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# A POWER FLOW METHOD FOR RADIAL AND MESHED DISTRIBUTION SYSTEMS INCLUDING DISTRIBUTED GENERATION AND STEP VOLTAGE REGULATOR MODELING

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**ABSTRACT**
This paper proposes a new power flow method for radial and meshed distribution systems. The method convert a meshed distribution system with distributed generators into an equivalent single source radial distribution system, by breaking out the tie lines and distributed generators connections, and by abstracting auxiliary buses to the distribution network. The approach goes beyond the previous works by formulating a power flow algorithm considering the combined modeling of the distributed generators, voltage dependency of static loads, line charging capacitances, shunt capacitors and the step voltage regulator. In addition, the representation of the automatic voltage regulation at the buses where the distributed generators are connected, as well as the manual and automatic operational modes of the step voltage regulator, are also contemplated in the formulation. The proposed approach has been tested using several distribution networks providing good performance and promising results. Some of these results are presented on case studies for a 27-bus distribution network, in order to illustrate the applicability of the developed approach, as well as foster discussions about future works.

**KEY WORDS**
Power flow analysis, power distribution, distributed power generation, step voltage regulator.

## 1. Introduction

Power flow analysis provides a steady state condition of a power system and is one of the main tools for modern power systems planning, operation, controlling and designing. Because of its importance, several efficient methods have been developed for the calculation of the power flow in high voltage transmission systems. These methods, however, do not preserve their efficiency and convergence proprieties when applied to power distribution systems, due to the particular topological structure and the low $x/r$ ratios of the distribution networks. As consequence, many research efforts have been recently performed to develop power flow methods suited to radial distribution system applications, most of them based on Cespedes' [1] classical approach.

Concomitant to these research efforts, power distribution delivery paradigm have changed considerably. Electricity market deregulation demanded utilities to adopt meshed configurations in order to improve service quality and reliability. By aiming to improve voltage profiles and customer's satisfaction, studies about optimal capacitor placement [2] and optimal step voltage regulator placement have become primary concern for the utilities. There is also an increase of distributed generation resources connected to the power distribution grid, reducing the load peaks, losses and postponing investments. The interconnection of the distributed generators also provides local automatic voltage regulation as well as the reverse power flow in the distribution feeders.

In this new scenario of electricity market restructuring, some researches worked on the formulation and calculation of the power flow for meshed distribution networks. In [3], D. Shirmohammadi et al. firstly approached the problem by formulating a compensation-based power flow method for weakly meshed distribution and transmission networks. This pioneer work was considerably improved by G. X. Luo and A. Semlyem, in [4], with the introduction of the concept of the Load Break Points (LBPs), assigning the opening of the loops by abstracting ficticious/auxiliary buses to the networks. M. Haque also presented good contributions on this area by formulating power flow algorithms including the representation of the line charging capacitance effect [5] and the networks with multiple feeding sources [6].

This paper approaches the power flow for radial and meshed distribution systems, going beyond the previous works by considering the combined modeling of the distributed generators, voltage dependency of static loads, line charging capacitances, shunt capacitors and the step voltage regulator. Furthermore, the formulation also comprises the representation of the automatic voltage regulation at the buses where the distributed generators are connected, as well as the manual and automatic operational modes of the step voltage regulator.

The document is divided into five sections. Section 2 and 3 present, respectively, the distribution systems modeling as well as the conversion of a meshed distribution system with distributed generators into an equivalent single source radial distribution system. In section 4, the developed new power flow method for radial and meshed distribution systems is formulated. Section 5 illustrates the application of proposed algorithm in case studies for a 27-bus distribution network. Finally, in section 6, conclusions and discussions are outlined by the authors.

## 2. System Modeling

This section approaches the representation of the line charging capacitances, capacitors, static voltage dependent loads, distributed generation and the step voltage regulator, in distribution networks power flow applications.

### 2.1 Line Charging Capacitance

The primary mission of a power delivery system is to transfer large blocks of power from the sources to points of consumption. The blocks of power flow across the line segments, transformers, as well as the control and protection devices (switches, relays, breakers, etc.). These components can be represented by $\pi$ equivalent models, de□ned by a series admittance $y_{ik}$ and a line charging admittance $y_{ik}^{sh}$, as shown in Figure 1. In the schematic, the complex voltages are denoted by $E_n = V_n \angle \theta_n, \forall n = 1, ..., N_{\text{buses}}$.

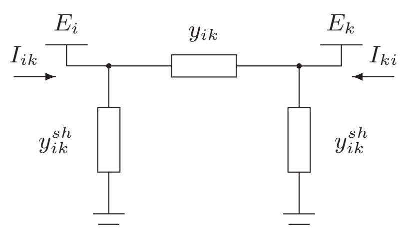


Figure 1. A distribution line equivalent model.

Although it is a common practice to ignore from modeling the line charging capacitance effect in distribution system analysis, this effect may not be neglected, mainly for underground distribution systems applications.

### 2.2 Capacitor

Capacitors are constant admittance components and they are usually installed in shunt configuration to provide reactive power compensation, power factor correction, improve voltage profiles, reduce losses and increase system capacity of the distribution networks. Normally rated at 50, 100, 150, 200, 300, and 400 kVAR, the corresponding reactive power supplied by these components are computed as follows.

$$Q_{C_k}^{pu} = y_{C_k} V_k^2 \tag{1}$$

where, $Q_{C_k}^{pu}$ denotes the per unit reactive power supplied by the capacitor at bus $k$, $y_{C_k}$ represents the per unit constant admittance modeling the capacitor at bus $k$, and $V_k$ is the per unit voltage magnitude at bus $k$.

### 2.3 Voltage Dependency of Static Loads

Static load models attempt to represent the aggregation of component devices in active and reactive demand functions of the systems voltage and frequency. In power flow studies, frequency deviations are not taken into account and load models for active and reactive power demand are usually expressed in the composed polynomial/exponential form

$$P_{L_k} = P_{L_k}^0 \left( \alpha_{P_k} + \beta_{P_k} V_k + \gamma_{P_k} V_k^2 + \epsilon_{P_k} V_k^{\eta_{P_k}} \right) \tag{2}$$

$$Q_{L_k} = Q_{L_k}^0 \left( \alpha_{Q_k} + \beta_{Q_k} V_k + \gamma_{Q_k} V_k^2 + \epsilon_{Q_k} V_k^{\eta_{Q_k}} \right) \tag{3}$$

where

$$\alpha_{P_k} + \beta_{P_k} + \gamma_{P_k} + \epsilon_{P_k} = 1 \tag{4}$$

$$\alpha_{Q_k} + \beta_{Q_k} + \gamma_{Q_k} + \epsilon_{Q_k} = 1 \tag{5}$$

In equations above, $P_{L_k}^0$ and $Q_{L_k}^0$ represent the active and reactive powers at nominal voltage at bus $k$, and $V_k$ is the the per unit voltage magnitude at bus $k$. The coefficients $\alpha_{P_k}, \beta_{P_k}, \gamma_{P_k}, \epsilon_{P_k}, \alpha_{Q_k}, \beta_{Q_k}, \gamma_{Q_k}, \epsilon_{Q_k}$ and the exponents $\eta_{P_k}$ and $\eta_{Q_k}$, are estimated values and depend on the load composition.

We bring out that the line charging capacitance effect as well as the installation of a capacitor bank can also be aggregated at a load model. In fact, by denoting $y_k^{sh}$ the equivalent effect of the line charging capacitances at a bus $k$, and $\Lambda_k$ the set of immediate downstream buses of bus $k$, the static active $P_{L_k}^c$ and reactive $Q_{L_k}^c$ demand functions can now be given by

$$P_{L_k}^c = P_{L_k} \tag{6}$$

$$Q_{L_k}^c = Q_{L_k}^0 \left( \alpha_{Q_k} + \beta_{Q_k} V_k + \gamma_{Q_k}^c V_k^2 + \epsilon_{Q_k} V_k^{\eta_{Q_k}} \right) \tag{7}$$

where

$$\gamma_{Q_k}^c = \gamma_{Q_k} + y_k^{sh} + y_{C_k} \tag{8}$$

$$y_k^{sh} = y_{ik}^{sh} + \sum_{n \in \Lambda_k} y_{kn}^{sh} \tag{9}$$

### 2.4 Distributed Generation

Distributed generation provide an important vehicle for promoting liberalization of the energy markets. By using distributed generation resources (wind turbines, fuel cells, photovoltaics, biomass, etc.), new producers generate electricity for their own needs and send the surplus to the power grid, decentralizing the production and forcing an inverse power flow across the distribution networks.

There are four basic operational modes for these generators, depending upon the contract and the control devices installed: a local generator supplying a large demand at a fixed active and reactive power; generator supplying the power grid at a specified power factor; a generator supplying the power grid at a specified terminal voltage; a combination of the previous cited operational modes.

### 2.5 Step Voltage Regulator

Step voltage regulators are auto-transformers with manual or automatic tap changer on their windings, installed to boost or buck the voltages along primary feeders. In the manual mode, the tap position $l$ can be manually changed on the regulator's control board to increase or decrease output voltage. Since the tap position is specified, a $\pi$ equivalent model can be used to represent these components, as shown in Figure 2.

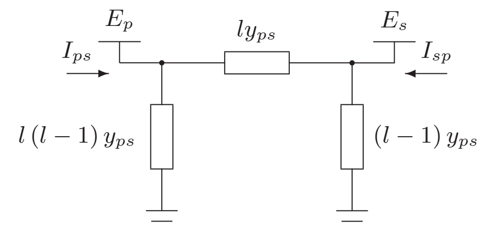


Figure 2. Step voltage regulator $\pi$ equivalent model.

In the automatic mode, the regulator control mechanism adjust the tap position to assure that the voltage being monitored is within certain range. The proposed approach uses an alternative step voltage regulator model to represent this operational mode, shown in Figure 3.

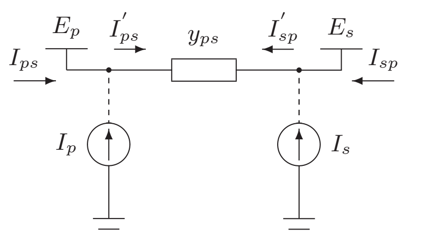


Figure 3. Step voltage regulator model with a tap independent series admittance.

This model is conveniently composed by two current sources and a tap independent series admittance. The current injections at the primary side $I_p$ and at the secondary $I_s$ are given by

$$I_p = \left(l^2-1\right) y_{ps} E_p - (l-1)\, y_{ps} E_s \qquad (10)$$

$$I_s = (1-l)\, y_{ps} E_p \qquad (11)$$

## 3. Meshed Configuration and Multiple Feeding Sources Modeling

The process to convert a distribution network with meshed configuration and multiple feeding sources into an equivalent single source radial system, aiming to preserve robustness of distribution systems recursive equations, was approached by M. Haque in [5, 6]. This technique is illustrated in Figure 4(a)-(b) and summarized as follows.

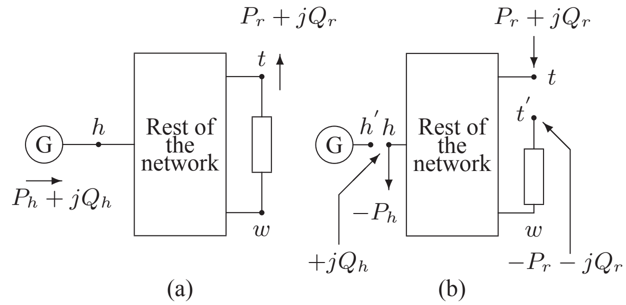


Figure 4. Conversion of a meshed network with distributed generation to a single source radial network.

Initially, the tie lines $w-t$ are eliminated abstracting auxiliary buses $t'$, where the injected power at buses $t$ and $t'$ are equal, of opposite signs, and calculated through the power flow iteration processes. Similarly, given a distributed generation connected at a bus $h$, an auxiliary bus $h'$ with equal voltage is abstracted, disconnecting the distributed generation from the system. The active power generation at the bus $h$ is assured and the reactive power generation depends on the specified terminal voltage. If the generation contract do not require automatic voltage regulation, the distributed generator can be modeled by a constant load, and the auxiliary bus $h'$ is no longer necessary. These break points $t-t'$ and $h-h'$ are named the Load Break Points (LBPs) and Generator Break Points (GBPs), respectively.

## 4. Power Flow Formulation

Since the distribution system with tie lines and distributed generations is transformed into a single source radial distribution system, the power flow problem can be partially described by a set of recursive equations. These equations can be solved through backward–forward iterative procedures, followed by the update of the power injections at the LBPs and GBPs, as well as the update of the tap position at the step voltage regulator.

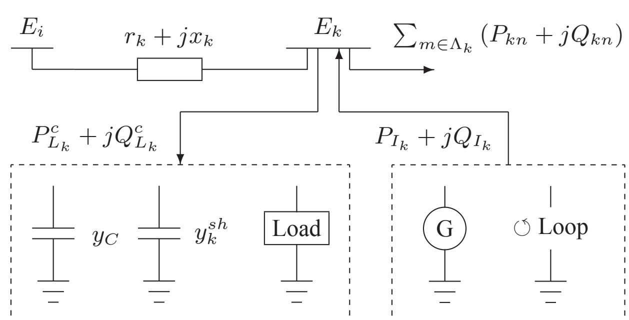


Figure 5. Distribution line segment schematic.

### 4.1 Power Flow Equations

Consider the schematic of the line segment $i-k$ shown in Figure 5, where the line series impedance and the accumulated injected power at bus $k$ are denoted by $r_k + jx_k$ and $S_k = P_k + jQ_k$, respectively. By assuming the load modeling described at Section 2.3, the accumulated injected power at bus $k$ can be written as functions of downstream power losses, power loads $\left(P_{L_k}^c, Q_{L_k}^c\right)$, and power injections $\left(P_{I_k}, Q_{I_k}\right)$. Separating this accumulated power at bus $k$ in its real and imaginary terms, we have

$$P_k = P_{L_k}^c - P_{I_k} + \sum_{n\in\Lambda_k}\left[P_n + r_n\left(\frac{P_n^2+Q_n^2}{V_n^2}\right)\right] \quad (12)$$

$$Q_k = Q_{L_k}^c - Q_{I_k} + \sum_{n\in\Lambda_k}\left[Q_n + x_n\left(\frac{P_n^2+Q_n^2}{V_n^2}\right)\right] \quad (13)$$

Once the accumulated powers at bus $k$ are known, the voltage magnitude and angle can be calculated by solving

$$V_k^4 + A_k V_k^2 + B_k = 0 \quad (14)$$

$$\theta_k = \theta_i + \tan^{-1}\left(\frac{P_k x_k - Q_k r_k}{P_k r_k + Q_k x_k - V_i^2}\right) \quad (15)$$

where

$$\begin{aligned} A_k &= 2\left(P_k r_k + Q_k x_k\right) - V_i^2 && (16)\\ B_k &= \left(P_k^2 + Q_k^2\right)\left(r_k^2 + x_k^2\right) && (17)\end{aligned}$$

### 4.2 Injected Power Update

The power injected at the break points can be straightforwardly obtained through operations in a reduced order impedance matrix, representing the sensitivities among buses associated to the break points. Consider the follow sets of buses [6].

- Set $a$: Composed by the original buses $h$ at the GBPs;
- Set $b$: Composed by the original buses $t$ at the LBPs;
- Set $c$: Composed by the auxiliary buses $t'$ at the LBPs;
- Set $d$: Composed by the rest of the system buses regardless the swing bus.

By choosing the bus ordering above, and by representing the loads as constant shunt admittances at nominal voltage, the node equation of the system can be written as $\mathbf{I} = \mathbf{YV}$. Since the currents injected at the buses in the Set $d$ are zero, these buses can be eliminated by Kron's reduction to obtain the impedance matrix below.

$$\begin{bmatrix}\mathbf{V_a}\\ \mathbf{V_b}\\ \mathbf{V_c}\end{bmatrix} = \begin{bmatrix}\mathbf{Z_{aa}} & \mathbf{Z_{ab}} & \mathbf{Z_{ac}}\\ \mathbf{Z_{ba}} & \mathbf{Z_{bb}} & \mathbf{Z_{bc}}\\ \mathbf{Z_{ca}} & \mathbf{Z_{cb}} & \mathbf{Z_{cc}}\end{bmatrix}\begin{bmatrix}\mathbf{I_a}\\ \mathbf{I_b}\\ \mathbf{I_c}\end{bmatrix} \quad (18)$$

Once bus voltage magnitudes are close to 1 pu and voltage angles for distribution systems are very small, by using the angle mismatches $\Delta\delta$ at the LBPs, as well as the voltage mismatches $\Delta V$ at the GBPs and LBPs, the incremental power injections required at the break points can be calculated by the system equation shown below[1].

$$\begin{array}{c}\text{GBPs + LBPs}\\ \text{LBPs}\end{array}\begin{bmatrix}\Delta Q\\ \Delta P\end{bmatrix} = \begin{bmatrix}J_{11} & J_{12}\\ J_{21} & J_{22}\end{bmatrix}^{-1}\begin{bmatrix}\Delta V\\ \Delta\delta\end{bmatrix}$$

where,

$$J_{11} = \Im\begin{bmatrix}\mathbf{Z_{aa}} & \mathbf{Z_{ab}} - \mathbf{Z_{ac}}\\ \mathbf{Z_{ba}} - \mathbf{Z_{ca}} & \mathbf{Z_{bb}} + \mathbf{Z_{cc}} - \mathbf{Z_{bc}} - \mathbf{Z_{cb}}\end{bmatrix}$$

$$J_{12} = \Re\begin{bmatrix}\mathbf{Z_{ab}} - \mathbf{Z_{ac}}\\ \mathbf{Z_{bb}} + \mathbf{Z_{cc}} - \mathbf{Z_{bc}} - \mathbf{Z_{cb}}\end{bmatrix}$$

$$J_{21} = \Re\begin{bmatrix}\mathbf{Z_{ca}} - \mathbf{Z_{ba}} & \mathbf{Z_{bc}} + \mathbf{Z_{cb}} - \mathbf{Z_{bb}} - \mathbf{Z_{cc}}\end{bmatrix}$$

$$J_{22} = \Im\begin{bmatrix}\mathbf{Z_{bb}} + \mathbf{Z_{cc}} - \mathbf{Z_{bc}} - \mathbf{Z_{cb}}\end{bmatrix}$$

Therefore, an update rule can be used to update the injections at the break points until the mismatches become zero. In equation below, $\alpha$ and $\beta$ are acceleration factors, used to speed up the algorithm.

$$Q_I^{iter+1} = Q_I^{iter} + \alpha\Delta Q \quad (19)$$

$$P_I^{iter+1} = P_I^{iter} + \beta\Delta P \quad (20)$$

### 4.3 Automatic Step Voltage Regulator Update

In the case of the step voltage regulator is operating at the manual mode, this equipment can be easily represented as a line segment in the power flow algorithm, where the series and shunt admittances are dependent of the tap position, and are calculated before the iterative procedures.

In the automatic mode, we can conveniently choose the step voltage regulator model presented in Figure 3, to avoid the recalculation of the reduced order impedance matrix in the power flow procedures. Furthermore, it must be assured that the monitored voltage ranges a specified voltage $V_{\text{SVR}}$, depending of a set of discrete tap positions $L = \{l_i, \forall i = 1, ..., N_{\text{taps}}\}$. This result can be obtained by the utilization of the following update rule.

$$l^{iter+1} = l^{iter} - \gamma\left(V_s^{iter} - V_s^{ref}\right) \quad (21)$$

where $\gamma$ is a discount factor and

$$V_s^{ref} = V_p^{iter}\left(\arg_l \min\left\{l^{iter} V_p^{iter} - V_{\text{SVR}}\right\}\right) \quad (22)$$

### 4.4 Proposed Algorithm

The algorithm proposed as the new power flow method for radial and meshed distribution system is presented in Figure 6. It is needed to emphasize that if the limits of the reactive power generation in a distributed generator is achieved, the distribute generator can be modeled by a load bus, setting the reactive generation on its limit. Similar procedures can be performed to handle the physical limitations of the step voltage regulator.

[1] A complete deduction and discussions about these equations can be found in [6].

Read system data and obtain the radial equivalent network using the approach described in section 3;
Initiate bus voltages assuming a flat start or using an approximated solution;
Set the initial power injection $(P_I, Q_I)$ at the break points equal to zero;
**while** $\left|v_i^{iter} - v_i^{iter-1}\right| <$ tolerance, $\forall i$ **do**
  **for** all $i$, following the backward direction **do**
    Calculate the accumulated active and reactive powers at the network buses using (12) and (13), respectively;
  **end for**
  **for** all $i$, following the forward direction **do**
    Obtain bus voltage magnitudes starting from the first bus using (14)–(17);
  **end for**
  Update the required power injections at the break points using (19) and (20);
  Update the tap position in the step voltage regulator using (21);
**end while**
Print result reports.

Figure 6. Proposed power flow algorithm.

## 5. Numerical Results

The proposed approach was implemented in a MATLAB scientific computing environment and tested with distribution networks obtained at literature and with Brazilian distribution networks. It's presented in this section, numerical power flow solutions for a 27-bus distribution network obtained in [7], on the several case studies described below. For all simulations, it was considered a tolerance of 0.000001, $\alpha = 1$, $\beta = 1$, $\gamma = 0.00001$, and voltage at the substation bus equal to 11kV. A summary of the equipments used on the case studies are indicated in Figure 7.

Case I: The original radial distribution network without any tie lines, distributed generators, line charging capacitance effect, capacitors or step voltage regulator;

Case II: The line charging capacitance effect is represented on the original radial distribution network. Similar to the approach developed in [6], it was considered that the shunt admittance of a branch with impedance $r + jx$ is $2.10^{-4}\left(r^2 + x^2\right)^{0.5}$;

Case III: Modeling the line charging capacitance effect, a tie line between buses 14 and 25 is added to the original network, with a $2 + j2\ \Omega$ impedance;

Case IV: Similar to the previous case, but including a step voltage regulator (SVR) at the bus 3, with internal impedance of $1.851 + j1.268\ \Omega$, and with output voltage adjusted to range near 1 pu;

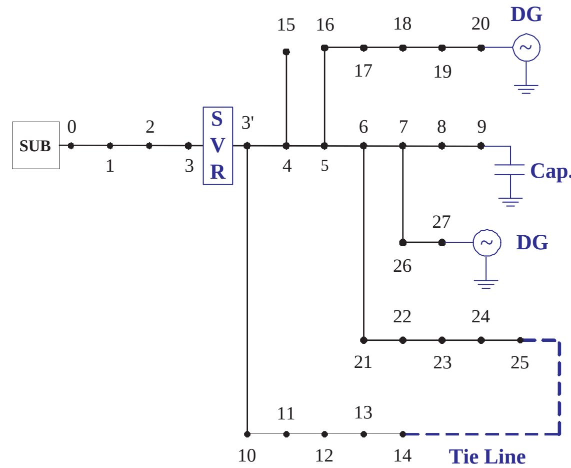


Figure 7. Case studies schematic for a 27-bus network.

Case V: Similar to the case III, but including distributed generators (DGs) at buses 20 and 27, with bus voltages specified at 0.9800 pu and 0.9700 pu, and generation assumed to be 200kW and 300kW, respectively;

Case VI: Similar to the case IV, but including distributed generators at buses 20 and 27, with bus voltages specifiedat 1.0150 pu and 1.0060 pu, and generation assumed to be 200kW and 300kW, respectively. It's also modeled a 50 kVAR capacitor bank at bus 9.

The power flow voltage magnitude solutions obtained for the cases I and II are shown in Table 1. In addition, active losses $L_P$ and reactive losses $L_Q$, as well as simulation runtime for all six cases are presented in Table 2. As expected, the representation of the line charging capacitive effect as well as the installation of a capacitor increase voltage pro□lesand reduced total active and reactive losses.

Table 1. Per unit voltage magnitudes for the cases I and II.

| Bus | Case I | Case II | Bus | Case I | Case II |
|---|---|---|---|---|---|
| 1 | 0.9862 | 0.9863 | 14 | 0.9428 | 0.9432 |
| 2 | 0.9665 | 0.9667 | 15 | 0.9371 | 0.9375 |
| 3 | 0.9524 | 0.9527 | 16 | 0.9259 | 0.9264 |
| 4 | 0.9382 | 0.9386 | 17 | 0.9249 | 0.9254 |
| 5 | 0.9277 | 0.9282 | 18 | 0.9232 | 0.9237 |
| 6 | 0.9185 | 0.9191 | 19 | 0.9224 | 0.9229 |
| 7 | 0.9160 | 0.9166 | 20 | 0.9217 | 0.9223 |
| 8 | 0.9158 | 0.9164 | 21 | 0.9156 | 0.9162 |
| 9 | 0.9155 | 0.9161 | 22 | 0.9141 | 0.9147 |
| 10 | 0.9462 | 0.9465 | 23 | 0.9129 | 0.9135 |
| 11 | 0.9444 | 0.9448 | 24 | 0.9126 | 0.9133 |
| 12 | 0.9433 | 0.9437 | 25 | 0.9125 | 0.9131 |
| 13 | 0.9431 | 0.9434 | 26 | 0.9155 | 0.9161 |
| - | - | - | 27 | 0.9154 | 0.9160 |

Table 2. Losses and simulation runtime for the case studies.

| | $L_P$ (kW) | $L_Q$ (kVAR) | Runtime (s) |
|---|---|---|---|
| Case I | 57.565 | 46.042 | 0.0465 |
| Case II | 56.408 | 45.274 | 0.0462 |
| Case III | 51.454 | 41.197 | 0.0785 |
| Case IV | 46.577 | 40.716 | 0.0905 |
| Case V | 15.930 | 17.918 | 0.0826 |
| Case VI | 3.686 | 12.040 | 0.1321 |

In Table 3, the power flow voltage magnitude solutions for the cases III–VI are presented. In these cases, the LBP was assigned by the abstraction of the auxiliary bus 25'. In fact, the voltage magnitudes at buses 25 and 25' are equal, demonstrating the correct operation of the developed method. Furthermore, the specified voltages at the distributed generators were assured. The reactive power injection obtained at the buses 20 and 27 were 48.5 and 58.6 kVAR, respectively for the case V, as well as 146.2 and 91.1 kVAR, respectively for the case VI. The specified voltages at the GBP were chosen to have reasonable values of reactive power injections. Finally, it was observed the taps positions obtained for the cases IV and VI were 1.0500 and 1.0275, respectively.

Table 3. Per unit voltage magnitudes for the cases III–VI.

| Bus | Case III | Case IV | Case V | Case VI |
|---|---|---|---|---|
| 1 | 0.9864 | 0.9864 | 0.9926 | 0.9939 |
| 2 | 0.9669 | 0.9670 | 0.9824 | 0.9858 |
| 3 | 0.9530 | 0.9531 | 0.9752 | 0.9801 |
| 3' | - | **1.0008** | - | **1.0070** |
| 4 | 0.9423 | 0.9907 | 0.9715 | 1.0046 |
| 5 | 0.9347 | 0.9834 | 0.9695 | 1.0036 |
| 6 | 0.9290 | 0.9781 | 0.9671 | 1.0021 |
| 7 | 0.9266 | 0.9758 | 0.9683 | 1.0041 |
| 8 | 0.9264 | 0.9756 | 0.9681 | 1.0041 |
| 9 | 0.9261 | 0.9753 | 0.9678 | 1.0042 |
| 10 | 0.9423 | 0.9907 | 0.9682 | 1.0006 |
| 11 | 0.9387 | 0.9873 | 0.9661 | 0.9987 |
| 12 | 0.9361 | 0.9848 | 0.9647 | 0.9976 |
| 13 | 0.9351 | 0.9838 | 0.9643 | 0.9972 |
| 14 | 0.9340 | 0.9828 | 0.9638 | 0.9969 |
| 15 | 0.9412 | 0.9896 | 0.9704 | 1.0035 |
| 16 | 0.9329 | 0.9818 | 0.9704 | 1.0047 |
| 17 | 0.9319 | 0.9808 | 0.9710 | 1.0054 |
| 18 | 0.9303 | 0.9793 | 0.9723 | 1.0068 |
| 19 | 0.9294 | 0.9785 | 0.9740 | 1.0087 |
| 20 | 0.9288 | 0.9779 | **0.9800** | **1.0150** |
| 21 | 0.9286 | 0.9777 | 0.9650 | 0.9997 |
| 22 | 0.9289 | 0.9779 | 0.9639 | 0.9985 |
| 23 | 0.9291 | 0.9782 | 0.9631 | 0.9977 |
| 24 | 0.9296 | 0.9786 | 0.9631 | 0.9975 |
| 25 | **0.9300** | **0.9790** | **0.9630** | **0.9974** |
| 26 | 0.9261 | 0.9754 | 0.9694 | 1.0053 |
| 27 | 0.9260 | 0.9752 | **0.9700** | **1.0060** |
| 25' | **0.9300** | **0.9790** | **0.9630** | **0.9974** |

## 6. Conclusion

This paper proposed a new power flow method for radial and meshed distribution systems modeling the distributed generators, voltage dependency of static loads, line charging capacitances, capacitors and the step voltage regulator. The approach convert a distribution system with meshed configuration and with distribution generators into an equivalent single source radial system, by breaking out the tie lines and distributed generator connections. The solution algorithm performs iterative forward-backward procedures and updates of the power injected at load and generator break points, as well as updates the tap position at the step voltage regulator. The proposed approach presented good performance and promising results. Future works will compare the developed algorithm with Newtonian based methods adjusted for distribution systems applications.

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