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AN IMPROVED NONLINEAR CONTROL FOR PEM FUEL CELLS

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ABSTRACT This paper presents a dynamic model of Polymer Electrolyte Membrane Fuel Cells (PEM FCs), and constructs a nonlinear control strategy for PEM FCs by using the exact linearization approach. By introducing additional outputs, the original multiple-input single-output (MISO) nonlinear model of PEM FC is transformed into a multiple-input multiple-output (MIMO) system so that the exact linearization approach can be directly utilized. The reformer is avoided in the control design for economical objective. Simulation results show that PEM fuel cells with nonlinear control have better transient and steady-state performances than linear controls.

KEY WORDS: Polymer Electrolyte Membrane Fuel Cells, nonlinear dynamic model, exact linearization.

I. INTRODUCTION

Fuel cells are electrochemical devices that convert chemical energy to electricity and thermal energy. At the beginning of 21st century, fuel cells, as a renewable energy source, is considered as one of the most promising alternative sources of electric power because they appeal to environment with nonpolluting energy generation and offer a wide size range application with high efficiency, such as from portable electronics to utility power plants. In addition to the fuel cell stack itself, a fuel cell system includes a fuel processor unit or reformer and subsystems to manage air, water, thermal energy and power. A PEM FCs produce water as by product and operating at low temperature and having many benefits, such as safe operational modes, lower maintenance costs due to less moving parts, fast start up, and activate a wide scope of applications in power systems [1,2,3]. PEM FCs are normally installed in distribution systems close to the loads. They often experience large and frequent disturbances due to load changes. The existing control approaches used for PEM fuel cells are based on linear models which are linearized at a specific operating point [10]. Due to large range of disturbances, such linear control approaches have difficulties to achieve satisfactory performances. [13] proposed a nonlinear control for PEM FCs by using exact linearization approach through a reformer which generates hydrogen and oxygen as the inputs to the nonlinear controller.

In this paper the nonlinear dynamic model for PEMFCs proposed in [13] is directly utilized to design a new control strategy for fuel cells without the use of reformer for the economical consideration. The exact linearization control approach transforms the original nonlinear dynamic model into a linear model by a diffeomorphism mapping, and then transforms back to the original nonlinear state-space the control law obtained from the exactly transformed linear system by linear optimal control approaches. The control law obtained from the exact linearization without the reformer is simpler than the one with reformer, and is expected to be more robust in the presence of large disturbances in a big range.

II. PEM FUEL CELL DYNAMIC MODEL

The following assumptions are applied to construct the simplified dynamic model for PEM FCs [3].

· The amount of Nitrogen in the cathode is constant in the FC model's state variable equation.
· The oxygen flow rate is determined by Nitrogen-oxygen flow ratio(79/21).
· The stack temperature is regulated at $80^{\circ}C$ by using an independent cooling system [4,5].
· The Nernst's equation is applied.

The dynamic nonlinear model developed in this paper is based on the fuel cell models provided by the Department of Energy (DoE) [2] and Ref.[6] are referred in this paper.

A. PEM FC Output Voltage Equation

According to the Nernst's equation and ohmic's law, the cell voltage equation is given as

$$V = N(E^0 + \frac{RT}{2F}\left\{\frac{pH_2(O_2/P_{std})^{0.5}}{pH_2O}\right\}) - L \quad (1)$$

where:
V : stack output voltage;
N : number of cells in the stack;
E^0 : cell open circuit voltage;
T : operating temperature;
L : voltage losses;

pH_2, pO_2, and pH_2O: the partial pressures of each gas inside cell;
R : gas constant (8.3144 J/mole*k)
F : Faraday's constant (96439 C/mole)
P_{std} : the standard pressure (101325 Pa)

The voltage losses L is given by

$$L = (i+i_n)r + a\ln(\frac{i+i_n}{i_o}) - b\ln(1-\frac{i+i_n}{i_l})$$

(2)

where
i : the output current density;
i_n : the internal current density to internal current losses;
i_o : the exchange current density related to activation losses;
i_l : the limiting current density related to concentration losses;
r : the area specific resistance related to resistive losses;
a, b : constants.

B. State Equations

Using the same derivation in [13], we obtain the following state equation:

$$\frac{d}{dt}pH_2 = \frac{RT}{V_A}\left[H_{2\_in} - 2K_rA_ci - (H_{2\_in} - 2K_rA_ci)\frac{pH_2}{p_{op}}\right]$$
$$\frac{d}{dt}pO_2 = \frac{RT}{V_C}\left[O_{2\_in} - K_rA_ci - (O_{2\_in} + H_2O_{C\_in} - K_rA_ci)\frac{pO_2}{p_{op}}\right]$$
$$\frac{d}{dt}H_2O_C = \frac{RT}{V_C}\left[2K_rA_ci - (O_{2\_in} + 2K_rA_ci)\frac{pH_2}{p_{op}}\right]$$

(3)

C. MIMO Nonilnear Dynamic Model of PEM FC

Consider the following multiple-input single-output (MISO) nonlinear system:

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \qquad i = 1,2,\ldots,m$$
$$y = h(x)$$

(4)

where $x \in X \subset R^n$ is the state, $u \in U \subset R^m$ is the input or control vector and $y \in Y \subset R^p$ is the output vector of the system.

Equations (3) and (1) imply the following nonlinear dynamic system model of PEM FC:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}\frac{RT}{V_A}(1-\frac{x_1}{p_{op}})\\ 0\\ 0\end{bmatrix}u_1 + \begin{bmatrix}0\\ \frac{RT}{V_C}(1-\frac{x_2}{p_{op}})\\ -\frac{RT}{V_Cp_{op}}x_3\end{bmatrix}u_2 + \begin{bmatrix}\frac{RT}{V_Ap_{op}}(-2K_rA+2K_rA_c\times x_1)\\ \frac{RT}{V_Cp_{op}}(-K_rA+2K_rA_c\times x_2)\\ \frac{RT}{V_Cp_{op}}(2K_rA-2K_rA_c\times x_3)\end{bmatrix}u_3$$

(4a)

$$y = N(E^0 + \frac{RT}{2F}\left\{\frac{pH_2(O_2/P_{std})^{0.5}}{pH_2O}\right\}) - L$$

(4b)

where

$$x = [pH_2 \quad pO_2 \quad pH_2O_C]^T$$
$$u = [H_{2\_in} \quad O_{2\_in} \quad i]$$
$$y = V$$

In the above nonlinear model, because the number of outputs is less than that of inputs and thus the decoupling matrix for exact linearization is not square, exact linearization regarding to multiple-input multiple output (MIMO) systems can not be directly applied. The problem of non-square can be solved by using an extended system [8, 11]. In other words, additional outputs are chosen and added in such a way that a square system appears as a result and the decoupling matrix is nonsingular. One possible way to make the decoupling matrix square and nonsingular, is to define *m-p* extra states:

$$\dot{x}_{n+1} = u_{i_1}$$
$$\vdots$$
$$\dot{x}_{n+m-p} = u_{i_{m-p}}$$

(5)

and to append the output vector with

$$y_{p+1} = x_{n+1}$$
$$\vdots$$
$$y_m = x_{n+m-p}$$

With the addition of two extra states x_4 and x_5 and two extra outputs, the MISO nonlinear system Eq. (4) can be converted into a MIMO system described by Eq. (6a) and Eq. (6b) below so that the decoupling matrix is nonsingular.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\end{bmatrix} = \begin{bmatrix}\frac{RT}{V_A}(1-\frac{x_1}{p_{op}})\\ 0\\ 0\\ 1\\ 0\end{bmatrix}u_1 + \begin{bmatrix}0\\ \frac{RT}{V_C}(1-\frac{x_2}{p_{op}})\\ -\frac{RT}{V_C}\frac{x_3}{p_{op}}\\ 0\\ 1\end{bmatrix}u_2 + \begin{bmatrix}\frac{RT}{V_A}(-2K_rA_c+2K_rA_c\frac{x_1}{p_{op}})\\ \frac{RT}{V_C}(-K_rA_c+K_rA_c\frac{x_2}{p_{op}})\\ \frac{RT}{V_C}(2K_rA_c-2K_rA_c\frac{x_3}{p_{op}})\\ 0\\ 0\end{bmatrix}u_3$$

(6a)

$$\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} = \begin{bmatrix}x_4\\ x_5\\ V\end{bmatrix} = \begin{bmatrix}h_1(x)\\ h_2(x)\\ h_3(x)\end{bmatrix}$$

(6b)

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D. Exact Linearization of MIMO Nonlinear System Model for PEM FC

An important property of a nonlinear system is its relative degree. In essence, the relative degree represents the number of times the output y must be differentiated with respect to time so that the input can be expressed explicitly. The relative degree for each input u_1, u_2 and u_3 in Eq. (6) is equal to 1 because the corresponding smooth vector field f in Eq. (4) is 0.

The approach to obtain the exact linearization of the MIMO systems is to differentiate the output y_i of Eq. (6) until the input shows up [9, 10]. By differentiating Eq. (6), we have:

$$\dot{y}_j = L_f h_j + \sum_{i=1}^{m} (L_{g_i} h_i) u_i \tag{7}$$

where L_f and L_g represent *Lie derivatives* of the smooth scalar function of $h(x)$ with respect to $f(x)$ and $g(x)$. If $L_g h_j(x) = 0$ for all *i*, then the inputs do not show up and we have to differentiate again. Assuming that r_j is the smallest integer such that at least one of the inputs appears in $y_j^{(r_j)}$, then

$$y_j^{(r_j)} = L_f^{r_j} h_j + \sum_{i=1}^{m} (L_{g_i} L_f^{r_j - 1} h_j) u_i \tag{8}$$

Where $L_{g_i} L_f^{r_j-1} h_j(x) \neq 0$ for at least one *i*.

Re-performing the above procedure for each y_j, we can obtain a total of *m* equations in the above form , which can be written compactly as

$$\begin{bmatrix} y_1^{r_1} \\ \cdots \\ \cdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x) \\ \cdots \\ \cdots \\ L_f^{r_m} h_m(x) \end{bmatrix} + E(x) \begin{bmatrix} u_1 \\ \cdots \\ \cdots \\ u_m \end{bmatrix} \tag{9}$$

where the $m \times m$ matrix *E(x)* is defined as

$$E(x) = \begin{bmatrix} L_{g_1} L_f^{r_1 - 1} h_1 & \cdots & \cdots & L_{g_m} L_f^{r_1 - 1} h_1 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ L_{g_1} L_f^{r_1 - 1} h_m & \cdots & \cdots & L_{g_m} L_f^{r_m - 1} h_m \end{bmatrix} \tag{10}$$

The matrix *E(x)* is called the *decoupling matrix* for the MIMO system. If E(x) is nonsingular, the nonlinear state feedback control law can be obtained

$$U = -E^{-1} \begin{bmatrix} L_f^{r_1} h_1(x) \\ \cdots \\ \cdots \\ L_f^{r_m} h_m(x) \end{bmatrix} + E^{-1} \begin{bmatrix} v_1 \\ \cdots \\ \cdots \\ v_m \end{bmatrix} \tag{11}$$

Based on Eq. (10), the decoupling matrix of MIMO system model for PEM FC is

$$E(x) = \begin{bmatrix} L_{g_1} L_f^{o} h_1 & \cdots & \cdots & L_{g_3} L_f^{0} h_1 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ L_{g_1} L_f^{0} h_3 & \cdots & \cdots & L_{g_3} L_f^{0} h_3 \end{bmatrix}$$

By the calculation of Lie derivatives, E(x) can be further substituted in Eq. (11).

Furthermore, since the Lie derivative of a scalar function *h(x)* with respect to a vector function *f(x)* is zero, the nonlinear feedback control law can be rewritten into Eq. (14) :

$$U = E^{-1}(x) \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{14}$$

where $E^{-1}(x)$ is given by Eq. (13).

Substituting Eq. (13) into Eq. (9) results in a linear differential relation between the output y and the new input v.

$$E(x) = \frac{NR^2T^2}{2F} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{V_A}(\frac{1}{x_1} - \frac{1}{p_{op}}) & \frac{1}{2V_C}\left(\frac{1}{x_2} - \frac{1}{p_{op}}\right) + \frac{1}{V_C p_{op}} & K_r A_C \left(\frac{2}{V_A}(-\frac{1}{x_1} + \frac{1}{p_{op}}) + \frac{1}{2V_C}(-\frac{1}{x_1} + \frac{1}{p_{op}}) - \frac{1}{2V_C}(\frac{2}{x_3} - \frac{1}{p_{op}}) \right) \end{bmatrix} \tag{12}$$

$$E^{-1}(x)=\frac{2F}{NR^2T^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \dfrac{-\frac{1}{V_A}(\frac{1}{x_1}-\frac{1}{p_{op}})}{C} & -\dfrac{\frac{1}{2V_C}\left(\frac{1}{x_2}-\frac{1}{p_{op}}\right)+\frac{1}{V_C p_{op}}}{C} & \dfrac{1}{C} \end{bmatrix}$$

$$\left(C=K_r A_C\left(\frac{2}{V_A}(-\frac{1}{x_1}+\frac{1}{p_{op}})+\frac{1}{2V_C}(-\frac{1}{x_1}+\frac{1}{p_{op}})-\frac{1}{2V_C}(\frac{2}{x_3}-\frac{1}{p_{op}})\right)\right)$$

(13)

$$u_3=\frac{2F}{NR^2T^2}\left[\frac{-\frac{1}{V_A}(\frac{1}{x_1}-\frac{1}{p_{op}})}{C}v_1--\frac{\frac{1}{2V_C}\left(\frac{1}{x_2}-\frac{1}{p_{op}}\right)+\frac{1}{V_C p_{op}}}{C}v_2+\frac{1}{C}v_3\right]$$

(19)

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\end{bmatrix}=\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}$$

(15)

Since v_1 and v_2 are the same as u_1 and u_2, only v_3 can be used for tracking control, so the new control input is obtained

$$v_3=\dot{y}_{3ref}-k_{31}e_3$$

(16)

where the tracking error $e_3=y_3-y_{3ref}$. In this form of the nonlinear control, a tracking error may exist due to parameter uncertainty. To obtain more robust control, an integral control term is added to Eq. (16) as so in [12].

$$v_3=\dot{y}_{3ref}-k_{31}e_3-k_{32}\int e_3 dt$$

(17)

Then the output error dynamics from Eq. (16) is described as follows

$$\ddot{e}_3+k_{31}\dot{e}_3+k_{32}e_3=0$$

(18)

The asymptotic tracking is achieved by selecting the gains k_{11} and k_{12} appropriately to place the desired closed-loop system poles located in the left hand plane. These control gains k_{11} and k_{12} are calculated by the desired poles which are located at -200 ± j20. Also, substituting Eq. (17) into Eq. (14), yields the control law for u_1 which is given above in Eq. (19).

The control law u_3 is fed backed to compare with the stack current calculated from load current, affecting inlet flow rates of hydrogen and oxygen keep supplying to PEM FCs . Figure 2 shows the block diagram of nonlinear PEM FCs model with exact linearization control.

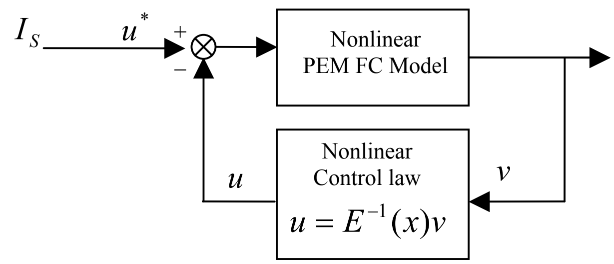

Fig.2. Block diagram of nonlinear PEM FCs model with exact linearization control

III. SIMULATION RESULTS

To demonstrate the performance of the proposed nonlinear control law, the system is simulated using the simplified models directly connected to a load consisted of R and L. The linear conventional PI controller is used for comparison purposes.

Model parameters used in our simulation are given as follows:

- Cell active area : $A_c=136.7\text{cm}^2$
- Volume of anode : $V_a=6.495\text{cm}^2$
- Volume of cathode : $V_c=12.96\text{cm}^2$
- Number of cells : $N=150$
- Operating cell temperature : 338.5 K
- Reference potential : $E_0=1.229\text{V}$
- Operating condition
  $H_{2in}=3{,}664$ ml/min
  $N_{2in}+O_{2in}=11{,}548$ ml/min
- Load resistance : 10 Ω
- Load reactance : 10 mH

The simulation has been conducted in SIMULINK environment. Figure 3 shows the design of PEM FC dynamic model with the nonlinear control, DC to DC converter and DC/AC inverter in SIMULINK.

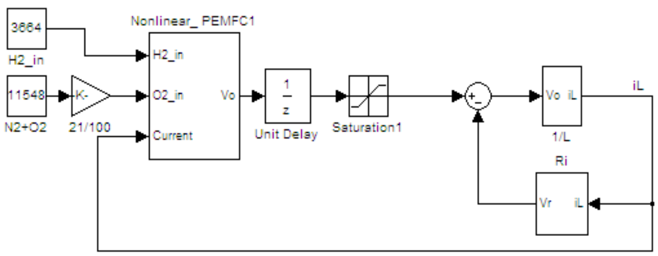

Fig. 3 PEM FCs with Nonlinear control connected to R-L load in SIMULINK

Figure 4 shows the dynamic model of PEM fuel cell with the implementation of nonlinear control. Inputs to the fuel cell are hydrogen, oxygen and load current, while the output is the voltage.

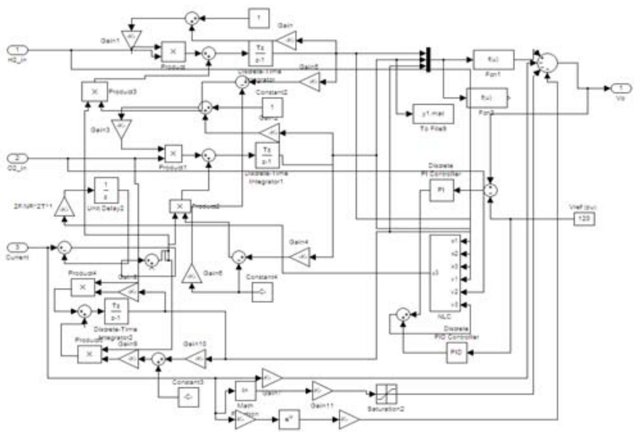

Fig. 4 PEM FC Dynamic Model with Nonlinear Control

To test the transient behaviors of fuel cell with nonlinear control, the value of resistance on the R-L load is changed from 10 Ω to 5Ω and 10mH to 5mH at time *t=1.0* second and from 5 Ω to 10Ω and 5mH to 1o mH at time *t=1.5* second. Figure 5 shows the fuel cell output voltage for the load step change. The transient response of fuel cell output voltage with nonlinear control is more stable than that of conventional PI controller.

Figure 6 shows the changes of fuel cell stack current for the load step change. In figure 6, the transient response of fuel cell stack current with nonlinear control can catch the reference than that of conventional PI controller for the load change.

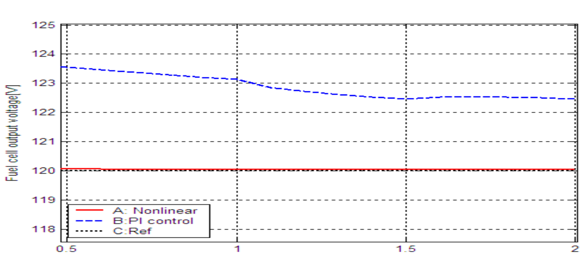

Fig. 5. Fuel cell output voltage for the load step change

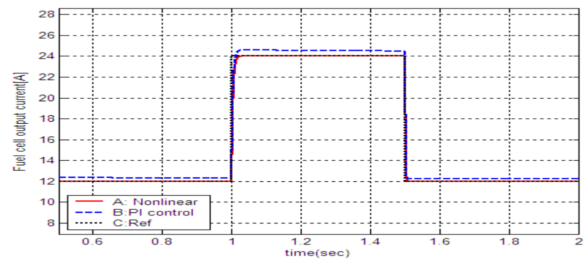

Fig.6 Fuel cell stack current for the load step change

Figure 7 shows the comparison of fuel cell power demand between nonlinear control and PI control, similarly to Fig 6, nonlinear control can catch the reference than that of conventional PI controller.

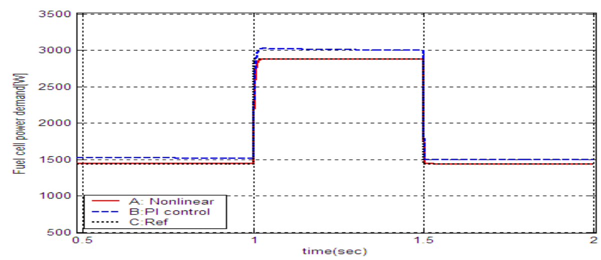

Fig. 7 Fuel cell power demand for the load step change

According to Fig.5, Fig.6 and Fig.7, it is obvious that the transient response of fuel cell with nonlinear control is better than that of conventional PI controller under the disturbance caused by the load step change. Fuel cell voltage and output current have a very good transient behavior when using nonlinear control.

IV. CONCLUSION

A design of nonlinear control for PEMFC by the exact linearization approach has been economically improved when only considering the feedback current as the control input. By introducing extra states and outputs, the original multiple-input single-output (MISO) PEMFC nonlinear system model is converted to a multiple-input multiple-output (MIMO) system model. By adding an integral control term to the state feedback control law, the steady steady-state error due to parameter uncertainty can be reduced. The control performance of the exact linearization control law has been tested. The results show that the fuel cells with the nonlinear control have very good transient behaviors for disturbances. The positive impact of PEM fuel cells with nonlinear control on the power systems transients needs to be conducted in further study.

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