FRACTIONAL ORDER PID CONTROL FOR IMPROVED LATERAL POSITION AND YAW ANGLE STABILITY FOR A VEHICLE

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ABSTRACT
With the growing development of autonomous automobiles, vehicle lateral control has caught the attention of researchers recently. Active vehicle front steering can enhance the handling performance and stability of sudden/emergency maneuvers. This research investigates a fractional order proportional-integral-derivative (FOPID) control technique that ensures the regulation of vehicle lateral dynamics behavior with the presence of uncertain parameters. A method for analyzing vehicle dynamics with the “bicycle model” is presented. To validate the proposed control algorithm, the system is linearized and Matlab simulation is used. FOPID is compared to conventional control methods such as PID. Simulation results confirm that the proposed control scheme improves lateral displacement and yaw angle of the vehicle to maneuver safely and potentially prevent a collision from occurring.

KEY WORDS
VEHICLE STABILITY, YAW ANGLE CONTROL, FOPID, TRANSIENT STABILITY.

1. Introduction
Lateral stability control is critical aspect of the vehicle performance, which can help a driver maintain control during a sudden/emergency maneuver. Data from Federal Highway Administration (FHWA) shows 18,779 fatalities in years between 2014-2016 resulted from lane departure which accounts for 53% of all traffic fatalities in the United States [1].

Varying factors affect the lateral dynamic performance of an automobile, such as road condition, vehicle parameter, structure, tire steering angle and the initial operation of the vehicle [2]. In recent years, the deployment for vehicle advanced control has increased with the high demand of the developing autonomous automobiles. Hence, it can be a promising solution to improve the stability, safety, emissions, passenger comfort and fuel economy [3]. Vehicle stability control (VSC) is an active safety system which can help in reducing wheel slip during acceleration. Additionally, VSC generally includes active front steering (AFS), direct yaw moment control (DYC), anti-lock braking systems (ABS) and active suspension system [4].

While to enhance vehicle handling and stability which is the primary concern of this research, active front wheel steering (AFS) systems has been heavily studied in both academic and industrial communities beginning in the late 1960’s by Kasselmann and Keranen [5]. AFS is one of the effective methods applied to achieve active safety control, since it can provide an additional steering angle component to the driver input which is not dependent on the input of the steering angle, thus it can be used to modify continuously the varying steering gear ratio. Consequently this ratio is decreased at high speeds for instance when driving at a highway in order to provide more accurate handling to the driver for harsh maneuvers, while the ratio is increased at slow vehicle speeds to diminish the steering efforts which is beneficial in parking situations [6].

There is two main challenges to achieve lateral dynamic stability of a vehicle. The first is the inheritance of nonlinearities in the dynamic model of the vehicle for instance the tire nonlinearity. While the second concern is the time varying longitudinal velocity for tracking the performance to control the lateral dynamics of the vehicle [7]. Other control methods have been applied such as gain scheduling [7], LQR [8], H\(_\infty\) [9] and adaptive control [10]. Hence these methods lack robust regulation when the system is complex and varying.

This paper proposes a control scheme based on fractional calculus theory; a fractional order control. Only limited literature has been conducted on this control area, thus more investigation is needed, in particular when applied to large vehicle lateral control. To achieve preliminary results a fractional order PID controller is applied to improve the performance of lateral vehicle dynamics. Fractional order calculus dates back to three centuries ago, it would provide a novel modeling approach for systems with extraordinary dynamical properties by introducing a non- integer fractional order system [11]. Further, fractional order control has acquired an increasing attention recently in the control community due to their better control performance when compared to traditional integer order controllers and only few studies show the benefits of applying this algorithm to vehicle dynamics.

Advantages of designing a fractional order controller include [12]:

[1] Federal Highway Administration (FHWA)
[2] Varying factors
[4] Active safety system
[8] LQR
[9] H\(_\infty\)
[10] Adaptive control
[12] Advantages

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• Improved output disturbance rejection due to the five tuning parameters ($K_p, K_i, K_d, \lambda, \mu$).
• Very low steady state error.
• Robustness against high frequency noise and plant gain variation.
• Attaining gain margins and phase cross over frequency specification.
• Improved control quality of dynamical systems.

This paper is organized as the follows: An overview of the vehicle lateral dynamics in section 2. Section 3 introduces fractional order control. Results and simulations are included in Section 4.

2. Vehicle Lateral Dynamics

The behavior of an autonomous land vehicle model is inherently nonlinear and uncertain, therefore, it is very challenging to obtain a precise dynamic model. The model for the system lateral motion is developed based upon system in [13], with the following assumptions:
• Only front wheels to steer is used by the automobile
• Slip angles can’t be disregarded since the longitudinal velocity is big.
• The lateral tire forces are proportional to the slip angles since they are so small
• The longitudinal force generated by the tires is disregarded.

Fig. 1 indicates the system lateral dynamics with two degrees of freedom. Table 1, define the symbols in Figs. 1-2.

Table 1. Vehicle dynamic parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$v_x$</td>
<td>Longitudinal velocity</td>
</tr>
<tr>
<td>$v_y$</td>
<td>Lateral velocity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Lateral displacement</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Lateral velocity</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Yaw angle</td>
</tr>
<tr>
<td>$\phi = r$</td>
<td>Vehicle yaw rate</td>
</tr>
<tr>
<td>$F_{yf}$</td>
<td>Lateral tire force of front wheel</td>
</tr>
<tr>
<td>$F_{rf}$</td>
<td>Lateral tire force of rear wheel</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Front tire cornering stiffness</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Rear tire cornering stiffness</td>
</tr>
<tr>
<td>$l_f$</td>
<td>Distance between the center of gravity and the front axle</td>
</tr>
<tr>
<td>$l_r$</td>
<td>Distance between the center of gravity and the rear axle</td>
</tr>
<tr>
<td>$m$</td>
<td>Vehicle mass</td>
</tr>
<tr>
<td>$I_z$</td>
<td>Yaw moment of inertia</td>
</tr>
</tbody>
</table>

While Fig. 2 shows the autonomous land vehicle model “bicycle model” (both right-handed ones) where xoy represents the coordinate system of the automobile, whereas XOY denotes the coordinate system based on the earth.

Fig. 2 Autonomous land vehicle bicycle model.

2.1 Active Front Steering System

AFS technology which was originally developed for passenger cars by BMW in order to make the front wheels maneuver in a particular angle to accommodate with the speed of the vehicle, with this technology over and under steering can be prevented. Fig. 3 shows a typical AFS configuration where the control module gathers information from the sensor package, then through a servo motor the angle is executed [6].

Fig. 3 Active front steering configuration.
2.2 Vehicle Linear Model System Model

The modeling would limit the performance of the control since the yaw moment of inertia, vehicle mass, front tire cornering stiffness and rear tire cornering stiffness are hard to model with accuracy, hence developing a robust control is crucial in real situation with these uncertainties and external disturbance. With small angle approximation $\cos(\psi) \approx 1, \sin(\psi) \approx 1$ the state-space representation of the system can be written as the following [13].

$$\dot{x} = Ax + Bu, \quad y = Cx$$

$$x_1 = y, x_2 = v_y, x_3 = \psi, x_4 = r$$

$$A = \begin{pmatrix} 0 & 1 & v & 0 \\ 0 & \eta_1 & 0 & -v + \frac{\eta_2}{v} \\ 0 & 0 & 0 & 1 \\ 0 & \eta_3 & 0 & \frac{\eta_4}{v} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

\[3. \text{ Fractional Order Calculus}\]

3.1 Theoretical Background

Fractional calculus is a generalization of integration and differentiation to non-integer order operates $aD_t^\alpha$ as in (1) [14].

$$aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & R(\alpha) > 0, \\ \frac{1}{\Gamma(\alpha)} \int_0^t (t^\alpha - \tau) \, d\tau & R(\alpha) < 0, \end{cases}$$

(1)

Where $\alpha$ and $t$ are the limits of the opration and $\alpha$ denotes the fractional order. It is assumed that $\alpha \in \mathbb{R}$, wheras $\alpha$ can also be a complex number $[15]$. One of three commonly used definitions to define a fractional differential and integration is Michele Caputos definition, where the derivative of a function $f(t)$ with respect to time is given as the following:

$$D_t^\alpha f(t) = \lim_{\varepsilon \to 0} \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} \, d\tau.$$  

(2)

Where $m=\alpha$ is an integer and $\Gamma$ is the Gamma function.

Aleksy Letnikov and Anton Grunwald, the Czech Republic and Russian mathematicians define fractional calculus as:

$$aD_t^\alpha f(t) = \lim_{\varepsilon \to 0} \frac{1}{\Gamma(m-\alpha)} \sum_{i=0}^{[\frac{t-a}{\varepsilon}]} (-1)^i \binom{\alpha}{i} f(t - ix).$$  

(3)

Where, $(-1)^i \binom{\alpha}{i} = \frac{(-1)^i \Gamma(i+1)}{\Gamma(\alpha-i+1)} \Gamma(i+1)$. With the assumption that the function $y(t) = 0$, when $t \leq t_0$ and $\Gamma$ is the Gamma function.

While Georg Riemann and Jospeh Liouville the German and French mathematicians defines fractional calculus with the assumption that $m - \alpha < \alpha \leq m$ as the following:

$$RL_a D_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-m}} \, d\tau.$$  

(4)

Where $\Gamma$ is the Gamma function and the power term in the integral $(\alpha + 1 - m)$ is ensured to be not less than the value -1.

The Laplace transform can be given as the following:

$$L\left[ aD_t^\alpha f(t) \right] = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k [ aD_t^\alpha ]_{t=0}.$$  

(5)

The differential equation of the fractional order $PI^\lambda D^\mu$ control is given by:

$$u(t) = K_p e(t) + \frac{K_i}{\lambda} \int_0^t e(\tau) \, d\tau + K_D D^\mu e(t).$$  

(6)

By applying a Laplace transform to equation (6) with null initial conditions, the transfer function can be expressed as the following [16].

$$G_c(s) = K_p + K_i s^{-\lambda} + K_D s^\mu.$$  

(7)

To demonstrate the control possibilities of using a fractional order $PI^\lambda D^\mu$ in a graphical way, Fig. 4 and 5 shows the four control points of a conventional PID range control points of the quarter-plane defined by selecting the fractional order values [16].

Using fractional order $PI^\lambda D^\mu$ controller the user has up to five parameters to tune, while for a fractional order PI or PD the user have four parameters to design. With $\lambda=1$ and $\mu=1$ a classical PID controller would be obtained, hence With $\lambda=1$ and $\mu=0$ a PI controller is acquired, while a PD controller is given when $\lambda=0$ and $\mu=1$.

![Fig. 4 Classical PID.](image)
In Fig. 6 a block diagram of an FOPID controller is shown. \( G_c(s) \) is the fractional order controller which is given in terms of a transfer function as the following:

\[
G_c(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu.
\]  

(8)

With the two additional adjustable parameters \( \lambda \) and \( \mu \), it allows more flexibility to control the system and at the same time would increase the control quality.

Several of optimization methods exist for tuning the five parameters of the FOPID controller, in this paper all five parameters are optimized using the Nelder-Mead (NM) method [17] based on the integral squared error (ISE). While the range of the order of integration and differentiation examined is in the range between \([0,1]\).

4. Simulation Results

The parameters used on the vehicle model are \( m = 1480 \) kg, \( C_f = 67500 \) N/rad, \( C_r = 47500 \) N/rad, \( I_x = 2350 \) kg m\(^2\), \( l_f = 1.05 \) m and \( l_r = 1.63 \) m. To validate the proposed controller, simulations results are done with Matlab/Simulink. Thus the vehicle dynamics are compared with a integer order PID and Fractional order \( PI^\lambda D^\mu \), by applying step response to the lateral displacement/yaw angle of the vehicle traveling at a speed of 55 mph and 75 mph. Results demonstrate that, with FOPID the vehicle position and yaw angle is improved compared to integer order PID controller. The controller values implemented are the following: \( K_p = 0.291, K_i = 1.65, K_d = 9.9, \lambda = 0.5, \mu = 0.67 \).
5. Conclusion

A Fractional order $PI^\alpha D^\mu$ control was implemented for a passenger vehicle dynamics. It was shown that by applying this control scheme the vehicle lateral displacement and yaw angle of the system has improved as compared to a classical PID control.

References