MAKING INVESTMENT DECISIONS UNDER UNCERTAINTY

Dulat N. Shukayev, Zhanar Bimurat, Darkhan Abdibekov, Nazgul O. Yergaliyeva
Kazakh National Research Technical University after K.I. Satpayev, 050013, 22a Satpayev street, Almaty, Kazakhstan
dshuk@mail.ru, bimuratzanhar@gmail.com, abdibekov.darkhan@mail.ru, naz_er@bk.ru

ABSTRACT
The investment promotion policies and strategies are currently at the center of attention in Kazakhstan. The government's primary goal is to achieve a favorable investment climate in the country and provide further incentives for an inflow of foreign direct investments into the economy. There is also an important focus on improving the methods of evaluating and selecting investment projects. Many authors have studied the issues of financial and economic evaluation of investment activities. In this article, we examine methods of making investment decisions and develop a specific procedure for selecting investment projects based on decision-making theory. The methods we present make it possible to determine the most effective investment strategy under uncertainty. An algorithm has been developed for selecting the most promising project from a set of projects under uncertainty.

KEY WORDS
Investment activities, investment projects, modeling.

1. Introduction

The global financial and economic crisis that caused a systemic shift in the financial sector of the world economy has exposed the shortcomings of the existing valuation systems used in investment activities. Many authors studied financial and economic evaluation of investment activities. These studies examine various aspects of selecting investment projects taking in account a multitude of different indicators [1-7]. Our analysis of these studies has shown that the focus is mainly on general approaches to organizing investment activities in accordance with the theory of Harry Markowitz, James Tobin and others. However, it is becoming increasingly important to take account of various unpredictable economic, social, environmental and other challenges when formulating an effective decision-making policy for selecting the most promising investment projects.

In this article we develop a concrete procedure for selecting investment projects based on decision-making theory, which has a good track record in organizational, social and other areas of human activity.

2. Methods of making investment decisions under uncertainty

Let there be a set of projects being considered as candidates for investment

\[ A = \{A_i | i = 1,2,...,n\} \]  \hspace{1cm} (1)

characterized by their forecasted rates of return \( b_i, i = 1,2,...,n \). The returns are uncertain due to incomplete or inaccurate information on the possible realizations of relevant factors, such as for example inflation or asset market conditions. Given this uncertainty, experts identify possible realizations \( S = \{ S_j | j=1,2,...,m\} \) of relevant macroeconomic variables or other asset market conditions affecting realized returns of the projects from the given set (1). This information may be represented the form shown in Table 1.

![Table 1](image)

Here, \( b_{ij} = f(A_i,S_j), i = 1,2,...,n, j = 1,2,...,m \) is the rate of return on project \( A_i \) under realized market conditions \( S_j \).

If determining \( b_{ij} = f(A_i,S_j) \) requires knowledge of the quantitative values of each condition \( S_j, j=1,2,...,m, \) the distribution laws for their possible values are usually defined in the form of discrete random variables

\[ S_j = \{S_{j1},S_{j2},...,S_{jL}\}, \hspace{1cm} j = 1,2,...,m \]
Theorem 1 is used in order to determine specific values $S_{jl}$ of each realization of the condition $S_j$, $j=1,2,\ldots,m$; "The value $S_{jl}$ occurs with probability $P_{jl}$ if $u \in \Delta_l$, where $\Delta_l = P_{jl}$ and $u$ is uniformly distributed in the interval $[0, 1]". The proof of this theorem may be found in [10], for example.

We use the following well-known criteria for decision-making under uncertainty in order to determine an effective investment strategy: Wald, Laplace, Hurwitz and Savage [8]. According to the Wald criterion, which in this case may be called the cautious investor criterion, project $A_p$ is selected on the assumption that the market will be in its most unfavorable condition. This is written analytically by the expression

$$b_{p,j}^* = \max_{A_p} \min_{S_j} f(A_p, S_j) \quad (2)$$

which yields the highest rate of return of all its minimum values.

According to the Hurwitz criterion, it is assumed that the market may be in the most unfavorable situation with probability $(1 - \tau)$, and in the most favorable situation with probability $(\tau)$, where $\tau$ is the confidence level. Then the project selection rule will take the form

$$b_{p,j}^* = \max_{A_p} \left[ \tau \cdot \max_{S_j} f(A_p, S_j) + (1 - \tau) \min_{S_j} f(A_p, S_j) \right], \quad \tau \in [0,1]. \quad (3)$$

For $\tau = 0$ from (3) we obtain an expression of the Wald criterion. For $\tau = 1$ we arrive at the strategy for an optimistic investor:

$$b_{p,j}^* = \max_{A_p} \max_{S_j} f(A_p, S_j).$$

The value of the confidence level itself is a random variable and may be described by various distribution laws depending on the type of asset market. Therefore, in order to determine its value, we may use the inverse function method of modeling continuous random variables, the principle of which is stated in the form of theorem 2 [9]: "Random variable $\tau$, realizations of which are calculated from the expression

$$F(\tau) = \int \varphi(\tau) d\tau = u \text{ or } \tau = F^{-1}(u),$$

where $u$ is uniformly distributed in the interval $[0, 1]$, has the distribution density $\varphi(\tau)$. " If the density function $\varphi(\tau)$ is a continuous function and obeys one of the known standard theoretical distribution laws, then the formulas given in Table 2 may be used to model the values $\tau$ [10].

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Density function</th>
<th>Modeling formula</th>
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<tbody>
<tr>
<td>Normal</td>
<td>$\varphi(\tau) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\tau-m)^2}{2\sigma^2}}$</td>
<td>$\tau = m + \sigma \left( \frac{12}{\pi} u - 6 \right)$</td>
</tr>
<tr>
<td>Uniform</td>
<td>$\varphi(\tau) = \frac{1}{b-a}$, $\tau \in [a,b]$</td>
<td>$\tau = a + u(b-a)$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$\varphi(\tau) = \lambda e^{-\lambda \tau}, \tau \geq 0$</td>
<td>$\tau = \frac{1}{\lambda} \ln u$</td>
</tr>
<tr>
<td>Linear</td>
<td>$\varphi(\tau) = \frac{\lambda}{2} (1 - \frac{\tau}{2})$, $\tau \in [0, \frac{2}{\lambda}]$</td>
<td>$\tau = -\frac{2}{\lambda} (1 - \sqrt{u})$</td>
</tr>
<tr>
<td>Gamma</td>
<td>$\varphi(\tau) = \frac{\alpha}{(k-1)!} \left( \frac{1}{\alpha} \right)^{k-1} e^{-\frac{1}{\alpha} \tau}$, $k &gt; 0$, $\tau \geq 0$</td>
<td>$\tau = -\frac{1}{\alpha} \ln(u_{1,2,\ldots,\alpha}^{<em>} * u_{1,2,\ldots,\alpha}^{</em>})$</td>
</tr>
</tbody>
</table>

The Laplace criterion is used in absence of information on the probabilities of possible future asset market situations, in which case all market situations are considered equally probable. This criterion has the corresponding analytical expression

$$b_{p,j}^* = \max_{A_p} \left\{ \frac{1}{m} \sum_{j=1}^{m} f(A_p, S_j) \right\}. \quad (4)$$

Finally, the Savage criterion is used to minimize the amounts of maximum losses for each decision made. When this criterion is used, Table 1 is converted into a loss (regret) table according to the formula

$$f_c(A_p, S_j) = \{ f(A_p, S_j) - \max f(A_p, S_j) \}. \quad (5)$$

Then the analytical expression of the Savage criterion has the form

$$b_{p,j}^* = \max_{A_p} \min_{S_j} f_c(A_p, S_j). \quad (6)$$

3. The algorithm for selecting a project under uncertainty

Let us build an algorithm for selecting a single project from a set of projects (1) under uncertainty

Step 1. Assign the elements of Table 1.

Step 2. Determine optimal project $A_{p*}$ according to the criterion (2).
Step 3. Determine optimal project $A_i*$ according to the criterion (3).

Step 4. Determine optimal project $A_i*$ according to the criterion (4).

Step 5. Compile a loss table according to formula (5).

Step 6. Determine optimal project $A_i*$ according to the criterion (6).

Step 7. For optimal projects selected according to each criterion, calculate one of the standard indices of their effectiveness (profitability index, payback period, etc.).

Step 8. Compare the results obtained according to various criteria and complete the comparison of the values of an additional standard index.

If several ($k$) projects must be selected for investment, we convert the algorithm to the following form.

Step 1. Assign the elements of Table 1.

Step 2. Select one of the criteria realized by expressions (2,3,4,6).

Step 3.1. Determine optimal project $A_i*$ according to the criterion (2).

Step 3.2. Determine optimal project $A_i*$ according to the criterion (3).

Step 3.3. Determine optimal project $A_i*$ according to the criterion (4).

Step 3.4.1. Compile a loss table according to formula (5).

Step 3.4.2. Determine optimal project $A_i*$ according to the criterion (6).

Step 4. From Table 1, delete the line corresponding to the selected project and proceed to implement step 3,* for projects selected previously in step 2, continuing doing so until determining $k$ projects suitable for investment.

According to the second algorithm, only one criterion selected in step 2 is used to determine all $k$ projects. However, in the presence of additional project indices that are not susceptible to market fluctuations, we may also make a comparative analysis of the results obtained according to different criteria.

4. Conclusion

There has recently been growing interest in procedures for analyzing the effectiveness of investment activities and for selecting promising investment projects. The reason for this renewed attention is the global financial and economic crisis, which exposed the shortcomings of methods used in the past. This article presents methods of making investment decisions and develops an algorithm for selecting promising projects under uncertainty. This algorithm can be used in the future for creating useful tools allowing a detailed comparative analysis of investment activities. The advantage of the methods and of the algorithm presented here is the ability to select the most effective option from alternative investment projects, while taking into consideration the uncertainty and risks associated with implementing the project.

References