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# SELECTION OF WIND GENERATOR CAPACITY FOR CAPTURING MAXIMUM WIND ENERGY IN WIND POWER GENERATION SYSTEMS

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**ABSTRACT**
This paper presents an approach for selecting capacity of wind turbine generators under different values of wind tower hub height using capacity factor, normalized average power, and the product of both quantities of a wind power generation system. In order to capture the maximum wind energy from wind, most current wind turbines have different hub heights and various values of rated capacity. This paper employs Weibull distribution to describe the probability distribution of wind speed while the important relationships among the expected value and standard deviation of wind speed as well as the scale parameter and the shape parameter of Weibull distribution function are also derived. Finally, a practical windfarm example in Taiwan is employed to examine the proposed method.

**KEY WORDS**
wind turbine generator, Weibull distribution, capacity factor, normalized average power, windfarm.

## 1. Introduction

The promotion, development, and increasing the use of new and renewable forms of energy are very important tasks for various countries in the whole world today. Among various renewable energy, wind power energy is currently one of the most popular and promising energy resources in the whole world [1]. Wind power is only an intermittent source of energy but it represents a useful energy resource. Many system integration studies completed in recent years have shown that intermittency of wind farms in the US has increased substantially in the last few years. Due to the increasing number of wind farms, the cost of wind power would obviously fall. Currently, the cost of wind power generation falls to about US$ 3 to 6 cents/kWh. By 2005, the cost will drop to US$ 2 cents/kWh and it will become one of the cheapest renewable resources available in the world.

At a specific site, the electricity generated by wind power generation system depends on the expected value and the standard deviation of the wind speed as well as the location of wind-tower installation. While year-to-year variation of annual mean wind speed remains hard to predict, wind speed variations during a year can be well characterized in terms of a probability distribution. Weibull distribution [2-4] has been utilized to represent the variations in hourly mean wind speed over a year at many typical locations and this paper employs this distribution to examine wind-speed characteristics. This paper also addresses the importance of the mean wind speed, its standard deviation, and two parameters of Weibull distribution, and then derives associated equations.

The generation of electricity by a wind turbine generator at a specific site depends upon several factors including different speed characteristics of the wind turbine such as cut-in wind speed ($v_C$), rated wind speed ($v_R$), cut-out wind speed ($v_F$), hub height ($h$), etc. However, the values of $v_C$ and $v_F$ of commercial wind turbines are respectively set to be 4 m/s and 25 m/s for most wind turbines control construction at different windfarms. This paper employs these two parameters, which comprise rated speed of wind turbine and hub height, to analyze how to capture maximum wind energy. The values of capacity factor CF, the normalized average power $P_n$, and the product of both CF and $P_n$ are also utilized. Finally, this paper uses wind turbine manufacture's specifications and wind speed probability distribution on a specific windfarm to roughly compute the generated average wind power. Through the above results of available analyses, a suitable capacity of wind turbines is selected at remote areas if a detailed planning and development staged of wind power station is under considered.

## 2. Probability distribution of wind

Let $\rho$ be the air density, $v$ the velocity of the wind, and $A$ the swept area of wind blades through which the wind passes normally. The mass of air passing in unit time is $\rho A v$ and the kinetic energy passing through the area in unit time is

$$P = \frac{1}{2} \cdot \rho A v \cdot v^2 = \frac{1}{2} \rho A v^3 \qquad (1)$$

where P is the total wind power available that can be extracted by a wind turbine. However, only a fraction of P can actually be extracted. The value of P varies linearly with the density of the air sweeping the blades and with the cube of the wind speed. Though most of the investigators employed simple wind speed distributions that were parameterized solely by the mean wind speed, it was reported in [5] that Weibull statistical model using cubic mean value of the wind speed would give better assessment of wind power potential at a installation site.

The data of electrical wind power obtained by wind turbines at the selected windfarm in Taiwan are given in Table 1. It shows reasonable results by using cubic mean wind speed to replace average wind speed according to a close check between generation and monthly average speed.

Table 1 Average electric power of wind turbine generator at a windfarm in Taiwan

| Month | Output Energy (kWh) | Duration of generation (hours) | Average power of generation (kW) | Mean wind speed (m/s) |
|---|---|---|---|---|
| January | 129,443 | 253.78 | 510.06 | 6.67 |
| February | 254,104 | 398.98 | 636.88 | 7.11 |

According to a statistical model, the mean wind speed is given by

$$\bar{v}_m = \int_0^\infty v f(v) dv \quad (2)$$

and the standard deviation of wind speed is given by

$$\sigma = \sqrt{\int_0^\infty (v - \bar{v}_m)^2 f(v) dv} \quad (3)$$

where f(v) is the probability density function (pdf) of wind speed $v$. These equations are used to compute theoretical values of mean and standard deviation for a wide variety of statistical functions that are used in various applications. This paper employs yearly mean wind speed and the associated standard deviation of wind speed distribution by using the following equations.

$$\bar{v} = \sqrt[n]{\frac{\sum_{i=1}^{N} f_i v_i^n}{\sum_{i=1}^{N} f_i}} \text{ and } \sigma = \sqrt{\frac{\sum_{i=1}^{N} f_i (v_i - \bar{v})^2}{\sum_{i=1}^{N} f_i}} \quad (4)$$

where $\bar{v}$ is the cubic mean wind speed, $v$ the actual wind speed, N the number of obtained wind speeds, $f_i$ the percentage of different cubic mean wind speeds during a year, $v_i$ the obtained mean wind speed, and n = 3 for obtaining cubic mean wind speed.

There are several probability density functions that can be used to describe the wind-speed variations. The two most common functions are Weibull and Rayleigh functions. While year-to-year variation in annual mean wind speeds remains hard to predict, wind speed variations during a year can be well characterized in terms of a probability distribution function (pdf). Weibull distribution function has been found to give a good representation of the variation in hourly mean wind speed over a year at many typical sites. Weibull distribution function has the form of

$$F(v) = \exp(-(\frac{v}{c})^k) \quad (5)$$

where $F(v)$ is the percentage of time for which the hourly mean wind speed exceeds $v$. Equation (5) is characterized by two parameters, a scale parameter $c$ and a shape parameter $k$ that describes the variability deviated from the mean value. The pdf can be derived by using (5)

$$f(v) = -\frac{dF(v)}{dv} = k\frac{v^{k-1}}{c^k}\exp(-(\frac{v}{c})^k) \quad (6)$$

where $k > 0$, $v > 0$, and $c > 1$. When a location has the scale parameter $c$ of 8, the pdf is shown in Fig. 1. A higher value of $k$, such as 2.5 or 3, indicates a location where the variation of hourly mean wind speed deviated from the annual mean is small. A lower value of $k$, such as 1.5 or 1.2, indicates greater deviation from the mean value. When a location has the shape parameter $k$ of 2, the pdf is shown in Fig. 2. A higher value of $c$, such as 12, indicates a greater deviation from the mean value.

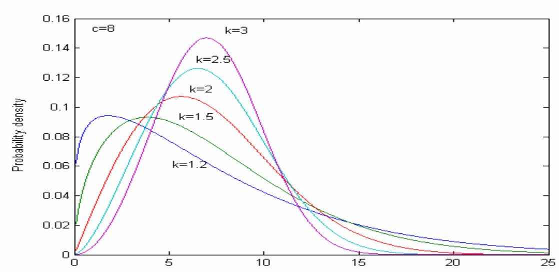


Fig. 1 Weibull distribution density versus wind speed under different constant scale parameter $k$.

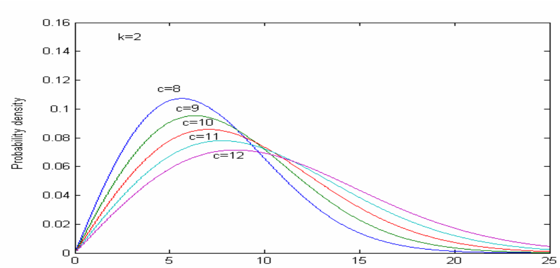


Fig. 2 Weibull distribution density versus wind speed under different constant shape parameter $c$.

Normally, the wind-speed data collected at a location can be used directly to calculate the mean wind speed $\bar{v}$. A good estimate for $c$ from the available data can be obtained quickly from $c = 1.12\bar{v}$, where $1.5 \le k \le 3.0$, by considering the value of $c/\bar{v}$ as a function of $k$, which is given in Fig. 3. For various values of $k$ below unity, the ratio $c/\bar{v}$ decreases rapidly. When the value of $k$ is greater than 1.5 and less than 3 or 4, the value of $c/\bar{v}$ is essentially a constant of 1.12. This means that the scale parameter $c$ is directly proportional to the mean wind speed for this range of $k$ and the mean wind speed is affected mainly by the scale parameter $c$. Most good windfarms have the shape parameter $k$ in this range and this estimate of $c$ in terms of $\bar{v}$ gives wide applications.

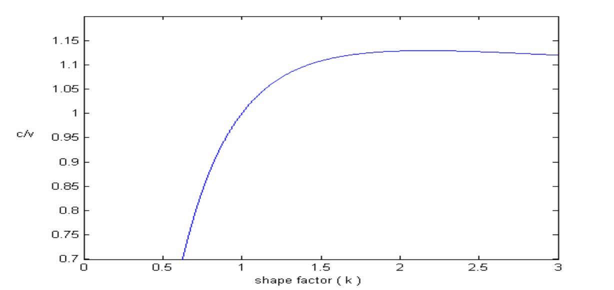


Fig. 3 The value of $c/\bar{v}$ versus shape parameter $k$.

## 3. Tower height and rated wind speed

In the some conditions when simple estimates of the distribution of mean wind speed with height are required, some engineers have favored using the empirical power law model.

$$\frac{v(z_2)}{v(z_1)} = \left(\frac{z_2}{z_1}\right)^{\alpha} \tag{7}$$

where $z_1$ is usually taken as the height of measurement of approximately 10 m, $z_2$ the height at which a wind-speed estimate is derived, and the friction coefficient $\alpha$ determined empirically. Typical values of $\alpha$ are listed in Table 2. The wind speed does not increase with height indefinitely as shown in Fig. 4, and it decreases when the height reaches about 450 m. The wind speed at 450 m height can be four to five times greater than the one near the ground surface.

Equation (7) can be made to fit the observed wind data reasonably well over the range of 10 m to perhaps 100 to 150 m if these are no sharp boundaries in the flow. The value of $\alpha$ varies with height, time of day, season of the year, nature of the terrain, wind speeds, and temperature [6]. The average value of $\alpha$ has been determined by many measurements around the world to be about one-seventh.

The average power output of a wind turbine is a very important parameter of a wind energy system since it determines the total energy production. It is a much better indicator of economics than the rated power, which can easily be chosen at too large a value. We can defined $P_{e,ave}$ as shown

$$P_{e.ave} = P_{eR}(CF) = \eta_o \frac{1}{2} \rho A v_R^{\,3} (CF) \tag{8}$$

where CF is the capacity factor defined as [5]

$$CF = \frac{1}{v_R^3} \int_{v_C}^{v_R} v^3 f(v)dv + \int_{v_R}^{v_F} f(v)dv \tag{9}$$

Table 2 Friction coefficient of various terrain

| Terrain Type | Friction Coefficient α |
|---|---|
| Lake, ocean and smooth hard ground | 0.10 |
| Foot high grass on level ground | 0.15 |
| Tall crops, hedges, and shrubs | 0.20 |
| Wooded country with many trees | 0.25 |
| Small town with some trees and shrubs | 0.30 |
| City area with tall buildings | 0.40 |

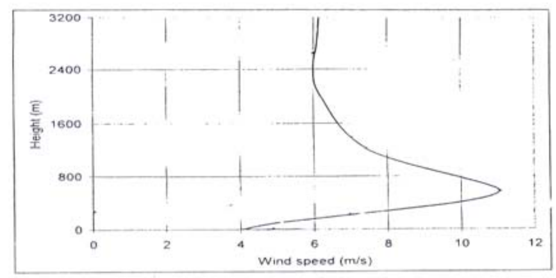


Fig. 4 Wind-speed variation with measured at Merida airport in Mexico.

The expression of CF can be replaced by using equation (9) and derived as shown.

$$CF = \left(\frac{v_C}{v_R}\right)^3 \varepsilon^{-(v_C/c)^k} + \frac{3\Gamma(\frac{3}{k})}{k(\frac{v_R}{c})^3} \times \left[\gamma\left(\left(\frac{v_R}{c}\right)^k, \frac{3}{k}\right) - \gamma\left(\left(\frac{v_C}{c}\right)^k, \frac{3}{k}\right)\right] - \varepsilon^{-(v_F/c)^k} \tag{10}$$

It can be seen that the selected rated wind speed $v_R$ is an important parameter for wind turbine design. For a given windfarm with known $c$ and $k$ parameters, we can select $v_R$, $v_C$ and $v_F$ to maximize the average power, and thereby maximize the total energy production.

According to the most current types of wind turbines, this paper sets $v_C$ to be 4 m/s, sets $v_F$ to be 25m/s, and lets $v_R$ be a variable. If the rated speed is chosen too low, the wind turbine may lose too much energy under higher wind speeds. If the rated speed is selected too high, the wind turbine seldom operates near rated capacity and may lose too much energy under low wind speeds. This means that the average power output may reach a maximum value at a specific value of rated wind speed. This paper proposes a novel analysis using CF for different values of height h and $v_R$ at the same site.

Because the choice of the rated wind speed does not depend on the rated overall efficiency, the air density, or the turbine area, these quantities can be normalized in terms of wind speed, it is convenient to do likewise in normalizing like the following equation (11) by dividing the expression by $c^3$ to get the term $(v_R/c)^3$. A normalized average power $P_n$ is defined as

$$P_n = \frac{P_{e.ave}}{\eta_o(\rho/2)Ac^3} = (CF)(\frac{v_R}{c})^3 \qquad (11)$$

This paper proposes a novel analysis on CF, $P_n$, and the product of both quantities under different values of tower height and rated wind speed.

## 4. A case study

Since the capacity of wind turbine with different hub heights is increased, how to select the optimal type of wind turbines for capturing maximum energy from wind is the most important aspect. This section analyzes a practical case study for capacity selection for wind turbines in Taiwan. This paper uses two parameters, i.e., hub height and rated wind speed, to perform wind generator capacity selection.

The data of wind-speed distribution during one year at the hub height of 65 m of the studied windfarm are listed in Table 3, where V1 is the wind speed range in m/s, V2 the mean wind speed in m/s, and V3 the percentage of wind speed in %. All data in Table 3 are obtained from meteorological observation or the measurement team of Taiwan Power Company (TPC) for over one year. In Table 3, the mean wind speed in column V2 is obtained by use of the average scheme of every wind speed range, and the results are reasonable when the number of measured wind-speed data is quite large. Parameters of Weibull distribution at the selected windfarm under different hub heights are given in Table 4.

This paper uses cubic mean of wind speed ($\overline{v}$) to replace mean wind speed ($v_m$) so as to match the electrical output power by using (11). Friction coefficient $\alpha$ of the studied windfarm is obtained by using the data at lower heights by means of Table 2 and (7). Other values such as scale parameter $c$ and shape parameter $k$ are obtained by using (5) and (6) in order to obtain more accurate results. Since our purpose is to capture maximum power from wind, the manufacturers of wind turbines such as ENERCON, VESTAS, and G.E. are developing various wind turbine types of different capacity including rated wind speed and tower height as listed in Table 5, where the value of $v_R$ is the rated wind speed when the value of ($CF \times P_n$) is the largest one. Table 6 lists the maximum values of capacity factor CF, normalized average power $P_n$, the product of CF and $P_n$, and rated wind speed $v_R$ under different tower heights h.

Fig. 4 shows that the characteristic curves of CF, $P_n$, and $CF \times P_n$ versus $v_R$ of the studied windfarm. The results under four different hub heights, i.e., 30, 45, 65, and 80 m, are compared. It is seen that the maximum value of CF occurs at lower rated wind speed about 4 m/s while the maximum value of $CF \times P_n$ occurs at medium rated wind speed about 12 m/s. The higher the value of rated wind speed, the larger the value of $P_n$. Fig. 4 also shows that the higher the value of h, the value of $P_n$ become smaller. Fig. 5 is obtained by enlarging the $CF \times P_n$ curves of Fig. 4. It is found from Fig. 5 that the higher the value of h, the lower the value of $CF \times P_n$ when the rated speed is less than 13 m/s. When the rated wind speed is higher than 13 m/s, the higher the value of h, the value of $CF \times P_n$ becomes larger. Hence, the value of hub height may have different effects on $CF \times P_n$ under various rated wind speed.

Table 3 Statistical data of annual wind speed v (m/s) at the studied windfarm in Taiwan.

| V1 | V2 | V3 | V1 | V2 | V3 |
|---|---|---|---|---|---|
| v <0.5 | 0.25 | 3.05 | 12.5-13.5 | 13.0 | 3.54 |
| 0.5-1.5 | 1.0 | 3.72 | 13.5-14.5 | 14.0 | 3.20 |
| 1.5-2.5 | 2.0 | 7.42 | 14.5-15.5 | 15.0 | 2.56 |
| 2.5-3.5 | 3.0 | 9.89 | 15.5-16.5 | 16.0 | 1.82 |
| 3.5-4.5 | 4.0 | 10.28 | 16.5-17.5 | 17.0 | 1.21 |
| 4.5-5.5 | 5.0 | 10.34 | 17.5-18.5 | 18.0 | 0.73 |
| 5.5-6.5 | 6.0 | 9.58 | 18.5-19.5 | 19.0 | 0.48 |
| 6.5-7.5 | 7.0 | 8.15 | 19.5-20.5 | 20.0 | 0.22 |
| 7.5-8.5 | 8.0 | 5.89 | 20.5-21.5 | 21.0 | 0.16 |
| 8.5-9.5 | 9.0 | 4.94 | 21.5-22.5 | 22.0 | 0.11 |
| 9.5-10.5 | 10.0 | 4.03 | 22.5-23.5 | 23.0 | 0.04 |
| 10.5-11.5 | 11.0 | 4.21 | 23.5-24.5 | 24.0 | 0.05 |
| 11.5-12.5 | 12.0 | 4.21 | v > 24.5 | 25.0 | 0.15 |

Table 4 Parameters of Weibull distribution at the studied windfarm.

| h (m) | $v_m$ (m/s) | $\overline{v}$ (m/s) | σ | k | c |
|---|---|---|---|---|---|
| 30 | 6.26 | 8.30 | 4.41 | 1.9639 | 9.3620 |
| 45 | 6.82 | 9.03 | 4.80 | 1.9631 | 10.1854 |
| 65 | 7.37 | 9.76 | 5.19 | 1.9622 | 11.0086 |
| 70 | 7.48 | 9.91 | 5.27 | 1.9626 | 11.1795 |
| 80 | 7.69 | 10.20 | 5.42 | 1.9637 | 11.5051 |
| 100 | 8.06 | 10.68 | 5.68 | 1.9626 | 12.0490 |

Table 5 Specifications of various wind power generation systems.

| Manufacturers | Wind speed (m/s) | | | Wind turbine | Generator | Tower heighth (m) |
|---|---|---|---|---|---|---|
| | $v_C$ | $v_R$ | $v_F$ | Blade diameter(m) | Rated power(kW) | |
| MICON | 4 | 14 | 25 | 30 | 200 | 30 |
| NEPC-MICON | 4 | 15 | 25 | 31 | 400 | 30.5 |
| ENERCON-E40 | 2.5 | 13 | 25 | 44 | 600 | 46 |
| VESTAS-V47 | 5 | 15 | 25 | 35 | 660 | 47 |
| VESTAS-V52 | 4 | 17 | 25 | 52 | 850 | 55 |
| GE-1.5S | 4 | 14 | 25 | 70.5 | 1500 | 64.7 |
| VESTAS-V88 | 3.5 | 13 | 24 | 82 | 1650 | variable |
| NEG-NICON | 3.5 | 16 | 25 | 60 | 1650 | 70 |
| VESTAS-V66 | 4 | 16 | 25 | 62 | 1750 | 60 |
| VESTAS-V80 | 4 | 16 | 25 | 76 | 1800 | 60 |
| ZEPHYROS-Z72 | 3 | 16 | 25 | 71.2 | 2000 | 65 |
| GAMESA EOLICA-G80 | 4 | 16 | 25 | 70 | 2000 | 67 |
| GE-2.3 | 3 | 14 | 25 | 94 | 2300 | 100 |
| GE-2.5 | 3.5 | 15 | 25 | 88 | 2500 | 85 |
| GE-2.7 | 3.5 | 16 | 25 | 84 | 2400 | 70 |

Table 6 Maximum values of capacity factor, normalized average power, and rated wind speed under different tower heights.

| h (m) | $CF_{max}$ | $P_{N,max}$ | $(CF\times P_N)_{max}$ | $v_R$ |
|---|---|---|---|---|
| 30 | 0.8274 | 1.3264 | 0.3489 | 11.70 |
| 45 | 0.8495 | 1.2969 | 0.3472 | 12.68 |
| 65 | 0.8651 | 1.2491 | 0.3419 | 13.61 |
| 70 | 0.8676 | 1.2368 | 0.3403 | 13.80 |
| 80 | 0.8718 | 1.2144 | 0.3369 | 14.15 |
| 100 | 0.8763 | 1.1646 | 0.3293 | 14.71 |

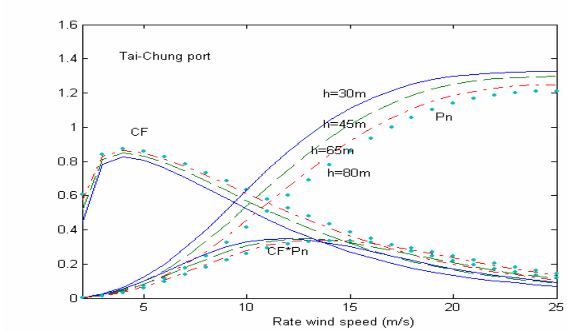

Fig. 4 Characteristic curves of CF, $P_n$, and CF×$P_n$ versus $v_R$ of the studied case.

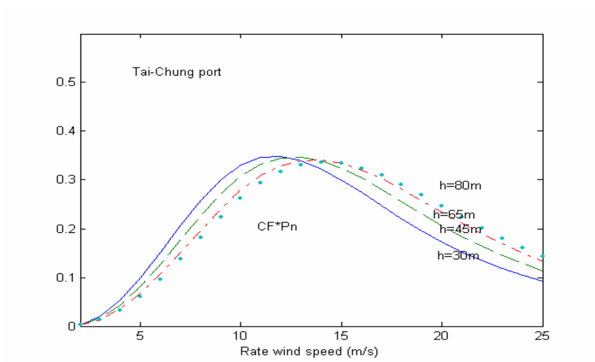

Fig. 5 Characteristic curves of CF×$P_n$ versus $v_R$ under various hub heights of the studied case.

Fig. 6 shows that the characteristic curves of CF, $P_n$, and CF×$P_n$ versus $v_R$ under a fixed value of $c$ ($c$ = 10) of the studied windfarm. The results of the same five different hub heights are also compared. It is discovered that the characteristic curves shown in Fig. 6 are similar to the ones of Fig. 4 except that the difference on the curves of CF, $P_n$, and CF×$P_n$ are obvious. Fig. 7 is obtained by enlarging the CF×$P_n$ curves of Fig. 6. Comparing the curves shown in Figs. 7 and 5, it is found that the maximum value of CF×$P_n$ in Fig. 7 is higher than the one shown in Fig. 5. Hence, the fixed value of $c$ may increase the maximum value of CF×$P_n$.

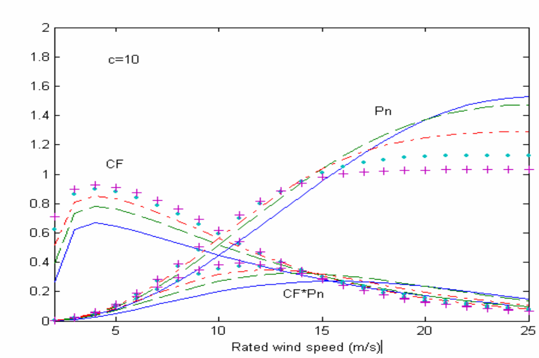

Fig. 6 Characteristic curves of CF, $P_n$, and CF×$P_n$ versus $v_R$ under the same value of c of the studied case.

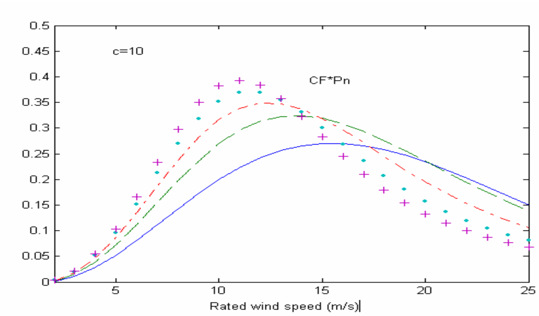

Fig. 7 Characteristic curves of CF×$P_n$ versus $v_R$ under the same value of c of the studied case.

Fig. 8 shows that the characteristic curves of CF, $P_n$, and CF×$P_n$ versus $v_R$ under a fixed value of $k$ ($k$ = 2) of the studied windfarm. It is observed that the characteristic curves shown in Fig. 8 are also similar to the ones of Fig. 6 except that the difference on CF, $P_n$, and CF×$P_n$ are more obvious. Fig. 9 is obtained by enlarging the CF×$P_n$ curves shown in Fig. 8. Comparing the curves shown in Figs. 9 and 7, it is found that the maximum value of CF×$P_n$ in Fig. 9 for each hub height is very close. Hence, the fixed value of $k$ may keep the maximum value of CF×$P_n$ to have nearly the same value under different hub heights.

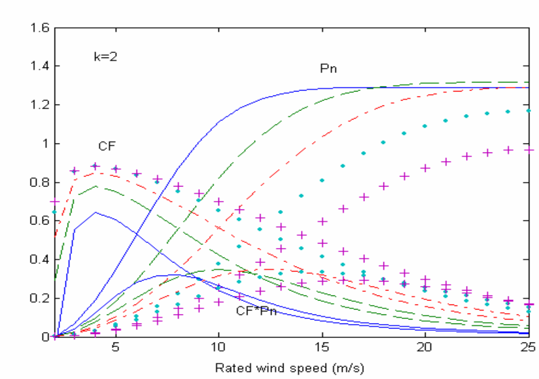

Fig. 8 Characteristic curves of CF, $P_n$, and CF×$P_n$ versus $v_R$ under the same value of k of the studied case.

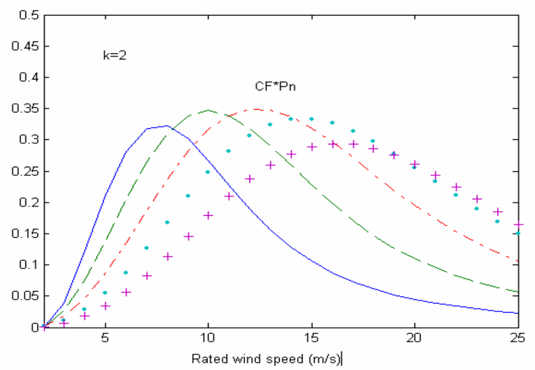


Fig. 9 Characteristic curves of CF×$P_n$ versus $v_R$ under the same value of k of the studied case.

Fig. 10 shows that the characteristic curves of CF, $P_n$, and CF×$P_n$ versus $v_R$ under variable values of *c* (*c* = 14-6) and *k* (*k* = 1.2-2.8) of the studied windfarm. The results of the same five different hub heights are also compared. It is observed that the characteristic curves shown in Fig. 10 are quite different from the ones of Figs. 4, 6, and 8. Fig. 11 is obtained by enlarging the CF×$P_n$ curves of Fig. 10. It is found from Fig. 11 that the maximum value of CF×$P_n$ for each hub height occurs at different rated wind speeds. Hence, the variable values of *c* and *k* cannot maintain the maximum value of CF×$P_n$ for a specified rated wind speed range under different hub heights.

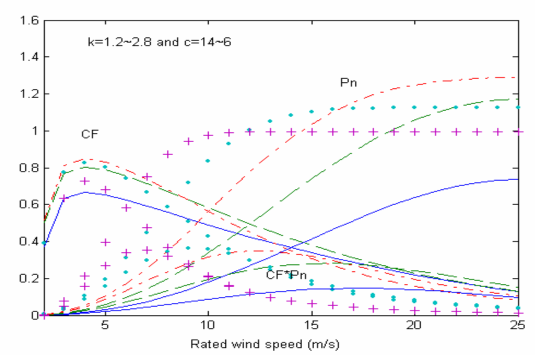


Fig. 10 Characteristic curves of CF, $P_n$, and CF×$P_n$ versus $v_R$ under variable values of c and k of the studied case.

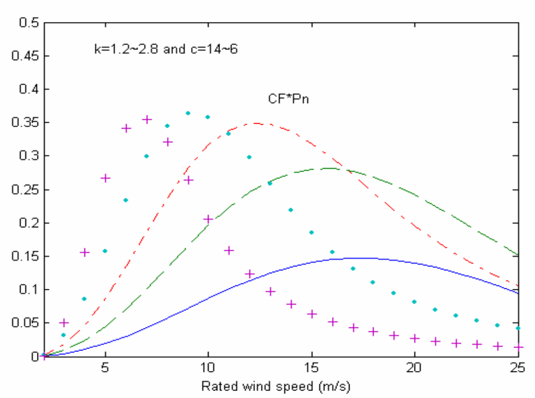


Fig. 11 Characteristic curves of CF×$P_n$ versus $v_R$ under variable values of c and k of the studied case.

## 5. Conclusion

This paper proposes two variable parameters, the hub height and rated wind speed capacity, to determine the location for installation of a wind turbine. Two constant parameters, cut-in wind speed of 4 m/s and cut-out wind speed of 25 m/s, are properly selected for wind turbine specification to obtain the accurate simulation results. This paper has presented comparative results under various values of capacity factor *CF*, normalized average power $P_n$, and $CF \times P_n$ under different hub heights.

It is found from the analyzed results of the studied windfarm that the capacity factor *CF*, the normalized average power $P_n$, and $CF \times P_n$ are not effected by a solely parameter. Normalized average power $P_n$ curve always becomes smaller when hub height becomes larger. The maximum value of CF×$P_n$ may occur at different rated wind speed range under different hub heights and both parameters *k* and *c* may effectively affect the maximum value and the variation of CF×$P_n$. Consequently, when selection of good sites for wind turbines installation, suitable scale parameter *c* and shape parameter *k*, and wind turbine capacity are identically important.

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