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OPTIMAL LOCATION FOR CAPACITOR INSTALLATION USING OPTIMAL POWER FLOW SENSITIVITY

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ABSTRACT

This paper presents the application of optimal power flow sensitivity in evaluating the optimal capacitor location for injecting reactive power into a power system. After the optimal power flow is solved, the sensitivity analysis is applied to evaluate the change of total generation cost with respect to the change of reactive power at any bus. The optimal capacitor location is the location where an increase in reactive power injection yields the lowest total generation cost due to decreasing of total system losses. This method greatly reduces the computational work of computing optimal power flow for several different systems in which capacitors are to be installed. The sensitivity technique is applied to 5-bus and 9-bus test systems. The results show that the technique gives the same answers as the simple technique in which more calculations are needed.

KEYWORDS

VAR compensation, optimal power flow, sensitivity analysis

1. Introduction

The reactive power injection is very useful for power system improvement [1],[2]. It increases power factor and reduces real power losses, which results in reducing total generation cost. Installing capacitor banks in the system is one of the techniques to inject reactive power into the system. The first step of installing capacitor banks into a power system is to determine the appropriate location. In this paper, the optimal location of the capacitor banks is considered the location where capacitor installation yields the largest savings in the total generation cost while the power system can still serve total loads. The Optimal power flow (OPF) problem is used to determine the optimal system operating point at the lowest total generation cost while enforcing a variety of operational constraints such as limits of bus voltages, line flows, real power generator and reactive power generator. The OPF problem has a long history [3] in power system research. A variety of numerical techniques developed for this problem are as followed:

1. Non-linear programming method (NLP) [4], [6] deals with problems involving nonlinear objective function and constrains.
2. Quadratic programming method [4], [7] is a nonlinear programming whose objective function is quadratic with linear constraints.
3. Linear programming method (LP) [5], [6] treats problems with constraints and objective function formulated in linear forms with non-negative variables. The simplex method is known to be quite effective for solving LP problems.
4. Interior point method [5], [7], [8] is applied to NLP and QP problems because of the enhanced performance and convergence properties.
5. Lagrange Newton method [5], [9], [10], [11] can solve nonlinear problems based on KKT conditions.

One way to evaluate the value of placing capacitor bank at any bus is by comparing the objective function (total generation cost) of the OPF problem of the system with a capacitor installed at that bus to that of the system without the capacitor (or so called “base case”). The bus that gives the largest difference in the objective functions is the optimal location for the capacitor. The method is called *differencing method*. An alternative method of evaluating the value of capacitor placement is achieved by applying sensitivity analysis to the base case OPF. Adding capacitor bank to any bus in a power system is equivalent to reducing reactive load at that bus. As a result, the optimal system operating point will be changed. The sensitivity analysis is therefore applied to the optimal solution after the base case OPF is solved, in order to evaluate the change of the objective function due to reducing one unit of reactive load at each bus. The bus that yields the biggest change, in other words, the largest incremental saving cost, is considered the optimal location of the capacitor bank. The *sensitivity method* requires less computational work since only one OPF is needed.

The sensitivity method is described in details in the next section, and applied to 5-bus and 9-bus test systems. The results from the test systems by sensitivity analysis give the same incremental saving costs, and thus the same optimal locations as the ones from the differencing method.

2. Methodology

From OPF methods mentioned above, the Lagrange Newton method is employed in this paper because sensitivity analysis needs the matrix of the second order derivatives of Lagrange function from OPF problem to determine the change of the objective function with respect to the change of injected reactive power.

2.1 Optimal Power Flow

Optimal power flow is used to obtain the optimal system operating point while minimizing total system generation cost subject to equality and inequality constraints. The equality constraints are the power balance equations (real and reactive power equations), and the inequality constraints include voltage limits, limits on transmission line flows, generator real power and reactive power limits and other control devices. The optimal power flow model can be written as followed:

Objective function: The total generation cost

$$\min_{P_G} C(P_G) = \sum_{i=1}^{N_G} C_i(P_{Gi}) \qquad (1)$$

Where $C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2$ is the cost of generation at generator bus i.

Subject to

Equality constraints : Power balanced equations

$$P_i(\theta, V, P_G / P_L) = 0 \qquad i = 1,2,...,N \qquad (2)$$

$$Q_i(\theta, V, Q_G / Q_L) = 0 \qquad i = 1,2,...,N \qquad (3)$$

where

θ	voltage angle
V	voltage magnitude
P_G	variable real power generator
Q_G	variable reactive power generator
P_L	real power load
Q_L	reactive power load
N	number of all buses in the system
N_G	number of generator buses.

Equations (2) and (3) represent vectors of power flow equations at bus i, where θ, V, P_G and Q_G are the variables to be solved, while P_L and Q_L are independent parameters.

Inequality constraints: Limitations of real and reactive power generations, bus voltages and line flows

$$V_{i,\min} \le V_i \le V_{i,\max} \qquad i = 1,2,...,N \qquad (4)$$

$$P_{Gi,\min} \le P_{Gi} \le P_{Gi,\max} \qquad i = 1,2,...,N_G \qquad (5)$$

$$Q_{Gi,\min} \le Q_{Gi} \le Q_{Gi,\max} \qquad i = 1,2,...,N_G \qquad (6)$$

$$\left|I_j\right| \le I_{j,\max} \qquad j = 1,2,...,N_l \qquad (7)$$

where

$\left\lvert I_j \right\rvert$	absolute current flow in line j
N_l	number of lines.

2.2 Lagrange function

To solve the optimal power flow problem by Newton method, the most common method of handling equality and inequality constraints is based on forming the Lagrange function for the problem defined as followed:

$$\begin{aligned} L = & \sum_{i=1}^{N_G}(\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \sum_{i=1}^{N} \lambda_{Pi} P_i(\theta, V, P_G / P_L) \\ & + \sum_{i=1}^{N} \lambda_{Qi} Q_i(\theta, V, Q_G / Q_L) + \sum_{j=1}^{N_l} \lambda_{line,j} I_j(\theta, V) \\ & + \sum_{i \in A_H} \mu_{Hi}(f_i(Y) - f_{Hi}) + \sum_{i \in A_L} \mu_{Li}(f_{Li} - f_i(Y)) \end{aligned} \qquad (8)$$

where

$f_i(Y)$	vectors of binding variable
f_{Hi}	upper limit of binding variable
f_{Li}	lower limit of binding variable
λ_{Pi}	Lagrange multiplier of real power balanced at bus i
λ_{Qi}	Lagrange multiplier of reactive power balanced at bus i
$\lambda_{line,j}$	Lagrange multiplier of line flow j
μ_H	Lagrange multiplier of binding variable at upper limit $(\mu_H \ge 0)$
μ_L	Lagrange multiplier of binding variable at lower limit $(\mu_L \ge 0)$.

From Lagrange function in equation (8), the optimal system operating point can be obtained by adjusting the Lagrange function to satisfy first order derivatives ignoring non-binding constraints as:

$$g(Z) = \nabla_Z L(Z) = \begin{bmatrix} \frac{\partial L}{\partial Y} & \frac{\partial L}{\partial \lambda} & \frac{\partial L}{\partial \mu_H} & \frac{\partial L}{\partial \mu_L} \end{bmatrix}^T = 0 \qquad (9)$$

where

Y	state vector $[\theta, V, P_G, Q_G, I_L]^T$
Z	vector of all variables $[Y, \lambda, \mu_H, \mu_L]^T$
λ	vector of Lagrange multipliers of equality constraints.

Equation (9) is called the Karush-Kuhn-Tucker (KKT) conditions solved by applying the Taylor's series expansion by ignoring the high-order terms and can be written as:

$$\frac{dg(Z)}{dZ} \cdot \Delta Z = -g(Z) \qquad (10)$$

From equation (10), $dg(Z)/dZ$ is the second order derivatives of Lagrange function with respect to vector of variables, Z, denoted by W which can be written in matrix form as:

$$\begin{bmatrix} H & J^T & A_H^T & A_L^T \\ J & 0 & \cdots & 0 \\ A_H & \vdots & \ddots & \vdots \\ A_L & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \Delta Y \\ \Delta\lambda \\ \Delta\mu_H \\ \Delta\mu_L \end{bmatrix} = - \begin{bmatrix} \frac{\partial L}{\partial Y} \\ \frac{\partial L}{\partial \lambda} \\ \frac{\partial L}{\partial \mu_H} \\ \frac{\partial L}{\partial \mu_L} \end{bmatrix}$$

where Hessian matrix: H, and Jacobian matrices: J, A_H and A_L are denoted as:

$$H = \frac{\partial^2 L}{\partial Y^2}, \qquad J = \frac{\partial^2 L}{\partial\lambda\partial Y}$$

and $$A_H = \frac{\partial^2 L}{\partial\mu_H \partial Y}, \qquad A_L = \frac{\partial^2 L}{\partial\mu_L \partial Y}$$

From equation (10), the Newton step can be obtained from solving

$$\Delta Z = -W^{-1} \cdot g(Z) \tag{11}$$

2.3 Sensitivity Analysis

Sensitivity analysis [12], [13] is used to find the optimal capacitor location in which yields the lowest total generation cost. The optimal system operating point from OPF problem changes as some parameters change. Sensitivity analysis evaluates a change of the optimal system operating point due to a change in a parameter ε by taking the first order derivatives of $g(Z)$ vector in equation (9) with respect to parameter ε as followed.

Elements of the first row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\theta}\right] = \frac{d}{d\varepsilon} g(\theta, V, \lambda_p, \lambda_Q, \lambda_{line}) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\theta}\right] = \frac{\partial g}{\partial\theta}\cdot\frac{\partial\theta}{\partial\varepsilon} + \frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial\varepsilon} + \frac{\partial g}{\partial\lambda_p}\cdot\frac{\partial\lambda_p}{\partial\varepsilon} + \frac{\partial g}{\partial\lambda_Q}\cdot\frac{\partial\lambda_Q}{\partial\varepsilon} + \frac{\partial g}{\partial\lambda_{line}}\cdot\frac{\partial\lambda_{line}}{\partial\varepsilon} = 0$$

$$\nabla^2_{\theta\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\theta V} L\cdot\nabla_\varepsilon V + \nabla^2_{\theta\lambda_p} L\cdot\nabla_\varepsilon\lambda_p + \nabla^2_{\theta\lambda_Q} L\cdot\nabla_\varepsilon\lambda_Q = 0$$
$$+\nabla^2_{\theta\lambda_{line}} L\cdot\nabla_\varepsilon\lambda_{line} \tag{12}$$

Elements of the second row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial V}\right] = \frac{d}{d\varepsilon} g(\theta, V, \lambda_p, \lambda_Q, \lambda_{line}) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial V}\right] = \frac{\partial g}{\partial\theta}\cdot\frac{\partial\theta}{\partial\varepsilon} + \frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial\varepsilon} + \frac{\partial g}{\partial\lambda_p}\cdot\frac{\partial\lambda_p}{\partial\varepsilon} + \frac{\partial g}{\partial\lambda_Q}\cdot\frac{\partial\lambda_Q}{\partial\varepsilon} + \frac{\partial g}{\partial\lambda_{line}}\cdot\frac{\partial\lambda_{line}}{\partial\varepsilon} = 0$$

$$\nabla^2_{V\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{VV} L\cdot\nabla_\varepsilon V + \nabla^2_{V\lambda_p} L\cdot\nabla_\varepsilon\lambda_p + \nabla^2_{V\lambda_Q} L\cdot\nabla_\varepsilon\lambda_Q = 0$$
$$+\nabla^2_{V\lambda_{line}} L\cdot\nabla_\varepsilon\lambda_{line} \tag{13}$$

Elements of the third row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial P_G}\right] = \frac{d}{d\varepsilon} g(P_G, \lambda_p) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial P_G}\right] = \frac{\partial g}{\partial P_G}\cdot\frac{\partial P_G}{\partial\varepsilon} + \frac{\partial g}{\partial\lambda_p}\cdot\frac{\partial\lambda_p}{\partial\varepsilon} = 0$$

$$\nabla^2_{P_G P_G} L\cdot\nabla_\varepsilon P_G + \nabla^2_{P_G\lambda_p} L\cdot\nabla_\varepsilon\lambda_p = 0 \tag{14}$$

Elements of the forth row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial Q_G}\right] = \frac{d}{d\varepsilon} g(\lambda_Q) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial Q_G}\right] = \frac{\partial g}{\partial\lambda_Q}\cdot\frac{\partial\lambda_Q}{\partial\varepsilon} = 0$$

$$\nabla^2_{Q_G\lambda_Q} L\cdot\nabla_\varepsilon\lambda_Q = 0 \tag{15}$$

Elements of the fifth row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial I_l}\right] = \frac{d}{d\varepsilon} g(\lambda_{line}) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial I_l}\right] = \frac{\partial g}{\partial\lambda_{line}}\cdot\frac{\partial\lambda_{line}}{\partial\varepsilon} = 0$$

$$\nabla^2_{I_l\lambda_{line}} L\cdot\nabla_\varepsilon\lambda_{line} = 0 \tag{16}$$

Elements of the sixth row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\lambda_p}\right] = \frac{d}{d\varepsilon} g(\theta, V, P_G / P_L) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\lambda_p}\right] = \frac{\partial g}{\partial\theta}\cdot\frac{\partial\theta}{\partial\varepsilon} + \frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial\varepsilon} + \frac{\partial g}{\partial P_G}\cdot\frac{\partial P_G}{\partial\varepsilon} + \frac{\partial g}{\partial P_L}\cdot\frac{\partial P_L}{\partial\varepsilon} = 0$$

$$\nabla^2_{\lambda_p\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\lambda_p V} L\cdot\nabla_\varepsilon V + \nabla^2_{\lambda_p P_G} L\cdot\nabla_\varepsilon P_G + \nabla^2_{\lambda_p P_L} L\cdot\nabla_\varepsilon P_L = 0$$
$$\nabla^2_{\lambda_p\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\lambda_p V} L\cdot\nabla_\varepsilon V + \nabla^2_{\lambda_p P_G} L\cdot\nabla_\varepsilon P_G + \nabla_\varepsilon P_L = 0$$
$$\nabla^2_{\lambda_p\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\lambda_p V} L\cdot\nabla_\varepsilon V + \nabla^2_{\lambda_p P_G} L\cdot\nabla_\varepsilon P_G = -\nabla_\varepsilon P_L \tag{17}$$

Elements of the seventh row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\lambda_Q}\right] = \frac{d}{d\varepsilon} g(\theta, V, Q_G / Q_L) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\lambda_Q}\right] = \frac{\partial g}{\partial\theta}\cdot\frac{\partial\theta}{\partial\varepsilon} + \frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial\varepsilon} + \frac{\partial g}{\partial Q_G}\cdot\frac{\partial Q_G}{\partial\varepsilon} + \frac{\partial g}{\partial Q_L}\cdot\frac{\partial Q_L}{\partial\varepsilon} = 0$$

$$\nabla^2_{\lambda_Q\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\lambda_Q V} L\cdot\nabla_\varepsilon V + \nabla^2_{\lambda_Q Q_G} L\cdot\nabla_\varepsilon Q_G + \nabla^2_{\lambda_Q Q_L} L\cdot\nabla_\varepsilon Q_L = 0$$
$$\nabla^2_{\lambda_Q\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\lambda_Q V} L\cdot\nabla_\varepsilon V + \nabla^2_{\lambda_Q Q_G} L\cdot\nabla_\varepsilon Q_G + \nabla_\varepsilon Q_L = 0$$
$$\nabla^2_{\lambda_Q\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\lambda_Q V} L\cdot\nabla_\varepsilon V + \nabla^2_{\lambda_Q Q_G} L\cdot\nabla_\varepsilon Q_G = -\nabla_\varepsilon Q_L \tag{18}$$

Elements of the eighth row are in the terms of

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\lambda_{line}}\right] = \frac{d}{d\varepsilon} g(\theta, V, I_l) = 0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial\lambda_{line}}\right] = \frac{\partial g}{\partial\theta}\cdot\frac{\partial\theta}{\partial\varepsilon} + \frac{\partial g}{\partial V}\cdot\frac{\partial V}{\partial\varepsilon} + \frac{\partial g}{\partial I_l}\cdot\frac{\partial I_l}{\partial\varepsilon} = 0$$

$$\nabla^2_{\lambda_{line}\theta} L\cdot\nabla_\varepsilon\theta + \nabla^2_{\lambda_{line}V} L\cdot\nabla_\varepsilon V + \nabla^2_{\lambda_{line}I_l} L\cdot\nabla_\varepsilon I_l = 0 \tag{19}$$

Elements of the binding variables at upper limit

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial \mu_H}\right]=\frac{d}{d\varepsilon}g(f(Y)/f_H)=0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial \mu_H}\right]=\frac{\partial g}{\partial f(Y)}\cdot\frac{\partial f(Y)}{\partial \varepsilon}+\frac{\partial g}{\partial f_H}\cdot\frac{\partial f_H}{\partial \varepsilon}=0$$

$$\nabla^2_{\mu_H f(Y)}L\cdot\nabla_\varepsilon f(Y)+\nabla^2_{\mu_H f_H}L\cdot\nabla_\varepsilon f_H=0$$

$$\nabla^2_{\mu_H f(Y)}L\cdot\nabla_\varepsilon f(Y)-\nabla_\varepsilon f_H=0$$

$$\nabla^2_{\mu_H f(Y)}L\cdot\nabla_\varepsilon f(Y)=\nabla_\varepsilon f_H \tag{20}$$

Elements of the binding variables at lower limit

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial \mu_L}\right]=\frac{d}{d\varepsilon}g(f(Y)/f_L)=0$$

$$\frac{d}{d\varepsilon}\left[\frac{\partial L}{\partial \mu_L}\right]=\frac{\partial g}{\partial f(Y)}\cdot\frac{\partial f(Y)}{\partial \varepsilon}+\frac{\partial g}{\partial f_L}\cdot\frac{\partial f_L}{\partial \varepsilon}=0$$

$$\nabla^2_{\mu_L f(Y)}L\cdot\nabla_\varepsilon f(Y)+\nabla^2_{\mu_L f_L}L\cdot\nabla_\varepsilon f_L=0$$

$$\nabla^2_{\mu_L f(Y)}L\cdot\nabla_\varepsilon f(Y)+\nabla_\varepsilon f_L=0$$

$$\nabla^2_{\mu_L f(Y)}L\cdot\nabla_\varepsilon f(Y)=-\nabla_\varepsilon f_L \tag{21}$$

Equations (12) to (21) can be expressed in a matrix form as followed:

$$W\cdot\nabla_\varepsilon Z^*(\varepsilon)=M \tag{22}$$

where W is the matrix of the second order derivative of Lagrange function with respect to Z from the OPF problem and is defined as:

$$\left[\begin{array}{cccccccc:cccc}
\nabla^2_{\theta\theta}L & \nabla^2_{\theta V}L & 0 & 0 & 0 & \nabla^2_{\theta\lambda_p}L & \nabla^2_{\theta\lambda_Q}L & \nabla^2_{\theta\lambda_{line}}L & 0 & 0 & 0 & 0\\
\nabla^2_{V\theta}L & \nabla^2_{VV}L & 0 & 0 & 0 & \nabla^2_{V\lambda_p}L & \nabla^2_{V\lambda_Q}L & \nabla^2_{V\lambda_{line}}L & B^T_{V*V} & 0 & 0 & 0\\
0 & 0 & \nabla^2_{P_GP_G}L & 0 & 0 & \nabla^2_{P_G\lambda_p}L & 0 & 0 & 0 & B^T_{P_G*P_G} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \nabla^2_{Q_G\lambda_Q}L & 0 & 0 & 0 & B^T_{Q_G*Q_G} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \nabla^2_{I_l\lambda_{line}}L & 0 & 0 & 0 & B^T_{I_l*I_l}\\
\nabla^2_{\lambda_p\theta}L & \nabla^2_{\lambda_pV}L & \nabla^2_{\lambda_pP_G}L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\nabla^2_{\lambda_Q\theta}L & \nabla^2_{\lambda_QV}L & 0 & \nabla^2_{\lambda_QQ_G}L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\nabla^2_{\lambda_{line}\theta}L & \nabla^2_{\lambda_{line}V}L & 0 & 0 & \nabla^2_{\lambda_{line}I_l}L & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\hdashline
0 & B_{V*V} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & B_{P_G*P_G} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & B_{Q_G*Q_G} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & B_{I_l*I_l} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

The element B_{V*V} in the W matrix is a matrix of binding limits on voltages. If the voltage is binding at its upper limit, the value of the element is 1. If it is binding at the lower limit, the value is -1. $B_{P_G*P_G}$, $B_{Q_G*Q_G}$ and $B_{I_l*I_l}$ are similarly defined. The other terms in equation (22) are:

$$\nabla^T_\varepsilon Z^*=\Big[\nabla^T_\varepsilon\theta\,\Big|\nabla^T_\varepsilon V\Big|\nabla^T_\varepsilon P_G\Big|\nabla^T_\varepsilon Q_G\Big|\nabla^T_\varepsilon I_{line}\Big|\nabla^T_\varepsilon\lambda_P\Big|$$
$$\nabla^T_\varepsilon\lambda_Q\Big|\nabla^T_\varepsilon\lambda_{line}\Big|\nabla^T_\varepsilon\mu_H\Big|\nabla^T_\varepsilon\mu_L\Big] \tag{23}$$

$$M^T=\Big[0^T_Y\Big|-\nabla^T_\varepsilon P_L\Big|-\nabla^T_\varepsilon Q_L\Big|0^T_{N_l}\Big|\ \nabla^T_\varepsilon f_{Hi}\Big|-\nabla^T_\varepsilon f_{Li}\Big] \tag{24}$$

where

Z^*	vector of the optimal system operating point
0_Y	zero vector of state vector (Y) dimension
$\nabla_\varepsilon P_L$	vector of dimension N with element $\nabla_\varepsilon P_{Li}$ for $i\in\{1,2,...,N\}$
$\nabla_\varepsilon Q_L$	vector of dimension N with element $\nabla_\varepsilon Q_{Li}$ for $i\in\{1,2,...,N\}$
0_{N_l}	zero vector of dimension N_l
$\nabla_\varepsilon f_{Hi}$	vector of upper binding variables dimension
$\nabla_\varepsilon f_{Li}$	vector of lower binding variables dimension

Optimal system operating point will change after a capacitor bank is added. Capacitor bank installation at any bus is equivalent to reducing reactive power load at that bus. Reactive power is therefore a parameter to be considered, denoted by ε. This method calculates the rate at which functions of the system operations change as a parameter ε changes given that the optimality is maintained.

From equations (22), (23) and (24), let ε be reactive power load at any bus j, that is $\varepsilon=Q_{Lj}$, we have

$$W\cdot\nabla_{Q_{Lj}}Z^*=M \tag{25}$$

The sensitivity of the optimal system operating point is

$$\nabla_{Q_{Lj}}Z^*=W^{-1}\cdot M \tag{26}$$

and

$$M^T=\Big[0^T_1\Big|0^T_2\Big|-\nabla^T_{Q_{Lj}}Q_L\Big|0^T_3\Big|0^T_4\Big]$$
$$=\Big[0^T_Y\Big|0^T_N\Big|0...0\ \underset{\substack{\uparrow\\ \text{bus j}}}{-1}\ 0...0\Big|0^T_{N_l}\Big|0^T_B\Big] \tag{27}$$

where

0^T_Y	zero vector of state vector (Y) dimension
0^T_N	zero vector of dimension N
$0^T_{N_l}$	zero vector of dimension N_l
0^T_B	zero vector of binding variables dimension

Since injecting reactive power into a system reduces total system losses, and thus reduces real power generation and total generation cost. Therefore, the interested function for sensitivity analysis is the total generation cost at the optimal operating point, which is the objective function of the base case OPF. The optimal total generation cost is expressed in terms of a parameter ε as:

$$C=\sum_{i=1}^{N_G}(\alpha_i+\beta_i P^*_{Gi}(\varepsilon)+\gamma_i P^{*\,2}_{Gi}(\varepsilon)) \tag{28}$$

where ε is the injected reactive power at any bus.

Therefore, the sensitivity analysis is done by taking the derivative of the total system generation cost with respect to ε .

$$\nabla_{\varepsilon} C = \sum_{i=1}^{N_G} (\beta_i \nabla_{\varepsilon} P_{Gi}^{*}(\varepsilon) P_{Gi} + 2\gamma_i P_{Gi} \nabla_{\varepsilon} P_{Gi}^{*}(\varepsilon)) \tag{29}$$

Injecting reactive power at bus j , by adding a capacitor bank, is equivalent to reducing reactive power load at that bus, then the parameter is $\varepsilon = Q_{Lj}$, and (29) becomes

$$\nabla_{Q_{Lj}} C = \sum_{i=1}^{N_G} (\beta_i \nabla_{Q_{Lj}} P_{Gi}^{*} + 2\gamma_i P_{Gi} \nabla_{Q_{Lj}} P_{Gi}^{*})$$

$$\nabla_{Q_{Lj}} C = \sum_{i=1}^{N_G} \nabla_{Q_{Lj}} P_{Gi}^{*} \left[\beta_i + 2\gamma_i P_{Gi}^{*} \right] \tag{30}$$

The sensitivity of the total generation cost with respect to the change in reactive power load, $\nabla_{Q_{Lj}} C$ in equation (30), is considered the *incremental saving cost* of capacitor installation at bus j , which can be obtained once the base case OPF is solved and $\nabla_{Q_{Lj}} P_{Gi}^{*}$ is evaluated from equation (26).

3. Case studies

The optimal location for capacitor bank is considered the bus that gives the largest savings in total generation cost when the capacitor is added. Therefore, there are two methods to compute such savings: the *differencing method* and the *sensitivity method*. The differencing method is achieved by comparing the objective functions of two solved OPF problems: the base case OPF and the OPF with 1 MVAr reactive load reduced at one load bus. For the system with N load busses, the OPF must be solved $N+1$ times. On the other hand, the sensitivity method solves base case OPF once, and then the sensitivity analysis is applied as discussed above in order to determine the savings.

In this study, the test systems are calculated by both differencing and sensitivity methods. The OPF program is written in MATLAB®. The results from two methods are compared. Finally, the optimal capacitor placement can be considered.

3.1 Case study 1

The model of 5-bus test system modified from the 14-bus IEEE test system is a simple power system with three generators connected at busses 1, 2 and 3 as shown in Fig. 1. The incremental saving costs of the 5-bus system determined from the differencing and the sensitivity methods are shown in Table 1.

From Table 1, incremental saving costs obtained from both methods are approximately the same. They give the highest values at bus 4, hence it is the optimal location for capacitor installation for this system. Although the results from two methods are the same, the sensitivity analysis uses much less calculation effort.

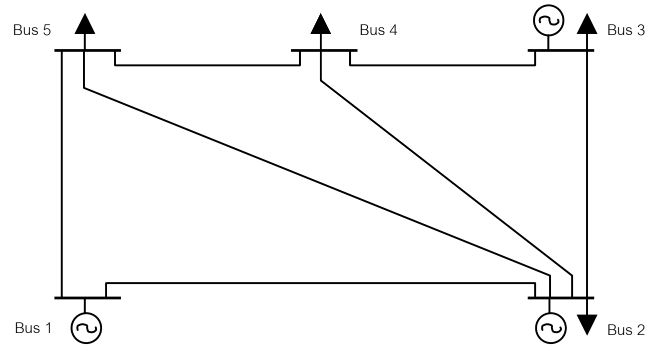

Fig. 1. The 5-bus test system

Table 1. Incremental saving costs for 5-bus test system

Bus Number	Incremental Saving Cost ($/MVAr-hr)	
	Differencing method	Sensitivity method
1	0	0
2	6.5305	6.5306
3	10.9381	10.9392
4	16.4360	16.4421
5	13.3827	13.3827

3.2 Case study 2

The system used in case study 2 is a 9-bus test system from Power System Engineering Research Center (PSERC) [14] of Cornell University. The system has three generators and nine transmission lines as shown in Fig. 2. The OPF results of the system using two methods discussed earlier are shown in Table 2.

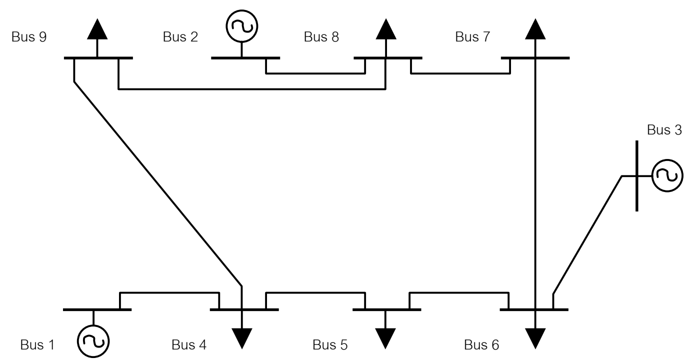

Fig. 2. The 9-bus test system of PSERC

The results of both methods give the same optimal location for capacitor installation with approximately same incremental saving costs, just like the 5-bus test system. The optimal capacitor location for the 9-bus system is therefore bus 8 with 30.43 $/MVAr-hr savings.

Note that the incremental saving costs of buses 1, 2 and 3 of the 9-bus system (and bus 1 of 5-bus system) are zeros since they are generator buses without loads.

Table 2. Incremental saving costs for 9-bus test system

Bus Number	Incremental Saving Cost ($/MVAr-hr)	
	Differencing method	Sensitivity method
1	0	0
2	0	0
3	0	0
4	22.0873	22.0952
5	21.1112	21.1475
6	11.5174	11.5162
7	26.3012	26.3012
8	30.4265	30.4271
9	27.8610	27.3638

4. Conclusions

The sensitivity method for evaluating the value of capacitor placement is achieved by applying sensitivity analysis to the base case OPF. OPF by Lagrange Newton method is employed in this paper since *W* matrix is needed to determine the change of the objective function with respect to the change of injected reactive power. The bus that yields the biggest change, in other words, the largest incremental saving cost, is considered the optimal location of the capacitor bank. The proposed sensitivity method is tested on 5-bus and 9-bus systems. The results are compared to the ones from the differencing method. Both methods give the same optimal location with approximately the same saving costs. However, the sensitivity method requires much less computational work since only one OPF computation is needed.

5. References

[1] W.J. Allen, *Power generation operation and control*(John Wiley & Son, Inc., United States of America.,1996).

[2] T.M. Bishop and A. Chaudhary, Primer on capacitor bank protection, *Petroleum and Chemical Industry Conference, Record of Conference Papers, Industry Applications Society 47th Annual* , 11-13 Sept. 2000, 199 -205.

[3] M. Huneault and F.D. Galiana, A survey of the optimal power flow literature, *Power Systems, IEEE Transactions* , *6*(2) , May 1991, 762 – 770.

[4] J.A. Momoh, R. Adapa and M.E. El-Hawary, A review of selected optimal power flow literature to 1993. I. Nonlinear and quadratic programming approaches," *Power Systems, IEEE Transactions*, *14*(1) , Feb. 1999, 96 – 104.

[5] J.A. Momoh, M.E. El-Hawary and R. Adapa, A review of selected optimal power flow literature to 1993. II. Newton, linear programming and interior point methods, *Power Systems, IEEE Transactions on*, *14*(1), Feb. 1999, 105 – 111.

[6] D. Pudjianto, S. Ahmed and G. Strbac, Allocation of VAR support using LP and NLP based optimal power flows, *Generation, Transmission and Distribution, IEE Proceedings, Volume: 149* Issue: 4 , July 2002, 377 –383.

[7] I.M. Nejdawi, K.A. Clements and P.W. Davis, An efficient interior point method for sequential quadratic programming based optimal power flow, *Power Systems, IEEE Transactions, 15*(4), Nov. 2000, 1179 – 1183.

[8] K. Xie and Y.H. Song, Dynamic optimal power flow by interior point methods, *Generation, Transmission and Distribution, IEE Proceedings, Volume: 148*, Issue: 1 , Jan. 2001, 76 – 84.

[9] D.I. Sun, B. Ashley, B. Brewer, A. Hughes and W.F. Tinney., Optimal power flow by newton approach," *IEEE Transactions on Power Apparatus and Systems, 103*(10), October 1984, 2864-2880.

[10] A.Santos and G.R.M. da Costa, Optimal-power-flow solution by newton's method applied to an augmented lagrangian function, *Generation, Transmission and Distribution, IEE Proceedings, Volume: 142* , Issue: 1 , Jan. 1995, 33 – 36.

[11] H. Ambriz-Perez, E. Acha and C.R. Fuerte-Esquivel, Advanced SVC models for newton-raphson load flow and newton optimal power flow studies, *Power Systems, IEEE Transactions* , *15*(1) , Feb. 2000, 129 – 136.

[12] P.R. Gribik, D. Shirmohammadi, S. Hao and C.L. Thomas, Optimal power flow sensitivity analysis, *Power System, IEEE Transactions, Volume: 53,* Aug.1990, 969-976.

[13] J.L. Bala, P.A. Kuntz and R.M. Taylor, Sensitivity-based optimal capacitor placement on a radial distribution feeder, *IEEE Technical Applications Conference and Workshop Northcon 95*, 10-12 Oct. 1995, 225 -230

[14] http://www.pserc.cornell.edu/matpower