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VALUE BASED TRANSMISSION EXPANSION PLANNING OF HYDROTHERMAL SYSTEMS WITH MULTIPLE SCENARIOS

Gerson C. Oliveira,PSR, Rua Voluntarios da Pátria 45 / 1401, Rio de Janeiro, Brasil, gerson@psr-inc.com
Silvio Binato, Rua Voluntarios da Pátria 45 / 1401, Rio de Janeiro, Brasil, silvio@psr-inc.com
Mario Pereira, Rua Voluntarios da Pátria 45 / 1401, Rio de Janeiro, Brasil, mario@psr-inc.com
Niko A. Iliadis, Rua Voluntarios da Pátria 45 / 1401, Rio de Janeiro, Brasil, niko@psr-inc.com

ABSTRACT
The network loading of a hydrothermal system is highly variable due to several factors. Hydro plants are usually located in different river basins, usually far from load centers. Diversity of streamflows along these basins lead to distinct generation dispatches, sometimes inverting energy interchanges between hydro based exporting regions, and also redistributing the power supplied to load centers. Transmission expansion planning criteria must reflect the trade-off between investments in transmission, inducing more competition in generation, at the expense of increasing customer costs and a higher reliability level due to these investments. Measuring these trade-offs for hydrothermal systems requires taking into account multiple dispatch scenarios, and assessing network reliability for each scenario by a contingency analysis for each circuit outage. The network design problem that aims at choosing the best reinforcements among many candidate routes and voltage levels must therefore represent the transmission constraints for relevant dispatch scenarios and circuit contingencies. In this work a mixed integer disjunctive model is extended so as to deal with this problem minimizing the sum of investment costs and network reliability worth, measured by average interruption costs due to contingencies. Real world case system applications show that by means of a judicious choice of scenarios and contingencies, despite the increase of problem size the model is applicable, achieving a balanced choice between network reliability and investment requirements.

KEY WORDS
combinatorial optimization, transmission expansion planning, network reliability

1. Introduction

The problem of determining the optimal set of candidate circuit additions for transmission network so as to supply the forecasted loads is usually formulated as a mixed nonlinear program. The nonlinearity is due to constraints related to the linearized power flow equations, where bus voltage angle variables are multiplied by circuit investment binary decision variables. The system generation is supposed capable of supplying the forecasted load, and candidate circuits are informed for all possible rights-of-way. The linearized power flow equations are usually used in planning studies of high voltage meshed networks, providing good approximations for the circuit flows, and avoids the need to iteratively solve the non-linear power flow equations. Inequality constraints are simple upper bounds on circuit flows.

In a previous work [1] a mixed integer disjunctive formulation was presented, where the nonlinear constraints are avoided by using a disjunctive form to which they are equivalent. The standard disjunctive formulation suffers from bad conditioning due to the use of large penalties in the disjunctive constraints. By using "optimal" penalty factors and a tighter representation of power flows on candidate circuits, a robust formulation with improved computational performance was achieved, and optimal solutions of moderate sized real world problems were found and proved for the first time. The resulting disjunctive model is solved by "branch-and-bound" (B&B).

In hydrothermal systems, the network design criterion should measure the benefits of increasing power interchanges through the network when different dispatch scenarios are considered; on the other hand transmission constraints related to single circuit contingencies also affect the power flows among regions, and differently along the load curve. Therefore, the network constraints must be represented for single circuit contingencies and relevant dispatch scenarios, in order to reflect likely operating points along the load curve.

Although the problem size (variables and constraints) grows linearly with the number of contingencies, scenarios and load levels, the number of binary variables remains the same since they are related to the transmission circuit candidates. Therefore, the combinatorial nature of the problem remains unaltered: only the size of the linear relaxation subproblem of the B&B algorithm increases.

The following notation will be used throughout this work:
n is the number of nodes (busses);
m is the number of candidate circuits (branches);
Ω^0_i is the set of existing circuits connected to bus i, i=1,n;

Ω_i^{+} is the set of candidate circuits connected to bus i, i=1,n;
$\Omega_i = \Omega^0_i \cup \Omega_i^{+}$;
f is the vector of circuit flows (existing and candidates);
f^{max} is the vector of circuit capacities (existing and candidates);
g is the vector of bus generations;
d is the vector of bus active loads;
θ is the vector of bus voltage angles (in radians);
r is the vector of bus load curtailments;
x is the binary vector of investments on candidate circuits;
c is the vector of candidate circuit annualized costs;
γ is the vector of circuit susceptances (the inverse of the reactance);
M is the penalty vector of candidate circuits.

The work is organized as follows. In section II the classical non-linear formulation is presented. Section III reviews the standard disjunctive formulation, and presents the alternative formulation. Section IV presents the extension for contingencies and multiple scenarios, applied for transmission planning of hydrothermal systems using a value based expansion criterion. A real world problem instance is solved in Section V and results are shown. Section VI concludes; also future works are discussed.

2. Formulation of the base case transmission expansion problem

The classical non-linear formulation [2] representing the linearized power flow equations, bounds on circuit flows, and integrality constraints for investment variables is shown below.

$\text{Min}_{\{x,f,g,\theta\}}\ c\,x$
s.t.

(power node balance equation – first Kirchoff's law)

$$\sum_{k=(i,j)\ j\in\Omega_i} f_k = d_i - g_i \ ,\quad i=1,n$$

(2nd Kirchoff's law for existing circuits)

$$f_k - \gamma_k\,(\theta_i - \theta_j) = 0 \ ,\quad k=(i,j)\ ,\ j\in\Omega_i^0\ ,\quad i=1,n$$

(2nd Kirchoff's law for candidate circuits)

$$f_k - x_k\,\gamma_k\,(\theta_i - \theta_j) = 0\ ,\ k=(i,j)\ ,\ j\in\Omega_i^+\ ,\quad i=1,n$$

(existing circuit flow upper and lower bounds)

$$-f_k^{max} \le f_k \le f_k^{max}\ ,\ k=(i,j)\ ,\quad j\in\Omega_i^0\ ,\quad i=1,n$$

(candidate circuit flow upper and lower bounds)

$$-f_k^{max}\ x_k \le f_k \le f_k^{max}\ x_k\ ,\ k=(i,j)\ ,\ j\in\Omega_i^+\ ,\ i=1,n$$

(integrality constraints of investment variables)

$$x \in \{0,1\}^m$$

Note that a non-linearity appears due to the product of variables θ and x in the 2nd Kirchoff's law for candidate circuits. Consider the 2nd Kirchoff's law for a candidate circuit k: if $x_k=0$, the corresponding flow must be null, while if $x_k=1$, equality is enforced, as required. This is a mixed integer non-linear program, not amenable to classical nonlinear optimization techniques.

3. The disjunctive mixed integer formulation of the transmission expansion problem

The non-linear constraints of the classical formulation are avoided by using a disjunctive form to which they are equivalent. The standard disjunctive mixed integer model is formulated as follows:

$\text{Min}_{\{x,f,\theta\}}\ c\,x$
s.t.

(power node balance equation – first Kirchoff's law)

$$\sum_{k=(i,j)\ j\in\Omega_i} f_k = d_i - g_i \ ,\quad i=1,n$$

(2nd Kirchoff's law for existing circuits)

$$f_k - \gamma_k\,(\theta_i - \theta_j) = 0 \ ,\quad k=(i,j)\ ,\ j\in\Omega_i^0\ ,\quad i=1,n$$

(2nd Kirchoff's law for candidate circuits, disjunctive form)

$$-M_k(1-x_k) \le f_k - \gamma_k\,(\theta_i - \theta_j) \le M_k(1-x_k)\ ,\ k=(i,j),\quad j\in\Omega_i^+,\ i=1,n$$

(existing circuit flow upper and lower bounds)

$$-f_k^{max} \le f_k \le f_k^{max}\ ,\ k=(i,j)\ ,\quad j\in\Omega_i^0\ ,\quad i=1,n$$

(candidate circuit flow upper and lower bounds)

$$-f_k^{max}\ x_k \le f_k \le f_k^{max}\ x_k\ ,\ k=(i,j)\ ,\ j\in\Omega_i^+\ ,\ i=1,n$$

(integrality constraints of investment variables)

$$x \in \{0,1\}^m$$

Note that the 2nd Kirchoff's law for each candidate circuit is now expressed as two linear inequalities. When a candidate circuit binary variable is set to zero, the corresponding disjunctive constraints enforce that no flow will go through the circuit, while if it is set one the flow will obey the second Kirchoff's law equation, as required.

This standard disjunctive formulation was independently proposed by [3,4]. It has been proven [5] that if a candidate circuit k is such that there is an existing circuit on the same branch, the minimum value of the penalty parameter M_k is given by f_k^{max} times the ratio of the candidate susceptance and the existing circuit susceptance. Also, if the candidate circuit is in a new right-of-way, its penalty parameter is the product of its susceptance times the solution value of a shortest path problem on the network between the branch's terminal nodes, where the "distance" between each pair of nodes is measured by the ratio of the branch's flow capacity and its susceptance. By using these optimal penalties, ill conditioning can be alleviated.

Better conditioning properties result when an alternative disjunctive formulation is adopted. In this formulation, Kirchoff's 2nd law for each candidate circuit k=(i,j) is represented by two inequalities, each one related to the possible flow direction (from i to j and from j to i), resulting in lower and upper bounds for flow in each direction:

(2nd Kirchoff's law for candidate circuit k=(i,j), upper bound)

$f_k^+ - \gamma_k \Delta\theta_k^+ \leq 0$, $j \in \Omega_i^+$, i=1,n

$f_k^- - \gamma_k \Delta\theta_k^- \leq 0$, $j \in \Omega_i^+$, i=1,n

(2nd Kirchoff's law for candidate circuit k=(i,j), lower bound)

$f_k^+ - \gamma_k \Delta\theta_k^+ \geq -M_k(1-x_k)$, $j \in \Omega_i^+$, i=1,n

$f_k^- - \gamma_k \Delta\theta_k^- \geq -M_k(1-x_k)$, $j \in \Omega_i^+$, i=1,n

The flow in each candidate circuit k is now expressed as the difference of two non-negative flow variables, f_k^+ and f_k^-:

(flow in each candidate circuit k=(i,j))

$f_k = f_k^+ - f_k^-$, k=(i,j) , $j \in \Omega_i^+$, i=1,n

Each candidate circuit angle difference is now expressed as the difference of two non-negative angle differences, $\Delta\theta_k^+$ and $\Delta\theta_k^-$:

(candidate circuit k=(i,j) angle difference $\Delta\theta_k$):

$\Delta\theta_k = \Delta\theta_k^+ - \Delta\theta_k^-$, k=(i,j) , $j \in \Omega_i^+$, i=1,n.

With the new candidate circuit flow variables, the flow bounds are now expressed as:

(candidate circuit flow upper and lower bounds)

$f_k^+ - f_k^{max} x_k \leq 0$, k=(i,j) , $j \in \Omega_i^+$, i=1,n

$f_k^- - f_k^{max} x_k \leq 0$, k=(i,j) , $j \in \Omega_i^+$, i=1,n

The objective function and other unmentioned constraints remain unaltered, as well as variables f^+, f^-, x and θ.

Comparing this formulation with the previous, it can be seen that:

- the upper bound is tighter since it doesn't include in the RHS the positive term with the penalty;
- the lower bound is exact when $x_k=1$, and the RHS is better than the one in the previous formulation when $x_k=0$.

The resulting formulation has more continuous variables, but being tighter should be better than the previous standard disjunctive formulation. Since it has the same number of binary variables as the original formulation, due to the tighter formulation the B&B algorithm tends to be more efficient. With this formulation, real world test system problem were solved to optimality for the first time [1] using a commercial state-of-the-art code [6].

4. Hydrothermal systems transmission expansion planning considering contingencies and multiple scenarios

An alternative network design approach models the trade-off between investment cost and reliability worth. Instead of complying with the security criterion, annual customer interruption costs are multiplied by the expected power not supplied (EPNS) and summed to annualized investment costs, so as to reach a balance between transmission reliability worth and economically justified network investments. The only change in constraints with respect to the previous formulation is the inclusion of a bus slack "load shedding" variable *r* in each bus power balance equation, as shown next.

(modified power node balance equation – first Kirchoff's law)

$$r_i + \sum_{k=(i,j)\, j \in \Omega_i} f_k = d_i - g_i \ , \quad i=1,n$$

This slack variable has an upper bound equal to the bus loads. Adding to the objective function a sum of all bus slack variables, penalized by a unit interruption cost *U*, provides the trade-off term. In order to take into account the load duration curve, it is discretized in peak and off-peak blocks. For each block, all network constraints are replicated. The objective function term reflecting the total interruption cost for each load block is weighted by its duration, so as to measure the unsupplied energy. Due to the load curve representation, the size of the problem doubles in terms of constraints and continuous variables, not affecting the number of binary investment variables.

It should be noted that if *U* is very high, circuit investments will be preferred instead of load sheddings, corresponding to the "N-1" expansion planning criterion. If any combination of candidate circuits cannot avoid all overloads, the resulting objective function second term, the minimum total load shedding is a better severity measure of network inadequacy then the sum of network overloads. On the other hand, if *U* is the unit load interruption cost, a trade-off solution with some unserved energy may provide a better compromise between investment and reliability worth, instead of avoiding circuit overloadings at all costs.

To deal with single circuit contingencies, network constraints and continuous variables are again replicated for each contingency. It should be noted that the *M* penalty coefficients of the disjunctive inequalities are recalculated for each contingency, since the network topology is changed. The circuit under contingency is disregarded in the corresponding contingency power flow constraints. In the interruption cost term of the objective term, for each contingency the load shedding variables are weighted by the circuit outage probability and their sum (the reliability index EPNS) is multiplied by the customer unit interruption cost so as to measure the network's annual reliability worth. Note that the base case probability is used to weight the corresponding load shedding term of the objective function.

Consideration of multiple dispatch scenarios follows the same reasoning. For each one a vector of bus generations is provided for each load block, such that total generation meets total demand. The interruption cost term of the objective function is averaged over all scenarios, since the dispatch scenarios are usually obtained by a stochastic hydrothermal scheduling model [7], whose hydro plant inflows are sampled.

The problem size also grows linearly with the number of scenarios, except for the binary variables. Since the investment binary variables affect all continuous variables, the resulting disjunctive model is coupled by the investment decisions. Once an investment decision is fixed, the problem decouples into as many problems as the number of load blocks times the number of scenarios times the number of contingencies. Each such problem is a linear program, measuring the severity of the proposed network reinforcement plan for each load block, dispatch scenario and contingency.

5. The disjunctive model with contingencies and multiple scenarios: A case study

The hydrothermal system of Bolivia will be used to illustrate the transmission network expansion for December 2013. The network has 47 nodes and 52 circuits. The peak load is 1079 MW, occurring 10% of the time. The off-peak load is 628 MW. There are 30 thermal plants and 28 hydro plants. New and existing routes were selected, totalizing 174 circuit candidates for 69kV, 115kV and 230kV voltage levels. Figure 1 presents the network configuration.

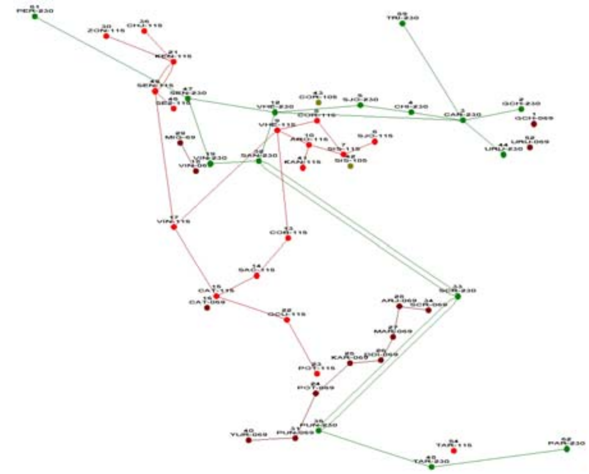

Figure 1: Bolivia network for 2013

The network has no overloads for base case, considering fifteen scenarios simulated by the hydrothermal scheduling model *SDDP* [7], for both load blocks. All non-radial circuits are considered summing 32 contingencies. Figures 2 and 3 present the total hydro and thermal generation along the scenarios and load blocks.

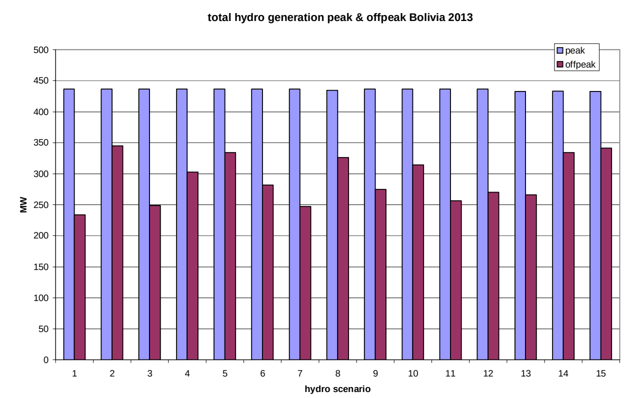

Figure 2: distribution of total hydro generation

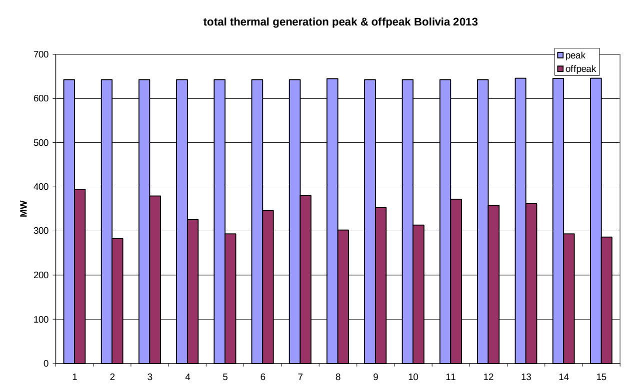

Figure 3: distribution of total thermal generation

Off-peak hydro and thermal generation are complementary and vary along the scenarios, both being constant during peak hours. Since hydro plants are far from the load and thermal units are near, the network loading during off-peak is different form peak hours, we need to represent both load blocks.

A severity analysis of each contingency for all scenarios and load blocks revealed that 6 are critical. The load shedding distribution of each one is shown in Figures 4-9.

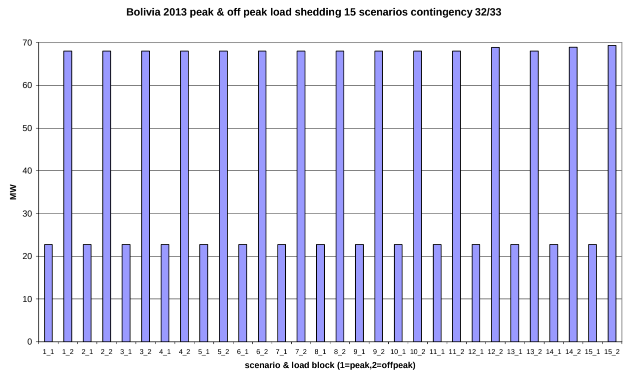

Figure 4: load shedding distribution, contingency 32/33

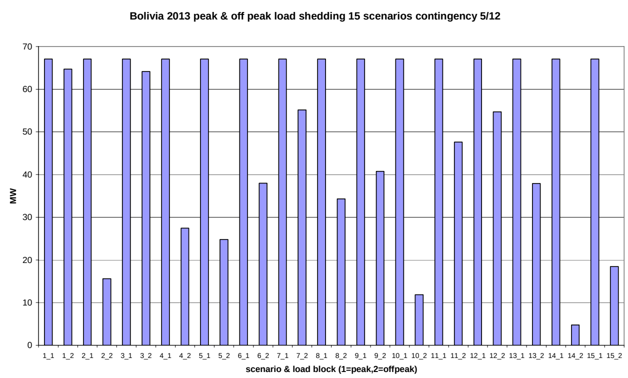

Figure 5: load shedding distribution, contingency 5/12

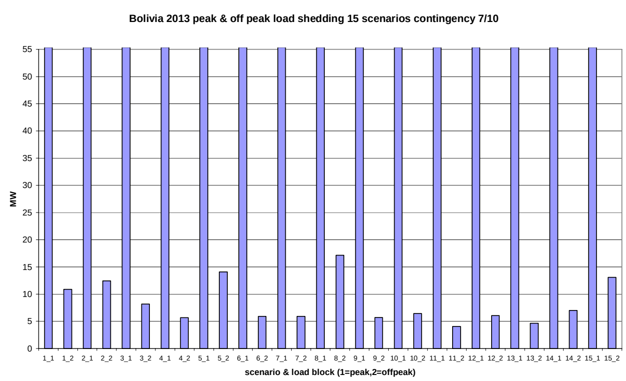

Figure 6: load shedding distribution, contingency 7/10

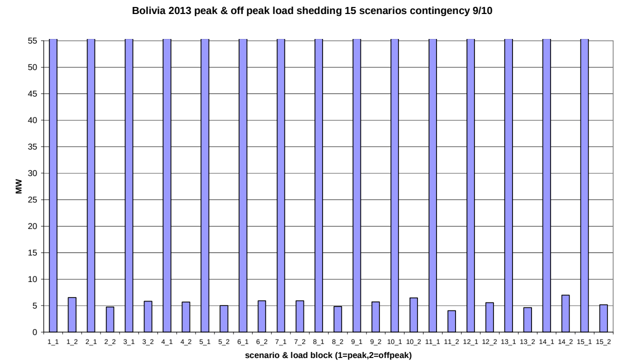

Figure 7: load shedding distribution, contingency 9/10

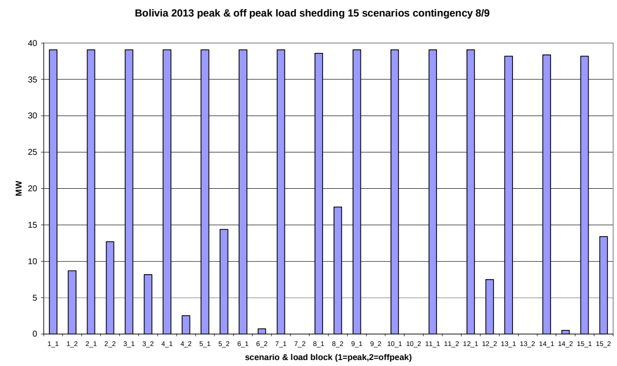

Figure 8: load shedding distribution, contingency 8/9

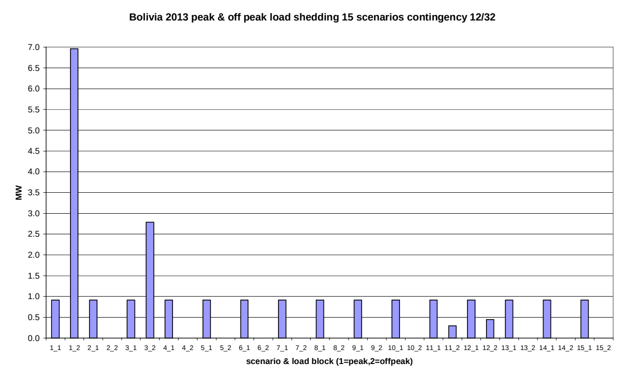

Figure 9: load shedding distribution, contingency 12/32

It can be noted that there is a large variation along scenarios of severities due to contingencies 5/12, 8/9 and 12/32. The 230kV line 32/33 is the most critical, and uniformly severe.

The average severity of the critical contingencies is shown in Figure 10.

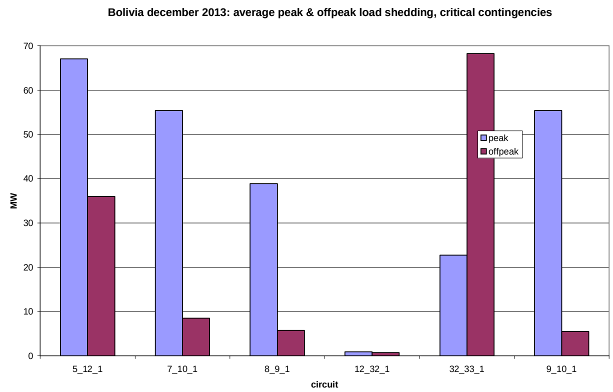

Figure 10: average severity of critical contingencies

The total average severity is 240 MW for peak hours, and 125 MW off-peak. Off peak severity is significant, and even more when estimating network reliability in terms of expected unserved energy (EENS), since the peak load lasts only 10% of the time. Off-peak hours account for 82% of total EENS.

If a very high interruption cost (equivalent to the “N-1” criterion) is used to solve the disjunctive model considering peak and off-peak loads, the optimal solution consists of reinforcements in circuits 5/6, 6/7, 7/10 and 32/33, totalizing US$ 38.27 million, of which US$ 29 million is due to circuit 32/33, the most critical contingency. For all scenarios circuit overloads are avoided. The CPU time was 16 seconds on a 2 GHz P-IV.

Suppose we adopt the traditional “N-1” planning criteria considering only the peak load condition. The optimal solution has 8 reinforcements: 5/6, 6/7, 7/10, 15/22, 22/23, 23/24, 24/31 and 35/31, totalizing US$ 31.5 million, being cheaper than the previous solution. Remember that the most critical contingency is 32/33, incurring in a network severity of 68MW off peak and 22 MW during peak. Since only the peak load is considered, the reinforcement 32/33 is no longer necessary (as shown in Figure 11, for scenario 1 no circuit overloads happen due to contingency in 32/33 for peak load but many are overloaded off-peak); it was not chosen since it alone is more expensive than 15/22, 23/23, 23/24, 24/31 and 35/31 together.

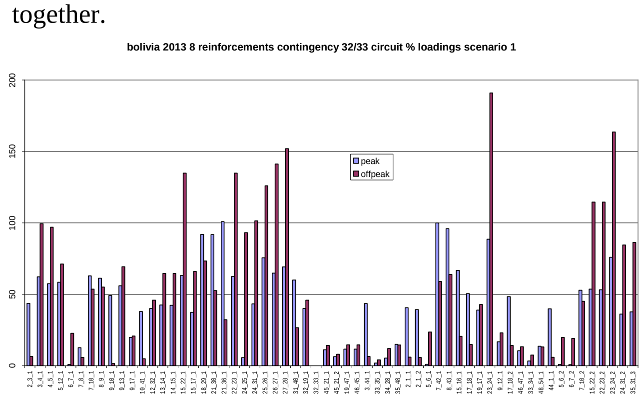

Figure 11: for 8 reinforcements chosen for peak load condition, circuit loadings for contingency 32/33

This result illustrates the relevance of representing network constraints for off-peak load hours in hydrothermal systems transmission expansion planning.

Considering only the first scenario, and using the long term energy shortage cost as the interruption cost (value based expansion criterion), the optimal solution has three reinforcements: 5/6, 6/7 and 7/10, with investment cost US$ 9.27 million. Note that the most expensive reinforcement of the “N-1” solution considering the load curve was avoided. The CPU time was 16 seconds. With this solution, the severities of the critical contingencies are shown in Figure 12.

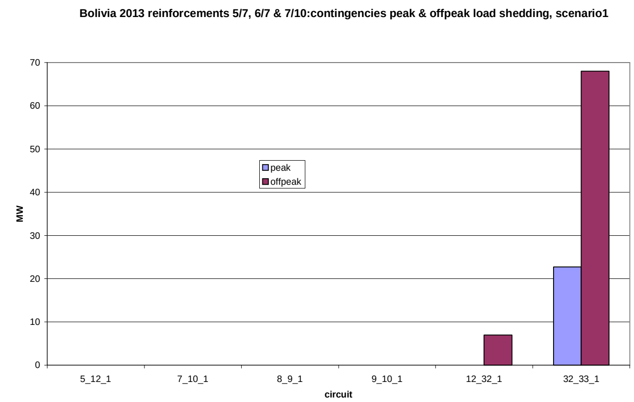

Figure 12: average severities of critical contingencies for expansion plan only considering the first scenario

Note that the remaining average severity is mostly due to the contingency 32/33, the rest being due to the nearby contingency 12/32. As expected, due to the trade-off between investment and reliability cost, the high cost of reinforcement 32/33 is not economically advantageous with respect to the reliability cost it avoids. The first scenario total severity for peak/off-peak load is 23 MW/75 MW, while the average severity is 24 MW/79 MW, very similar since the severity of the major contingency 32/33 is uniform along scenarios.

If all scenarios are considered, and the same interruption cost, the optimal solution consists of the single reinforcement 7/10, costing US$ 4 million. This problem instance is large scale, having 1.1 million constraints and 0.8 million variables. The B&B solver took 16.2 CPU minutes to find and prove optimality. Figure 13 presents the distribution of severities for the still remaining critical contingencies 5/12, 12/32 and 32/33.

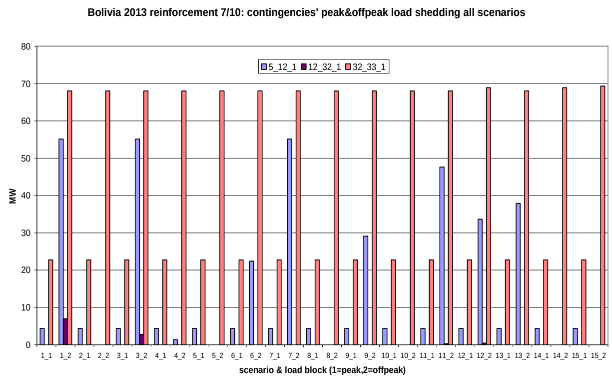

Figure 13: severity due to contingencies, 7/10 reinforcement

The average severity for peak/off-peak load is 27MW/91MW. Comparing these values with the average severities resulting from the previous solution, one can see that there is a slight increase for peak load, and a 32% increase off-peak, mainly due to contingency 5/12. Since the investment cost was cut by more than half, this reduction is worthwhile when compared to the slight increase of reliability cost with respect to the previous solution. One can therefore conclude that for hydrothermal systems transmission expansion planning, it pays to represent the scenario dispatch diversity when using the reliability worth approach. Also, this approach provides a clear economic justification of transmission investment needs, while allowing a quantitative evaluation of network reliability.

A final question is the time required to solve the disjunctive model when dealing with contingencies and multiple scenarios. As seen, when all scenarios were considered instead of only one, the problem size increase had a great impact on computing time. If many contingencies are critical, problem size will also increase. Since the trade-off in the objective function is greatly affected if any critical contingency is neglected, there is room only for approximations regarding the dispatch scenarios. By applying a severity analysis of the initial network configuration for the critical contingencies, one can apply a statistical multidimensional clustering technique to aggregate similar scenarios in terms of the total severity of each contingency.

Using such a technique, 3 scenario clusters with probabilities 0.2, 0.27 and 0.53 were obtained. The disjunctive model was again solved, where the objective function reliability term was evaluated by taking the weighted average over scenario clusters. It turns out that the same previous solution was obtained, taking only 27 CPU seconds to find and prove optimality. By using this “scenario reduction” preprocessing, one can cope with the diversity of dispatch scenarios which are common in hydrothermal systems, and still deal with the increasing problem complexity due to the representation of network constraints for each contingency.

5. Conclusion

The proposed approach for transmission expansion planning of hydrothermal systems has a sound economic foundation. Investments in transmission reinforcements required to deal with contingencies can be justified in term of avoided customer interruption costs, taking into account critical contingencies’ severity and outage probability.

It was shown that in order to measure the network reliability, transmission constraints must be enforced for all load conditions, dispatch scenarios and critical contingencies. The mixed integer disjunctive model for the expansion planning problem has been extended to this framework, considering a value based transmission expansion objective function which reflects the tradeoff between investment and reliability cost.

Although the size of this model is sensitive to the number of dispatch scenarios and contingencies, a “scenario reduction” preprocessing allows selecting a few representative dispatch scenarios, measuring the severities of critical contingencies. The case study results show that this approach, when applied to real world hydrothermal systems, provides grounds for transmission planning with a sound economic base.

References

[1] L. Bahiense, G. C. Oliveira, M. Pereira, S. Granville,” A mixed integer disjunctive model for transmission netwrk expansion”, *IEEE Transactions on Power Systems*, Vol 16, #3, 2002.
[2] M. Pereira, L. Pinto, S. Cunha, G. Oliveira, “A decomposition approach to automated generation-transmission expansion planning”, *IEEE Transactions on PAS*, Vol. Pas-104, #11, Nov 1985
[3] R. Villanasa, “Transmission network planning using linear and mixed linear integer programming”, PhD Thesis, Ressenlaer Polythechnic Institute, 1984.
[4] M. Pereira, S. Granville, “Analysis of the linearized power flow model in Benders decomposition”, Technical Report SOL 85-04, SOL Lab, Dept. of Oper. Research, Stanford University, 1985
[5] S. Binato, “Optimal expansion of transmission networks by Benders decomposition and cutting planes”, PhD Thesis, Federal University of Rio de Janeiro, 2000 (in Portuguese).
[6] DASH Associates, “XPRESS-MP User Guide and Reference Manual”, http://www.dashopt.com
[7] Power Systems Reseach, “SDDP hydrothermal stochastic scheduling model”, http://www.psr-inc.com