A NUMERICAL ANALYSIS OF MAGNETO-ACOUSTO ELECTRICAL TOMOGRAPHY WITH A SIMPLIFIED BREAST MODEL

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ABSTRACT
In this study, Magneto-Acousto Electrical Tomography with magnetic measurements technique is investigated using two dimensional (2D) simplified numerical breast model. This technique comprises of the electrical current induction and ultrasound in the static magnetic field. 2D numerical simplified breast geometry is modeled with an ultrasonic transducer (linear phased array (LPA)) placed on the top side of the simplified breast model. A rectangular loop coil is encircled the breast. The sensitivity matrix is obtained for this transducer-coil configuration. LPA transducer is steered with eleven angles (\(-25^\circ\) to \(25^\circ\) at intervals of \(5^\circ\)). The characteristics of this modality is shown by the singular value decomposition (SVD) method. The reconstruction of the images is performed by truncated SVD approach. This study shows that perturbations (5 mm x 5 mm) up to a depth of 3 cm can be detected.

KEY WORDS
Electrical Impedance Tomography, Ultrasound, Hall Effect Imaging, Magneto-Acousto-Electrical Tomography

1 Introduction
Electrical conductivity imaging is an essential area in medical imaging field. Different approaches are focused on imaging the conductivity of the biological bodies. Magneto-Acousto-Electrical Tomography (MAET) [1–11] is one of the latest hybrid imaging approaches on the imaging of the electrical conductivity of bodies. This approach is depended on the electrical impedance tomography and ultrasound. If an ultrasonic pulse is propagated into the conductive body in the static magnetic field, an electric current is induced in the body. The electrical impedance of the object is reconstructed by using two measurement methods. In the first one, the resultant surface voltage is measured through the electrodes and in the second one, the magnetic field intensity occurred from induced currents is measured by a coil configuration.

In [11], the theory on the mathematical basis for Magneto-Acousto Electrical Tomography with magnetic field measurement was considered. The relation between the measurements and the conductivity distribution was defined with reciprocity theorem. For data acquisition a new coil configuration (x-and y-coils) was introduced. The feasibility and performance of this approach were investigated numerically.

In this study, we reviewed this theory with a simplified 2D numerical breast model. A rectangular loop coil is encircled the model for data acquisition. A LPA transducer with 16 element is placed on the top side of the breast model. The sensitivity analysis for loop coil-LPA transducer configuration is done. Twelve identical inhomogeneities are located in the imaging model and the reconstruction of the conductivity distributions are investigated with simulations.

2 Theory
2.1 Forward Problem
The forward problem of this modality consists of two problems, namely, acoustic and electromagnetic (Fig.1). In [11] the relation between the magnetic measurements and acoustic waves was investigated.

This relationship is given as a final formulation [11]:

\[
v_{ab}(t) = \int_{V_{body}} \frac{\partial}{\partial t} (\sigma(\vec{v}(t) \times \vec{B})) \cdot \vec{E}_R^b(\sigma) \ dV \quad (1)
\]

where \(\vec{E}_R^b(\sigma)\) is the reciprocal electric field, \(\vec{v}(t)\) is the particle velocity, \(\vec{B}\) is the magnetic flux density, \(\sigma\) is the conductivity of the tissues, and \(v_{ab}(t)\) is the induced voltage in the detector coil (Fig.2) [11].

2.1.1 Numerical Solution of the Forward Problem
The feasibility of this imaging approach was shown with different numerical simulations in [6, 11]. This multiphysics problem is solved by employing different modules (acoustic, piezoelectric, and electromagnetic) of the COMSOL Multiphysics software. The numerical geometry consists of a LPA transducer with 16-element, a simplified breast model, tumorous tissues and surrounding air. Simplified breast model consists of subcutaneous fat, gland, tumor, muscle and thoracic wall [12] (Fig.3). Since the thoracic wall mainly consisting of muscle, it is also assumed as muscle [13]. The conductivity density and
Figure 1. Geometry of forward problem. The conductive body and transducer are placed in a static field \( \vec{B}_0 \) (z-axis). The transducer generates pressure distribution \( p(t) \) and particle velocity \( \vec{v}(t) \). The electric field \( \vec{E}(t) \) is generated due to the particle velocity \( \vec{v}(t) \) and the magnetic flux density \( \vec{B}(t) \) [11].

Figure 2. The magnetic field measurement approach of MAET problem. The reciprocal current is shown as \( I_R(t) \). A coil encircled the body is used to measure the magnetic field intensity generated due to induced currents [11].

The speed of sound values of the breast tissues are given in Table 1. To excite the transducer a sinusoidal voltage (\( V(t) = 100\sin(2\pi ft) \)) with a period of resonance frequency \( f = 1 \text{ MHz} \) is applied to the surface of a piezoelectric crystal [11].

A rectangular loop coil is used to detect the response of velocity current density induced in the conductive body (Fig. 4) [11]. The pick-up voltages are obtained with the following equation:

\[
v_{ab}(t) = -\int_{\text{coil}} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad (2)
\]

Figure 3. 2D simplified breast geometry. The geometry consists of subcutaneous fat, gland, tumor, muscle and thoracic wall [12].

<table>
<thead>
<tr>
<th>Tissues</th>
<th>Conductivity (S/m)</th>
<th>Density (kg/m³)</th>
<th>Speed of sound (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>0.0257</td>
<td>980</td>
<td>1520</td>
</tr>
<tr>
<td>Gland</td>
<td>0.603</td>
<td>1041</td>
<td>1505</td>
</tr>
<tr>
<td>Muscle</td>
<td>0.503</td>
<td>1090</td>
<td>1588</td>
</tr>
<tr>
<td>Tumor</td>
<td>0.8221</td>
<td>1040</td>
<td>1564</td>
</tr>
</tbody>
</table>

Table 1. Electrical conductivity at 1 MHz [14–16], density and speed of sound values [17] of tissues.

The electric field generated in the detector coil by a reciprocal current is termed as lead field vector. The detector coil (rectangular loop coil) is excited with the change of unit current (\( \frac{\partial I}{\partial t} = 1 \text{ A.s}^{-1} \)) (Fig. 5) to solve the lead field vectors [11].

2.2 Inverse Problem

In this imaging modality the electrical impedance distribution \( \sigma \) is reconstructed. The relation between the voltage in the coil and conductivity distribution is given in (1). The matrix form of this relation was expand on in [11] for a transducer-coil structure:

\[
\Delta V = S \Delta \sigma \quad (3)
\]

here \( S \) is called ad the \( M \times N \) sensitivity matrix.

2.2.1 Image Reconstruction

In this study, two electromagnetic problems are solved to obtain the sensitivity matrix. First, electric fields in the detector coil and second, Lorentz electric fields. The sensitivity matrix is obtained with the dot product of the first fields and time derivative of the second fields [11]. This matrix is calculated for transducer-simplified breast model-rectangular coil configuration. This configuration is placed in a 4 Tesla static magnetic field. The LPA is located on the upper edge of the breast model and steered with eleven angles, \(-25^\circ\) to \(25^\circ\) at intervals of \(5^\circ\). The period of data acquisition is 42.1 \(\mu\)s. To show the performance of
Figure 4. A rectangular coil encircles the conductive body. The reciprocal current is shown as $I_R(t)$ [11].

Figure 5. Electric field distribution due to rectangular loop coil.

Figure 6. 2D simplified breast model and transducer configuration. Twelve identical inhomogeneities are placed in the simplified breast model. A LPA transducer with 16-element is on the top boundary of the breast model. It is assumed that the uniform static magnetic flux density is $B_0 = 4$ T (in z-direction).

Figure 7. The conductivity inhomogeneities are reconstructed using a rectangular loop coil with 80 dB SNR.

this imaging modality, twelve inhomogeneities are located in the simplified breast model (Fig. 6). The SNR in measurements is set to be 80 dB with the Gaussian noise. The reconstructed conductivity distributions are obtained with the truncated singular value decomposition (SVD) method [19].

2.2.2 Resolution matrix analysis

The analysis of the sensitivity matrix with the SVD is performed to investigate the performance of the imaging system [21]. An image sensitivity map of the system can be constructed to analyze the resolution matrix [20, 21]. The resolution matrix is obtained with 80 dB SNR. The number of basis vectors are limited to 3033. The image sensitivity map for rectangular loop coil measurements is shown in Fig. 8.

3 Discussion

The acoustic and electromagnetic problems of MAET are solved simultaneously in COMSOL Multiphysics. Since a rectangular loop coil is used for data acquisition, the inhomogeneities close to the body center are not possible to be reconstructed (Fig. 7). If the center of the coil is shifted to different sides, the inhomogeneities in the center can also be reconstructed. Furthermore, image sensitivity map can be improved by shifting the center of coil and placement of transducer.

4 Conclusion

In this study, a simplified breast model is used for the first time numerically to investigate the feasibility of magnetic field measurement approach of MAET. The forward and inverse problems of this imaging approach are solved numerically. In numerical simulations, simplified breast model consists of fat, gland, and muscle. A LPA transducer with 16-element was placed on the top side of the breast surface. For excitation of the transducer, a single cycle sinusoidal voltage with an amplitude of 100 V (peak) is used. The static field of the medium is assumed as 4 Tesla. The rectangular loop coil is placed encircling the body and transducer. The sensitivity matrix and image sensitivity maps are analyzed. According to this analysis, a conductivity
distribution with 5 mm x 5 mm size up to a depth of 3 cm can be detected.

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References


