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APPLICATION OF FINITE-ELEMENT SENSITIVITIES TO POWER CABLE THERMAL FIELD ANALYSIS

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ABSTRACT
In this paper, the authors describe a new approach to cable thermal field and ampacity computations using a proposed concept of perturbed finite-element analysis. The proposed model provides a fast sensitivity methodology, based on the finite element model, to assess the cable thermal performance subjected to variations in the cable thermal circuit parameters. The sensitivity information is useful not only in determining the important and non-important parameter variations in term of their relative effect on the cable temperature and ampacity, but also in evaluating the cable amapacity subject to various parameter changes in vary fast and compact scheme. An application of six directly buried cables is presented to show the applicability of the derived model in actual practical systems.

KEY WORDS
Underground cables, ampacity, finite-element, thermal field.

1. Introduction

The maximum cable ampacity is a function of all internal and external cable system components which comprise the thermal circuit of the cable and its boundaries. The parameters of the thermal circuit of power cable are subjected to geographical and seasonal changes which affect the allowable loading level of any particular cable [1-3]. The proposed perturbed finite-element analysis technique provides useful sensitivity information of the cable ampacity, with respect to fluctuations in the cable circuit parameters, to assess the effects on the permissible cable loading caused by these fluctuations without repeating the whole thermal analysis for each possible parameter change. In addition, the technique applies both to the design phase and the operational aspects of power cables buried in complex media of soil, heat sources and sinks and variable boundary conditions [4-6]. Because the simulation procedure for various parameter variations does not involve repeated finite element analyses, the results can be obtained by very fast and compact scheme and displayed in efficient, straightforward manner. Since only one nominal finite element analysis is required, larger element grid can be employed allowing even more accurate modeling. The sensitivity information is useful not only in evaluating the cable amapacity subject to various parameter changes but also in determining the important and non-important parameter variations in term of their relative effect on the cable temperature and ampacity. The finite element sensitivity technique can be applied to complex cable configurations and boundaries to assess the effect of variations of various soil and boundary parameters. Unfortunately, in finite element technique an explicit expression of the cable temperature in terms of cable thermal circuit parameters is not possible. However, the sensitivity coefficients associated with various cable parameters of the interest provides the cable engineers with important information about relative effects of such parameters variations on the cable performance, which helps in design of a new system and improving performance of existing one.

This paper describes the analytical and computational aspects of the sensitivity methodology and demonstrates the applicability and usefulness of the developed methodology via applications to actual cable systems under different loading, soil and atmospheric conditions.

2. Problem Formulation

The thermal filed in the cable medium is governed by the differential equation [7,8]

$$\nabla \cdot \left(k \ \nabla T\right) = - Q + c \ \frac{\partial T}{\partial t} \tag{1}$$

Where T denotes the temperature at any point, k and c represent, respectively, the thermal conductivity and capacity, Q is the heat generation per unit of area and t denotes the time. In steady state thermal analysis of two dimensional media, equation (1) reduces to

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + Q = 0 \tag{2}$$

For any homogenous region of given thermal conductivity and heat generation rate, equation (2) can be solved for the temperature at any point in the region subjected to specified boundary conditions. The power cable thermal circuit includes various regions of complicated shapes having different values of thermal conductivity and heat generation and equation (2) should be solved for the overall cable medium. The finite element method exploits the theory that the solution of (2), namely $T(x,y)$ is that which minimizes the functional given by

$$I = \tfrac{1}{2}\iint \left[k_x \left(\frac{\partial T}{\partial x} \right)^2 + k_y \left(\frac{\partial T}{\partial y} \right)^2 \right] dx\,dy \; - \iint Q\;T \;\; dx\,dy \tag{3}$$

The minimization of (3) is performed over finite element mesh resulted from partitioning the cable system medium, yielding a set of sparse equations of the form

$$\mathbf{H}\,\mathbf{T} = \mathbf{b} \tag{4}$$

where **H** is the heat conductivity matrix, **T** is a vector of temperature at the finite element nodes and **b** is a vector which normally contains the heat generation associated to each node. Both the matrix **H** and the vector **b** are adjusted to accommodate the boundary conditions of thermal circuit. The conductivity matrix and the right hand side vector of equation (4) are functions of the cable loading, circuit parameters and boundary conditions. Whether heat transfer solution is implemented using finite element modeling or any other tool, solvable heat conduction should also have boundary conditions well defined. In the finite element analysis the following cable parameters are independent and can be freely specified:

- ii Cable current I (*amperes*).
- iii Ambient temperature T_a at all part of the soil surface ^oC.
- iiii Thermal conductivity k of any portion of the cable circuit including backfill and duct bank ($W/(^oC.\ m)$).
- ivi Heat generation Q in certain circuit portions (W/m).
- vi Heat flux coefficient q at certain circuit boundaries (W/m^2)
- vii Heat convection loss coefficient h at a certain circuit boundaries ($W/(^oC.\ m^2)$).
- viii Emissivity radiation coefficient ε at a certain circuit boundaries.

Unfortunately, in finite element technique an explicit expression of the cable temperature in terms of cable thermal circuit parameter is not possible. However, using sensitivity coefficients associated with various cable parameters of the interest, an accurate representation of the cable temperature can be accomplished. In order to achieve that, the basic finite element sensitivity equation has to be extracted by differentiating equation (4) with respect to thermal circuit parameters described above.

$$\mathbf{H(p)}\left(\frac{\partial \mathbf{T(p)}}{\partial \mathbf{p}} \right) = \left(\frac{\partial \mathbf{b(p)}}{\partial \mathbf{p}} \right) - \left(\frac{\partial \mathbf{H(p)}}{\partial \mathbf{p}} \right) \mathbf{T(p)} \tag{5}$$

where **p** represent the subset of thermal circuit parameters to be considered arranged as the column vector. Consequently, the finite element sensitivity equation (5) is solved for the temperature sensitivity vector [$\partial\mathbf{T}(\mathbf{p})/\partial\mathbf{p}$].

The change in the temperature vector **T** resulting from change in some or all parameters is given, to a first order approximation, by

$$\mathbf{T} = \mathbf{T^o} + \left(\frac{\partial \mathbf{T}}{\partial \mathbf{p}} \right)_{\mathbf{p^o}} \left[\mathbf{p} - \mathbf{p^o} \right] \tag{6}$$

where $\mathbf{T^o}$ denotes the vector of nodes temperatures at base-case (nominal) parameter values $\mathbf{p^o}$. The use of the cable temperature sensitivities with respect to various cable circuit parameters is the key factor in replacing the numerous temperature evaluations for all possible parameter by fewer temperature evaluations which, in conjunction with the temperature sensitivities, are sufficient to describe the cable temperature. The presented novel finite element based sensitivity technique enables cable temperature sensitivity with respect to all parameters to be evaluated in efficient and fast way. In the finite element sensitivity technique, complex cable, soil and boundary conditions can be simulated accurately in the thermal model leading to more accurate temperature prediction.

3. Applications

The development of a sensitivity methodology based on the finite element model was accomplished in which various cable environmental parameters are modeled accurately, including the heat transfer mechanisms at boundaries, backfill, duct banks and other media surrounding the cable. The temperature sensitivities with respect to variations in the cable thermal parameters represent the foundation of performance indices of interest to cable designers and operators. In case when the cable operating in fluctuating conditions, which is mostly the practical case, the associated disturbance in some system parameters will result in a defragging temperature profile. The only way to figure out the new temperature profile - in the lack of any information about the sensitivity of the temperature to the system parameters variation- is to recalculate the system temperature at the new operating condition. In this section, we will show who is the sensitivity model can be utilized to lead to the desired new profile with much less efforts and with acceptable accuracy. This section represents some useful applications of the sensitivity techniques to variety of practical cable systems.

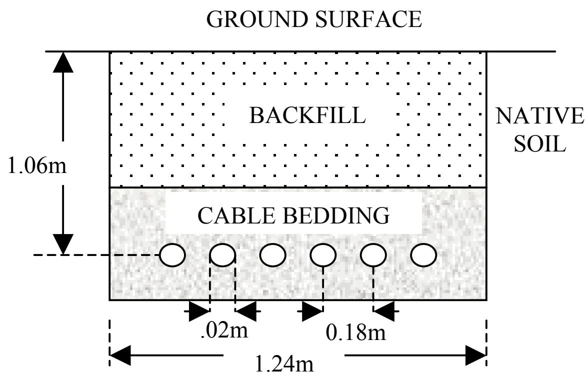

Fig. 1 Six direct buried underground cable system

Figure 1 shows the arrangement of 161kv six direct buried cables with rating given in Table 1. The trench width is 1.246m and the cable bedding extends 0.12m above and below the cable surface. The cables are buried at distance of 1.060m between the cable center and the ground surface which is modelled as convective boundary with heat convection loss coefficient of 5 $W/(^oC.m^2)$.

TABLE 1: CABLE RATINGS

Cable size (mm^2)	400
XLPE insulation (mm)	18
Rated voltage (kv)	161
Rated frequency (Hz)	60
Operating current (A)	650
Conductor losses (W/m)	26.246
Dielectric losses (W/m)	0.424
Shield losses (W/m)	1.968

For this cable, the temperature distribution profile was evaluated using the finite element grid containing 1520 nodes which shown in Figure 2.

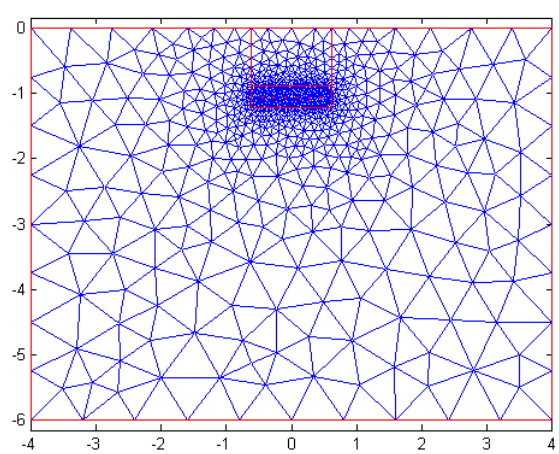

Fig. 2 Finite element mesh imposed on the model solution domain

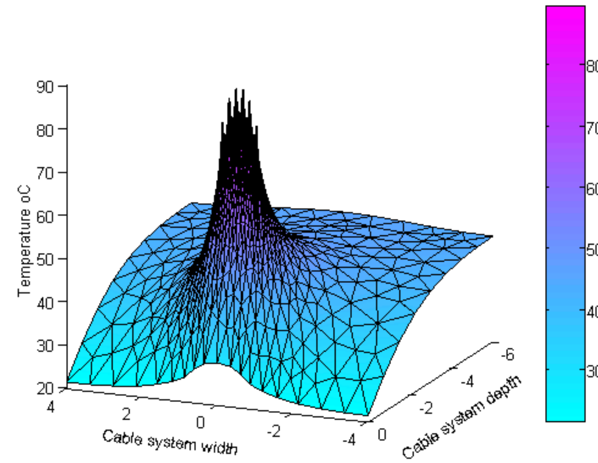

Fig. 3 3D Temperature distribution

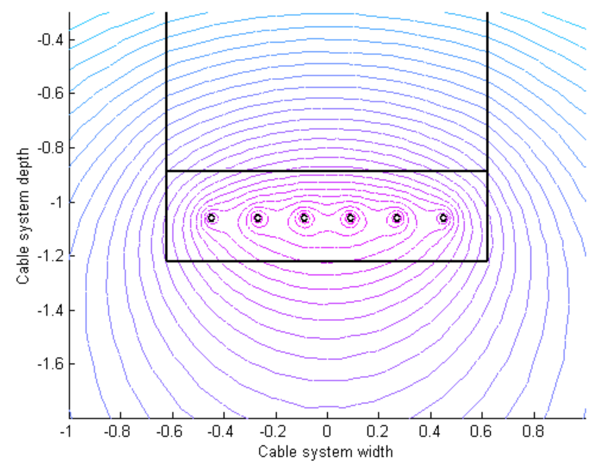

Fig. 4 Temperature contour

The nominal value of the ambient air temperature and the heat convection loss coefficient are 20 ^oC and 5 $W/(^oC. m^2)$ respectively. The nominal values of the thermal conductivities of the cable bedding, trench backfill and native soil are, respectively 1.053 $W/(^oC. m)$, 1.25 $W/(^oC. m)$, 0.833 $W/(^oC. m)$ and they are all subjected to variations about their nominal values. Figure 3 shows the temperature distribution profile calculated by finite element technique for the nominal values of the ambient temperature and soil thermal conductivities indicated above and for cable loading of 650A per cable. Figure 4 shows the temperature contours.

Table 2 shows the sensitivities of the each cable maximum surface temperatures with respect to various parameters of the thermal circuit. The sensitivity values of Table 2 show that the thermal conductivities have the dominant impact on determining the cables temperatures over all other parameters. The heat dissipation increases with the increase of soils conductivities surrounding the cables, and consequently results in markedly temperatures reduction.

Furthermore, it can be noted that the mother soil thermal conductivity has the greatest effect among the backfill and the bedding thermal conductivities. This can be demonstrated by considering the large geometrical area occupied by the mother soil. On the other hand, the nearness of a geometrical area specified by such thermal conductivity from the cables plays an important role on determining the sensitivity of the cable temperatures with respect to the thermal conductivity too. Both bedding and backfill are centered in the middle of the cable system, the bedding is containing the cables while the backfill occupying large area, and as result, both associated temperature sensitivities increase in each case due to the mentioned characteristic factor to end up with close sensitivities indicated in Table 2. Increasing the thermal conductivities and heat convection loss coefficient reduce the cables temperature, both aid in dissipation more heat away from the cables spots.

of 1%, 5%, and 10% increasing in the mother soil thermal conductivity, respectively.

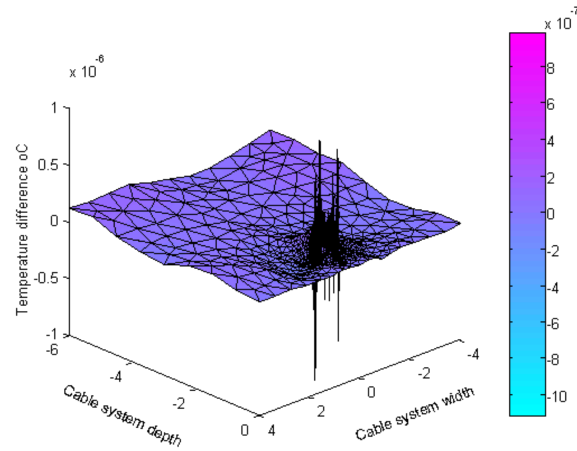

Fig. 5 Temperature comparison between PFE sensitivity analysis and CFE analysis

TABLE 2: SENSITIVITIES OF CABLES TEMPERATURES AT NOMINAL VALUES

Parameter(p) \ Sensitivity $\partial T/\partial p$	Cable #1	Cable #2	Cable #3	Cable #4	Cable #5	Cable #6
Mother soil thermal conductivity $W/(^{o}C.\ m)$	-30.4886	-30.4414	-30.5571	-30.5575	-30.4392	-30.4845
Backfill thermal conductivity $W/(^{o}C.\ m)$	-13.4097	-15.2159	-16.1620	-16.1623	-15.2172	-13.4100
Bedding thermal conductivity $W/(^{o}C.\ m)$	-12.9561	-16.1046	-17.1241	-17.1277	-16.1002	-12.9706
Ambient temperature ^{o}C	1	1	1	1	1	1
The heat convection loss coefficient $W/(^{o}C.\ m^2)$	-1.1248	-1.1459	-1.1567	-1.1567	-1.1459	-1.1248
The generated heat in the cables $W/\ m$	2.1454	2.3421	2.4261	2.4263	2.3420	2.1458

On the other hand, the cable losses increase the amount of heat generation, and cause more temperature rise. However, In addition to the assessment of the relative effects of the variations of various parameters, the sensitivity values can be used to estimate the changes in the temperatures profile in the domain of the cable system due to variations of one or more of the parameters. Consequently, the most critical temperatures spots such as cable temperatures can be obtained by very fast scheme around the nominal operating point in case of fluctuations of cable system parameters without repeating the whole thermal analysis for each possible parameter change.

Figure 5 shows the difference between the temperature profile using the perturbed finite element sensitivity analysis due to 10 % increase in all the cable system parameters indicated in Table 2, and the temperature profile from the conventional finite element method at this new operating point. This figure shows a very small difference between the exact convectional method and the proposed approach, which does not exceed $1\times10^{-7}\ ^{o}C$ according to 10 % of all parameters disturbance from the base-case. Table 3 shows the cables hottest temperatures at the nominal case and the new temperatures for the case

TABLE 3: CABLES SURFACE TEMPERATURES AT NOMINAL AND NEW CASE DUE TO MOTHER SOIL THERMAL CONDUCTIVITY VARIATIONS

cable #	Nominal value ^{o}C	New values due to parameter variation (convectional finite element)		
		1% $k_{\text{mother-soil}}$	5% $k_{\text{mother-soil}}$	10% $k_{\text{mother-soil}}$
1	81.4390	81.1866	80.2073	79.0480
2	87.0741	86.8219	85.8429	84.6818
3	89.4791	89.2260	88.2426	87.0751
4	89.4834	89.2302	88.2468	87.0793
5	87.0691	86.8170	85.8381	84.6771
6	81.4514	81.1990	80.2198	79.0607

Table 4 shows the percentage deviation of the cable temperatures calculated using the perturbed finite element sensitivity analysis, from the convectional finite element temperatures illustrated in Table 3.

TABLE 4: PERCENTAGE DEVIATION OF THE PFE SENSITIVITY CABLES SURFACE TEMPERATURES FROM THE CFE TEMPERATURES

Cable #	Temperature percentage deviation (%)		
	1% $k_{\text{mother-soil}}$	5% $k_{\text{mother-soil}}$	10% $k_{\text{mother-soil}}$
1	0.0019	0.0481	0.1893
2	0.0018	0.0433	0.1706
3	0.0017	0.0415	0.1634
4	0.0017	0.0415	0.1634
5	0.0018	0.0433	0.1706
6	0.0019	0.0481	0.1892

The effect of changing the backfill and bedding thermal conductivities were studied and the temperature deviations of the perturbed finite element sensitivity from the convectional finite element were carried out.

Tables 5 and Table 6, respectively, show the percentage deviation of the perturbed finite element sensitivity cables surface temperatures from the convectional finite element temperatures, for the case of changing the backfill and bedding thermal conductivities of backfill and bedding by 5% and 10% decreasing and increasing respectively.

TABLE 5: PERCENTAGE DEVIATION OF PFE SENSITIVITY CABLES SURFACE TEMPERATURES FROM CFE TEMPERATURES (BACKFILL)

Cable #	Temperature percentage deviation (%)			
	-5% k_{backfill}	-10% k_{backfill}	5% k_{backfill}	10% k_{backfill}
1	0.0325	0.1328	0.0311	0.1217
2	0.0351	0.1437	0.0336	0.1318
3	0.0366	0.1495	0.0350	0.1373
4	0.0366	0.1495	0.0350	0.1373
5	0.0351	0.1436	0.0336	0.1318
6	0.0324	0.1328	0.0311	0.1217

TABLE 6: PERCENTAGE DEVIATION OF PFE SENSITIVITY CABLES SURFACE TEMPERATURES FROM CFE TEMPERATURES (BEDDING)

Cable #	Temperature percentage deviation (%)			
	-5% k_{bedding}	-10% k_{bedding}	5% k_{bedding}	10% k_{bedding}
1	0.0431	0.1799	0.0397	0.1526
2	0.0475	0.1979	0.0439	0.1694
3	0.0479	0.1996	0.0444	0.1714
4	0.0479	0.1996	0.0444	0.1714
5	0.0475	0.1978	0.0439	0.1694
6	0.0431	0.1801	0.0397	0.1527

4. Conclusion

The work presented in this paper was intended to show the capability of the perturbed finite element sensitivity technique to deal with complicated system configurations. Because the simulation procedure for various parameter variations does not involve repeated finite element analyses, the results can be obtained by very fast and compact scheme and displayed in efficient, straightforward manner. Since only one nominal finite element analysis is required, larger element grid can be employed allowing even more accurate modeling. The sensitivity results show that the variations of the thermal conductivity of the mother soil would affect the cables temperatures more than variations of other parameters. Special manipulation of the total mother soil thermal conductivity by the mean of the bedding soil will result in a big reduction of cable temperatures as indicated in this paper results.

In this paper compassion was shown between the convectional and proposed finite element technique applied on the presented cable system for a various cable system parameters variations. The maximum noted deviation was note to be less that 0.2% for 10% increasing in mother soil thermal conductivity. This shows the capability of the proposed perturbed finite element sensitivity analysis on capturing the trend of the cables temperatures with respect to the thermal parameters variations, even in the case of complicated cable system and boundary conditions. Consequently the perturbed finite sensitivity method solve the thermal field just once (at the nominal case) and use the derived sensitivities to calculate the new thermal fields over various sets of circuit parameters changes within ±10% from the nominal values. It worth mentioning that this range is based on the most sensitive parameters such as thermal conductivities, extended range can be applied for parameters such as those representing boundary conditions.

References

[1] P.Vaucheret, R.A Hartlein, and W.Z Black,.; "Ampacity derating factors for cables buried in short segments of conduit" IEEE Transactions on Power Delivery, Volume 20, No.2, April 2005, pp.560 – 565.

[2] G. J. Anders, and E. Dorison "Derating factor for cable crossings with consideration of longitudinal heat flow in cable screen" IEEE Transactions on Power Delivery, Vol.19, No.3, July2004,pp.926–932.

[3] M. A. El-Kady, G. A. Anders, J. Motlis, and D. J. Horrocks, "Modified values for geometric factor of external thermal resistance of cables in duct banks," *Power Delivery, IEEE Transactions on*, vol. 3, pp. 1303-1309, 1988.

[4] C. C. Hwang, "Calculation of thermal fields of underground cable systems with consideration of structural steels constructed in a duct bank," Generation, Transmission and Distribution, IEE Proceedings-, vol. 144, pp. 541-545, 1997.

[5] M. A. El-Kady, "Optimization of power cable and thermal backfill configurations," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-101, pp. 4681-4688, 1982.

[6] J. H. Neher, and McGrath, "The calculation of the temperature rise and load capacity of cable system" Transaction AIEE, Vol. 76, Part III, PAS. 1957.

[7] M. A. El-Kady, "calculation of the sensitivity of power cable ampacity to variations of design and environment parameters," in *IEEE Transaction on power apparatus and Systems*, vol. PAS-103, August 1984, pp. 2043-2050.

[8] N. Flatabo, "Transient heat conduction problem in power cables solved by the finite element method," *IEEE Transactions on Power Apparatus and Systems,* vol. PAS-92, pp. 56-53, 1973.