Proceedings of the Sixth IASTED International Conference
EUROPEAN POWER AND ENERGY SYSTEMS
June 26–28, 2006, Rhodes, Greece

COMPARISON OF COMPLETE AND REDUCED ORDER MODELS OF DOUBLE FED INDUCTION MACHINE

Deepak Nagaria, G. N. Pillai, H. O. Gupta
Indian Institute of Technology, Roorkee
India

ABSTRACT
Wind turbines equipped with a Doubly-Fed Induction Generator are increasingly popular in the power range above 1 MW. For power system stability studies it is desirable to apply reduced models of the machine in order to limit the computation time. In this paper, the slow and fast modes direct decomposition method is used to derive a reduced order model of double fed induction machine. A comparison is made with other known methods to test the performance of the technique. Satisfactory results are obtained with slow and fast modes direct decomposition method.

KEY WORDS
Double fed induction machine, Reduced order model, Slow and fast modes, Decomposition method, Step response

1. Introduction

As a result of increasing environmental concern, more and more electricity is generated from renewable sources. One way of generating electricity from renewable sources is to use wind turbines. The doubly fed induction generator (DFIG) wind turbines are nowadays widely used in large wind farms. The main advantage of electricity generation from this machine is wide operating range from sub-synchronous to super-synchronous speeds [1,2]. The DFIG concept also provides a possibility to control the overall system power factor.

To facilitate the investigation of the impact of a wind farm on the dynamics of the power system to which it is connected, an adequate model of the wind turbines is required. Although personal computers become faster and faster, computational speed is still one of the limiting factors in (dynamic) simulation of power systems. One of the problems is the complexity of the models that limits the computational speed. When reduced models are used simulation can be done much faster, but the results may be less accurate [3].

The electromechanical behavior of electrical machine is difficult to analyze. The dynamic model of double-fed induction machine is of high order. In the study of small perturbation it is realized that there exist both large and small time constants. To obtain a reduced order model, the electromechanical quantities split into two sets: fast varying quantities and slow varying quantities. In this paper, the approach of slow and fast modes direct decomposition is derived by transformation to diagonal form. A comparison is made with other known methods to test the performance of the technique.

2. Doubly Fed Induction Machine Model

The basic configuration of a DFIM is sketched in Fig. 1. The most significant feature of this machine is that it has to be fed from both stator and rotor side. Normally, the stator is directly connected to the grid and the rotor is interfaced through a variable frequency bi-directional power flow converter in order to cover a wide operation range from sub-synchronous to super-synchronous speeds.

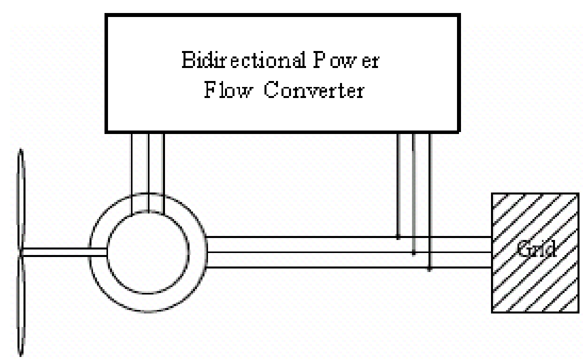

Fig.1. DFIM Configuration

The operating principle of a DFIM can be analyzed using the classic theory of rotating fields and the well known d-q model. The phasor diagram showing the reference axis is shown in Fig. 2. The basic equations of DFIM are considered here. The equations describing a doubly fed induction machine can be found in literature [4 - 9]. We consider three phase balanced voltage supply of both stator and rotor. The reference frame of the machine will be chosen in such a way that the quadrature stator voltage

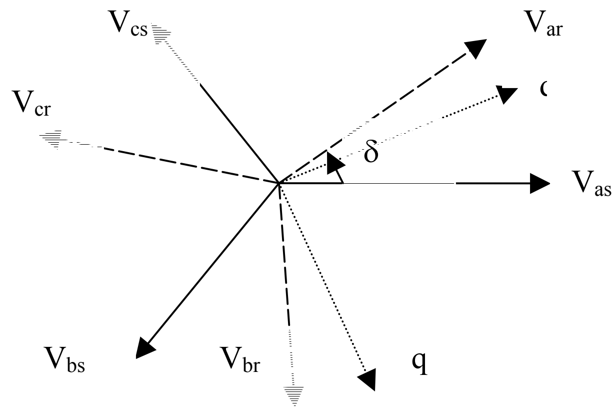

Fig.2. Phasor diagram showing the reference axis

is zero. Using the above mentioned convention, the following set of equations in complex form are:

$$\begin{aligned} \overline{V}_s &= -\overline{Z}_s \overline{I}_s - \overline{Z}_m \overline{I}_r \\ \overline{V}_r &= -\overline{Z}_d \overline{I}_s - \overline{Z}_r \overline{I}_r \end{aligned} \tag{1}$$

in which the complex operational impedances are contain a derivative operator *p*

$$\begin{aligned} \overline{Z}_s &= R_s + (L_s + L_m)(p + j\omega_s) \\ \overline{Z}_m &= L_m (p + j\omega_s) \\ \overline{Z}_d &= L_m (p + j\omega_r) \\ \overline{Z}_r &= R_r + (L_r + L_m)(p + j\omega_r) \end{aligned} \tag{2}$$

with *V,R,L* and *I* stands for per unit voltage, resistance, inductance and current; ω_s the stator angular frequency and is the rotor electrical angular speed ; s slip, L_m the mutual inductance, L_s and L_r the stator and rotor leakage inductance respectively and ω_m is the mechanical frequency of the generator. The indices s and r indicate stator and rotor quantities and in the later equations d and q indicate the direct and quadrature axis components.

The instantaneous torque equation is

$$T_m = J\dot{\omega} m + T_e \tag{3}$$

where

$$T_e = L_m \operatorname{Im}(\overline{I}_s \overline{I}_r^{\,*}) \tag{4}$$

The d-q component of currents are obtained as solution of the following system:

$$\begin{aligned} V_{ds} &= -R_s I_{ds} - \omega_s \psi_{qs} - \frac{d\psi_{ds}}{dt} \\ V_{qs} &= -R_s I_{qs} + \omega_s \psi_{ds} - \frac{d\psi_{qs}}{dt} \end{aligned} \tag{5.1}$$

$$\begin{aligned} V_{dr} &= -R_r I_{dr} - (\omega_s - \omega_m)\psi_{qr} - \frac{d\psi_{dr}}{dt} \\ V_{qr} &= -R_r I_{qr} + (\omega_s - \omega_m)\psi_{dr} - \frac{d\psi_{qr}}{dt} \end{aligned} \tag{5.2}$$

where Ψ stands for per unit flux and

$$\begin{aligned} \psi_{ds} &= (L_s + L_m) I_{ds} + L_m I_{dr} \\ \psi_{qs} &= (L_s + L_m) I_{qs} + L_m I_{qr} \\ \psi_{dr} &= (L_r + L_m) I_{dr} + L_m I_{ds} \\ \psi_{qr} &= (L_r + L_m) I_{qr} + L_m I_{qs} \end{aligned} \tag{6}$$

For mechanical parts the equations are:

$$\begin{aligned} \dot{\delta} &= \omega_m - \omega_s + \omega_r \\ J\dot{\omega} m &= T_m - T_e \end{aligned} \tag{7}$$

$$T_e = \psi_{ds} I_{qs} - \psi_{qs} I_{ds} \tag{8}$$

The Voltages are given by:

$$\begin{aligned} V_{ds} &= V_s \quad , \quad V_{qs} = 0 \\ V_{dr} &= V_r \cos\delta \quad , \quad V_{qr} = -V_r \sin\delta \end{aligned} \tag{9}$$

which define δ.

Assume a small perturbation around a steady state operating point. This can be described by:

$$\begin{aligned} \Delta V_{ds} &= -\frac{R_s L_{rr}}{K_1}\Delta\psi_{ds} + \frac{R_s L_m}{K_1}\Delta\psi_{dr} - \omega_s \Delta\psi_{qs} \\ &\quad - \Delta\omega_s \psi_{qs} - \frac{d\Delta\psi_{ds}}{dt} \\ \Delta V_{qs} &= -\frac{R_s L_{rr}}{K_1}\Delta\psi_{qs} + \frac{R_s L_m}{K_1}\Delta\psi_{qr} + \omega_s \Delta\psi_{ds} \\ &\quad + \Delta\omega_s \psi_{ds} - \frac{d\Delta\psi_{qs}}{dt} \\ \Delta V_{dr} &= -\frac{R_r L_{ss}}{K_1}\Delta\psi_{dr} + \frac{R_r L_m}{K_1}\Delta\psi_{ds} - (\omega_s - \omega_{m0})\Delta\psi_{qr} \\ &\quad - (\Delta\omega_s - \Delta\omega_m)\Delta\psi_{qr} - \frac{d\Delta\psi_{dr}}{dt} \\ \Delta V_{qr} &= -\frac{R_r L_{ss}}{K_1}\Delta\psi_{qr} + \frac{R_r L_m}{K_1}\Delta\psi_{qs} + (\omega_s - \omega_{m0})\Delta\psi_{dr} \\ &\quad + (\Delta\omega_s - \Delta\omega_m)\Delta\psi_{dr} - \frac{d\Delta\psi_{qr}}{dt} \end{aligned} \tag{10}$$

$$\text{where} \quad \begin{aligned} &L_{ss} = L_s + L_m, \quad L_{rr} = L_r + L_m, \\ &K_1 = L_{ss} L_{rr} - L_m^{\,2} \quad and \quad \Delta\omega_s = 0 \end{aligned} \tag{11}$$

$$\dot{X} = AX + BU \quad (12)$$

where $\dot{X} = [\Delta\psi_{ds}, \Delta\psi_{qs}, \Delta\psi_{dr}, \Delta\psi_{qr}, \Delta\delta, \Delta\omega_m]^T$ is perturbed state vector. The concrete expression of A is classical, and is omitted.

The numerical values of the parameters of DFIM are given in table1.

Table 1 The Parameters of DFIM

Symbol	Quantity	Per unit value
R_s	stator resistance	0.02
R_r	rotor resistance	0.03
L_s	stator leakage inductance	0.1
L_r	rotor leakage inductance	0.1
L_m	mutual inductance	2.7
I	Inertia	5
J	moment of inertia	100π I

Let us consider the steady state operating point defined by:

$$V_s = 1, \quad V_r = 0.1, \quad \omega_s = 1,$$

$$s = 0.1, \;\; \delta_0 = -\pi/6, \;\; \omega_{m0} = (1-s)\omega_s = 0.9$$

First we have made a complete state model of the sixth order system without any simplification. We have used the MATLAB for this purpose.

The eigenvalues of matrix A are:

-0.1011 ± 0.9837i
-0.1504 ± 0.1216i
-0.0031 ± 0.0367i

The output of the system are considered as:

$$Y = [\Delta T_e] \quad (13)$$

3. Slow and Fast Modes Direct Decomposition Method

In the slow and fast modes direct decomposition of a system based on the large and small time constants of the system.

A system $G(s)$ can be written in decomposition form as:

$$G(s) = [G(s)]_S + [G(s)]_F \quad (14)$$

where $[G(s)]_S$ represents the slow part of $G(s)$
$[G(s)]_F$ represents the fast part of $G(s)$

Transformation to Diagonal Form: A transformation that results in a diagonal form of the system matrix A can provide insight into the internal structure of a system.

Consider a system with distinct eigenvalues $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ and a modal matrix M, formed by adjoining columns of eigenvectors. Let Z be the transformed state vector, defined by $X=MZ$, so that the new set of state and output equations are[10,11]:

$$\begin{aligned} \dot{Z} &= (M^{-1}AM)Z + (M^{-1}B)U \\ Y &= (CM)Z + DU \end{aligned} \quad (15)$$

The new system matrix is $(M^{-1} AM)$. The product AM may be written in terms of the eigenvalues and eigenvectors

$$\begin{aligned} AM &= [Am_1 \,|\, Am_2 \,\ldots\ldots\ldots\ldots |\, Am_m] \\ &= [\lambda_1 m_1 \,|\, \lambda_2 Am_2 \,\ldots\ldots\ldots\ldots |\, \lambda_n m_m] \end{aligned} \quad (16)$$

because $Am_i = \lambda_i m_i$ is the relationship that defined the eigenvalue λ_i. Equation (16) can be rearranged and written

$$AM = [m_1 | m_2 \ldots\ldots\ldots\ldots | m_m] \begin{bmatrix} \lambda_1 & 0 \;\ldots\ldots\ldots\ldots\ldots\ldots 0 \\ 0 & \lambda_2 \ldots\ldots\ldots\ldots\ldots\ldots 0 \\ . & \\ . & \\ 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \lambda_n \end{bmatrix}$$

$$= M\Lambda \quad (17)$$

where Λ is the diagonal nxn square matrix containing the system eigenvalues on the leading diagonal

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \;\ldots\ldots\ldots\ldots\ldots\ldots 0 \\ 0 & \lambda_2 \ldots\ldots\ldots\ldots\ldots\ldots 0 \\ . & \\ . & \\ 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \lambda_n \end{bmatrix} \quad (18)$$

If both sides of Eq. (17) are pre-multiplied by M^{-1}

$$M^{-1}AM = M^{-1}M\Lambda = \Lambda \quad (19)$$

The transformed state equations are

$$\dot{Z} = \Lambda Z + B^{'}U \quad (20)$$

where $B^{'} = (M^{-1}B)$. Equation (20) represents a set of n uncoupled first-order differential equations, each of which may be written as

$$\dot{z}_i = \lambda_i z_i + \sum_{j=1}^{r} b_{ij}^{'} u_j \quad (21)$$

and does not involve any cross coupling from other states. The homogeneous state equations $Z=\Lambda Z$ are simply $z_i = \lambda_i z_i$.

After the transformation to diagonal form, we get

$$\Lambda = M^{-1}AM = \begin{bmatrix} \Lambda_{11} & 0 \\ 0 & \Lambda_{22} \end{bmatrix} \tag{22}$$

Based on the style of ordered reduction

$\left|\lambda_i(\Lambda_{11})\right| \langle \left|\lambda_i(\Lambda_{22})\right|$.

we get the state-space projections for *[G(s)]*$_S$ and *[G(s)]*$_F$

$$\left[G(s)\right]_S = (\Lambda 11, B'1, C'1, D1) \tag{23}$$

and

$$\left[G(s)\right]_F = (\Lambda 22, B'2, C'2, D2) \tag{24}$$

where

$$\begin{bmatrix} B'1 \\ B'2 \end{bmatrix} = B' = (M^{-1}B) \tag{25}$$

and

$$\begin{bmatrix} C_1' & | & C_2' \end{bmatrix} = C' = CM \tag{26}$$

The systems with repeated eigenvalues may not be reducible to a diagonal form, but may be represented in a closely related form, known as the Schur form. A unitary matrix V can be found via the ordered Schur decomposition and get

$$V^H AV = \begin{bmatrix} \hat{A}11 & \hat{A}12 \\ 0 & \hat{A}22 \end{bmatrix} \tag{27}$$

4. Reduced Order Response

The original double fed induction machine model has six distinct eigenvalues. The proposed method of direct decomposition method is applied on the model to get fourth order and second order reduced models. The dynamic performance of the DFIM is analyzed due to the step variation in T_m. To observe the performance of the method the results are compared with the modes decomposition method based on Schur form and Balance model truncation via square root method based on hankel S.V.[12-14].

Fig. 3 shows the output response of the original system and fourth order reduced system. In Fig. 3, E1 shows the response of the reduced order system by the proposed method. E2 and E3 show the responses of the reduced order system by Schur form and balanced truncation method respectively. As seen from the figure, the response of the reduced system is indistinguishable from the response of the original system. The magnified view of the response is also shown in Fig. 3. The balanced truncation method gives inferior results compared to other methods. Fig. 4 shows responses of the original system and second order reduced systems. The magnified view of the response shows that balanced truncation method gives better results when the system is reduced to a second order system.

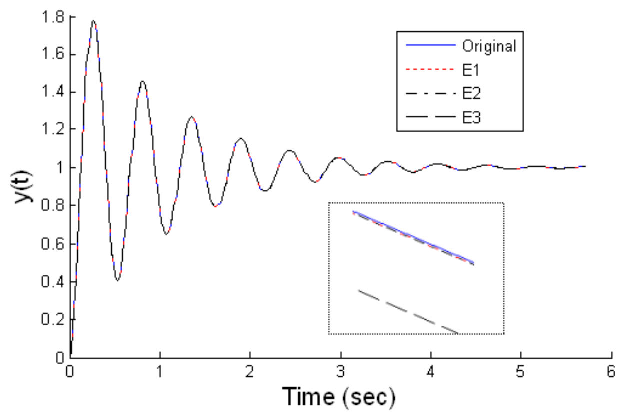

Fig.3. Response of the original system and the fourth order reduced systems

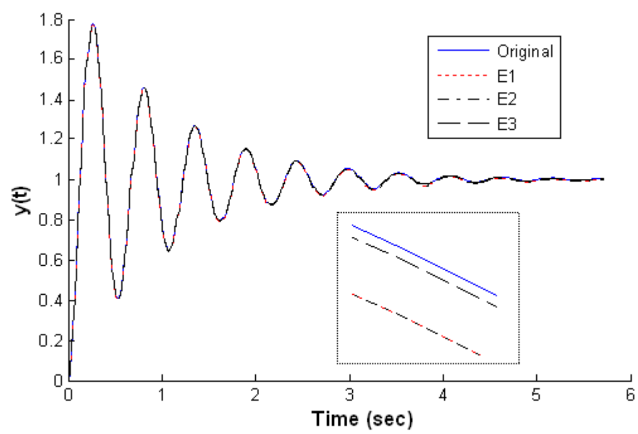

Fig.4. Response of the original system and the second order reduced systems

For the sake of global evaluation of various methods, it is reasonable to introduce an error index. If *Y(t)* is the output response of the original system and *Ŷ(t)* is the output of reduced system, let us consider the error criterion *e* as follows:

$$e = \frac{\int_0^\tau \sqrt{(Y-\hat{Y})^2}\, dt}{\int_0^\tau Y dt} \tag{28}$$

The comparison of above results on different values of slip *s* and per unit inertia *I* is shown in tables 2 and 3. The proposed method (E1) is compared with the other two methods (E2 and E3) mentioned earlier. Table 2 shows the above defined error for fourth order reduced order model. As slip and per unit inertia increase, the error is decreasing. Balanced model truncation method (E3) gives inferior results compared to other two methods. Table 3 shows the error for second order reduced order model. In this case, method 3 gives better results.

Table 2
Error Criterion or Fourth Order Reduced Order Model

s \ I		1	2	5	10
0.02	E1	3.0352E-05	2.4819E-05	6.3658E-06	2.6772E-06
	E2	3.0352E-05	2.4807E-05	6.3738E-06	2.6801E-06
	E3	2.9250E-04	3.0321E-04	1.1444E-04	4.6457E-05
0.1	E1	7.0135E-06	3.2587E-06	9.9529E-07	3.3428E-07
	E2	7.0140E-06	3.2586E-06	9.9529E-07	3.3646E-07
	E3	5.8611E-05	4.2805E-05	2.2068E-05	8.8844E-06
0.12	E1	3.5207E-06	1.9317E-06	3.6231E-07	1.1733E-07
	E2	3.5204E-06	1.9315E-06	3.6348E-07	1.1714E-07
	E3	4.3179E-05	3.8490E-05	1.4354E-05	6.0245E-06
0.15	E1	1.0042E-06	7.3886E-07	1.5752E-07	6.1716E-08
	E2	1.0044E-06	7.5033E-07	1.6430E-07	9.9698E-08
	E3	2.1193E-05	2.4052E-05	6.4501E-06	2.6446E-06

Table 3
Error Criterion for Second Order Reduced Order Model

s \ I		1	2	5	10
0.02	E1	7.0455E-02	1.7540E-01	1.5310E-02	5.7284E-03
	E2	2.4970E-01	7.4422E-02	1.5310E-02	5.7284E-03
	E3	5.4348E-03	1.2587E-02	8.5286E-03	4.3645E-03
0.1	E1	6.2580E-04	4.3386E-04	3.2486E-04	1.4859E-04
	E2	6.2580E-04	4.3385E-04	3.2486E-04	1.4859E-04
	E3	2.4810E-03	8.0913E-04	7.4974E-05	9.1442E-05
0.12	E1	1.6434E-03	1.6298E-03	6.8395E-04	3.0078E-04
	E2	1.6433E-03	1.6298E-03	6.8394E-04	3.0078E-04
	E3	2.1588E-04	8.4581E-04	5.6272E-04	2.7501E-04
0.15	E1	1.3578E-03	1.7178E-03	5.0203E-04	2.1258E-04
	E2	1.3578E-03	1.7178E-03	5.0202E-04	2.1258E-04
	E3	1.0328E-03	1.5582E-03	4.8716E-04	2.0972E-04

The error plots for fourth order model are plotted on a suitable logarithmic scale with respect to slip at different operating conditions. The plots are shown in Fig.5

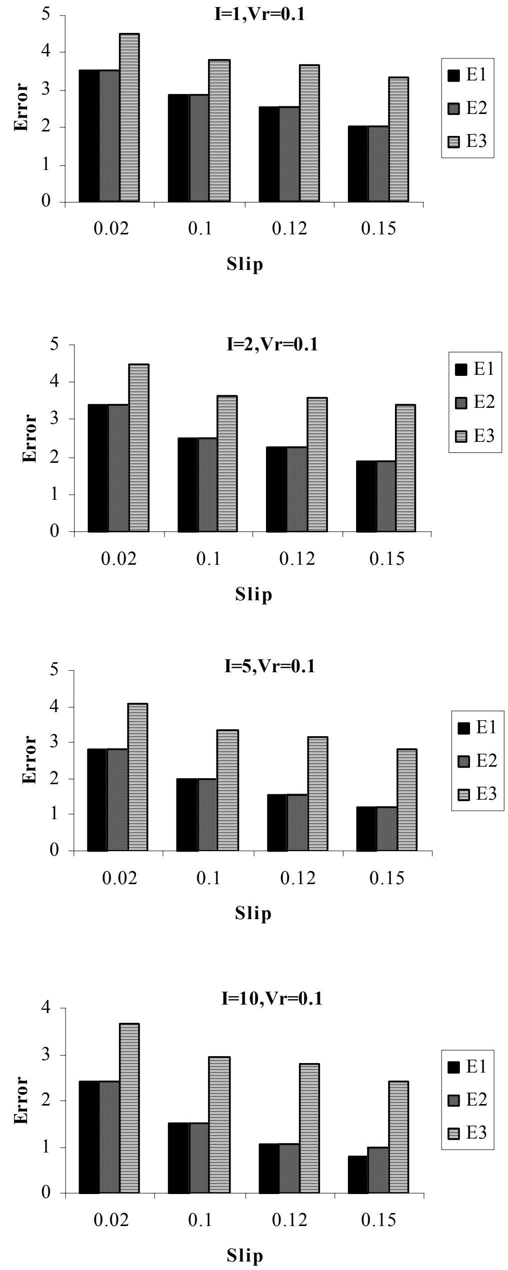

Fig.5. Error plots for fourth order model with respect to the slip at different operating conditions

The above results are dependent on the operating point. It is observed that an increase in slip increases the absolute value of rotor eigenvalues. In such cases both stator and rotor quantities can be considered as rapidly varying quantities. It is clear from the analysis that the fourth order response by slow and fast mode direct decomposition yields good results, while in the case of second order response, the third method shows good results. It is concluded that when the difference between the modes are more the slow and fast mode decomposition method will present good results.

5. Conclusion

For power system stability studies of power systems including wind turbines it is desirable to apply reduced models of the turbines in order to limit the computation time. In this paper, an approach based on decomposition of slow and fast mode is used to derive a reduced order model of double fed induction generator. The accuracy of the model is verified by comparing the step responses of the reduced order system and the original system. The proposed method is compared with standard model reduction techniques. This method leads generally good results and also requires less mathematical computation.

References

[1] J.G. Slootweg, H. Polinder, and W.L. Kling, Dynamic modeling of a wind turbine with doubly fed induction generator, *IEEE Power Engineering Society Summer Meeting 2001,* July 15-19, 2001, 644 - 649.

[2] A. Tapia, G. Tapia, J.X. Ostolaza, and J.R. Saenz, Modeling and control of a wind turbine driven doubly fed induction generator, *IEEE Trans. on Energy Conversion, 18(2)*, June 2003, 194-204.

[3] V. Akhimatov, Modelling of variable-speed wind turbines with doubly-fed induction generators in short-term stability investigations, in: *Proc. 3rd Int. Workshop on Transmission Networks for Offshore Wind Farms*, April 11-12, 2002, Stockholm, Sweden.

[4] M.S. Vicators and J.A. Ieqopouios, Steady state analysis of a doubly-fed induction generator under synchronous operation, *IEEE Trans. on Energy Conservation*, *4(3)* , September 1989, 495 - 501.

[5] J. Tamura, T. Murata, I. Takeda, J. Hasegawa, and H. Fujiwara, New approach to the steady state stability analysis of synchronous machines, *IEEE Trans. on Energy Conservation, 3(2),* June 1988, 323 - 329.

[6] C. Concordia, S.B. Crary, and G. Kron, The doubly fed machine, *IEEE Trans. on Electrical Energy, 61*, May 1942, 286 – 289.

[7] N. Derbel and M. Poloujadoff, Two step three time scale reduction of doubly fed machine models, *IEEE Trans. on Energy Conversion,9(1),*March1994,77-84.

[8] M.G. Ioannides, Doubly fed induction machine state variables model and dynamic response, *IEEE Trans. on Energy Conservation, 6*, March 1991, 55 - 61.

[9] H.M.B. Metwally, F.E. Abdel-Kader, H.M. El-Shewy, and M.M. El-Kholy, Optimum performance characteristics of doubly fed induction motors using field oriented control, *Elsevier, Energy Conversion* And *Management, 43,* 2002, 3-13.

[10] P. Kundur*, Power system stability and control* (McGraw-Hill. NY, USA, 1994).

[11] T. Kailath, *Linear systems* (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1980).

[12] K. Glover, All Optimal Hankel Norm Approximation of Linear Multivariable Systems, and Their Lμ -error bounds, *International Journal of Control*, *39(6),* 1984, 1145-1193.

[13] M.G. Safonov and R. Y. Chiang, A Schur Method for balanced model reduction, *IEEE Trans. on Automat. Contr.*, *34(7),* July 1989, 729-733.

[14] M. G. Safonov, E. A. Jonckheere, M. Verma, and D. J. N. Limebeer, Synthesis of Positive Real Multivariable Feedback Systems, *International Journal of Control, 45(3),* 1987, 817-842.