conjunction with a separate frequency analysis. The MAC gives a solid statistical indicator and a degree of consistency between mode shapes. Finally, a frequency comparison enables to determine the correlated mode pairs. The MAC is bounded between 0 and 1: values close to 0 indicate a non-consistent mode and a value around 1 indicates a fully consistent mode shape. A value of 1 generally signifies that the two mode shapes compared are the same, especially if these are found very close together in frequency. While in an analytic modal analysis (i.e. finite element method) is easy to make the distinction between the characteristics of each separate mode, experimentally this task becomes much
harder. Due to imperfect spatial resolution, frequency discretization or imperfect choice of mode extraction algorithm, it is hard to distinguish between the same mode estimated twice and different modes found at close to one another in the frequency domain. That is why it is useful to develop a quantitative method needed to assess the orthogonality of estimated modes. The MAC:

- Assures uniqueness and orthogonality of the experimental modes.
- May be used to compare an experimentally-found mode to an analytically-found one.

The MAC plot seen in Figure 8 was generated by the software. From the plot in Figure 8 it was observed that in general the correlation between two adjacent modes was much less than 1; thus the modes were independent of one another. A summary of the list of modes found is presented in Table 1. Further Figures will depict the actual mode shapes.

The suspended modes of the aircraft should be the most relevant ones to an aeroelastic analysis; unfortunately any suspension mechanism introduces a change in the boundary conditions, no matter how compliant the suspension method may be (see ropes used to suspend the VAR in Figure 1). However, it was interesting to see how the same modes would be affected by the different boundary condition (landing gear on ground). The more shapes corresponding to an aircraft placed on ground would be more difficult to analyze due to the coupling introduced by the landing gear. The advantage of having the aircraft placed on the ground would be that the tail and wings would be free. Figure 12 shows the first bending mode of the wing on the ground. Some tail empennage bending was also involved in the mode shape.

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**Table 1 Mode Shape Comparison for Aircraft On-Ground vs. Suspended with Extended Configuration**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Suspended</th>
<th>ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st bending</td>
<td>10.7 Hz</td>
<td>7.8 Hz</td>
</tr>
<tr>
<td>Antisymmetric/torsion</td>
<td>8.5 Hz</td>
<td>13.8 Hz</td>
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</tbody>
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Figure 10 shows a mode shape at 10.7 Hz which corresponds to a symmetric flapping of wing extensions combined with tail bending (arrows indicate deflection from initial position). An antisymmetric bending coupled with stabilizer torsion was observed at 8.5 Hz.
Figure 13 shows the torsion/antisymmetric wing bending at 13.8 Hz.

Figure 13 First Torsion around Fuselage of Wing and Empennage at 13.8 Hz on Ground

While the first bending (in phase bending of the wing extensions) combined with empennage bending in the vertical plane was actually higher in the suspended condition than on the ground condition, one probable reason was that the wing was suspended at the edge of the wing extensions. The antisymmetric tail and wing torsion mode was higher in frequency when the aircraft was on the ground, which would be expected since the landing gear provides additional stiffness.

At 27 Hz, Figure 14, there was an additional mode shape corresponding to wing extension torsion around the lateral (pitch) axis. This mode would be very important for aeroelastic investigations along with the bending modes.

Figure 14 First Wing Torsion around Lateral Pitch Axis at 27 Hz on Ground

This mode was found in the on-ground configuration, which allowed the wing to be free.

Yet another mode of interest was the torsion of the empennage around the long thin tail. This occurred at 5 Hz and although some motion of the wing extensions was found, the predominant motion in the mode was that of the empennage (Figure 15).

Figure 15 First Torsion around fuselage of empennage at 5 Hz on Ground

4. Conclusion

The results have shown that the natural mode frequencies and mode shapes differ upon the boundary condition: aircraft grounded vs. suspended (mimicking the free flight condition). The modal analysis provided a reference set of modes and aeroelastic investigations would follow to determine flutter speed. More investigations need to be conducted in order to compare similar modes in different boundary condition: for example the aircraft could be suspended by tail and nose. A comparison must be conducted in order to determine as well as possible the modes that may be first excited. Flutter could occur first on the empennage or on the wings extensions due to their less stiff attachments to the wings.

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References

