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MODELLING AND SIMULATION OF A PHOTOVOLTAIC MODULE

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ABSTRACT
A new model for current-voltage characteristics and maximum power operation of a photovoltaic (PV) module is presented which aims to model the effect on I-V and P-V curves of varying climatic conditions. This model has been implemented using the Matlab/Simulink program and is used to investigate the effect of meteorological conditions on the performance of a PV module generator. In order to validate the developed simulation model, simulated results from the model under standard condition are compared with specifications given by the manufacturer. There is a very good agreement between the results. In addition, it is shown that the maximum power available at low temperatures is higher than that at higher temperatures, and the maximum power is directly proportional to irradiance. Afterwards, a PV array model is derived. The benefit of improved curve fitting parameter is investigated as well.

KEY WORDS
Off-grid Photovoltaic Systems, Solar Cells, Maximum Power Point, Modelling, Simulation

1. Introduction

An off-grid photovoltaic (PV) system generally uses a solar array to supply electric power. The array consists of several solar modules which also consist of a number of solar cells in parallel and series.

Designing an efficient system means designing each component to operate at its maximum efficiency [1]. Therefore the study of off-grid PV systems' performances is well done by investigating the performance of each of its components. By modelling and simulating the PV module, which is the heart of off-grid PV systems, its electrical performances under various meteorological conditions, namely ambient temperature and solar irradiation, are investigated.

The main purpose of the study is to develop an accurate simulation model to predict the electrical characteristics of a PV module under varying climatic conditions. The model should improve the results presented in Hansen et al. [2], where the short circuit current and open circuit voltage of PV cell under operating conditions exclusively depends on sun intensity and temperature, respectively.

Many researchers have developed models for PV module. Ulleberg [3] and Wang [4] proposed a one diode model with assumption of independency of series resistor with respect to temperature and irradiance. The PV cell model described in Metwally [5] does not require additional parameters, such as number of cells in parallel and number of cells in series, to be extended to PV module model.

In this paper, a model of PV module is developed combining the findings of the above researchers. Starting with the empirical model of PV cell, empirical model of PV module is derived taking into account the effect of both temperature and irradiance on short circuit current and open circuit voltage. To improve the model, a new thermal voltage, which is the curve fitting parameter for the module, is introduced. The new model is then implemented in Matlab/Simulink environment for simulation. The simulated results are verified against specifications given in manufacturer's data sheet. The agreement between these results and PV module electrical characteristics given in literature is investigated as well. Finally, a PV array model is derived from the PV module model.

2. PV Cell Model

Figure 1 depicts the equivalent circuit of the PV cell empirical model. The circuit consists of a current source I_{ph}, a parallel-connected diode D and a series resistor R_s.

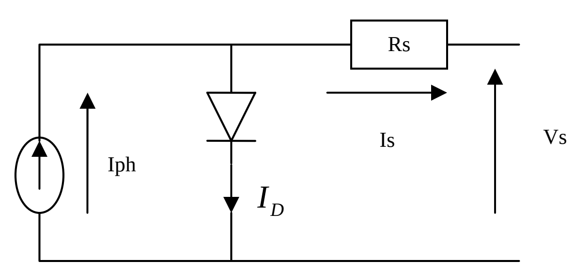


Figure 1. Equivalent circuit of PV cell

The equation describing the I-V curve of the PV cell is derived using Kirchoff current's law as follows:

$$I_s = I_{ph} - I_D \qquad (1)$$

The normal diode current is given by Lorenzo [6]:

$$I_D = I_o * \left[\exp \frac{e*\left(V_s + R_s * I_s\right)}{m*k*T_j} - 1 \right] \quad (2)$$

Substituting equation (2) into (1) yields:

$$I_s = I_{ph} - I_o * \left[\exp \frac{\left(V_s + R_s * I_s\right)}{V_t} - 1 \right] \quad (3)$$

where: I_o is the saturation current, [A]

$V_t = \frac{m*k*T_j}{e}$ is the thermal voltage, also called curve *fitting parameter*, with

$m \in [1\ 2]$-ideal factor of the PV cell

$k = 1.38*10^{-23}[J / {}^oK]$-Boltzmann's constant

$e = 1.6*10^{-19}[C]$-electron charge

The following assumptions can be made for modelling practical solar cell [6]:

- $I_{ph} = I_{sc}$ (4)
- $\exp\left(\frac{V_s + R_s * I_s}{V_t}\right) >> 1$ (5)

Putting (4) and (5) into (3):

$$I_s = I_{sc} - I_o * \exp\left(\frac{V_s + R_s * I_s}{V_t}\right) \quad (6)$$

where: I_{sc} -is the short-circuit current, [A]

At $I_s = 0$, $V_s = V_{oc}$ (7)

Substituting (7) into (6), and after simple mathematical manipulation, the following equation is found:

$$I_s = I_{sc} * \left[1 - \exp\left(\frac{V_S - V_{oc} + R_s * I_s}{V_t}\right)\right] \quad (8)$$

where: V_{oc} -is the open-circuit voltage, [V]

Equation (8) characterises the I-V curve of a PV cell. However, the commercial form of PV generator is the PV module. Hence, it is necessary to derive the I-V relation for PV module. This is done in the following section.

3. PV Module Model

The model for PV module is derived from the PV cell's model. Consider a PV module consisting of N_{pm} cells in parallel and N_{sm} cells in series. Thus, all currents in (8) are multiplied by N_{pm} and all voltages are multiplied by N_{sm}, that is:

$$\begin{aligned} I_{sm} &= N_{pm} * I_s \ \& \ I_{scm} = N_{pm} * I_{sc} \\ V_{sm} &= N_{sm} * V_s \ \& V_{ocm} = N_{sm} * V_{oc} \\ R_{sm} &= \left(\frac{N_{sm}}{N_{pm}}\right) * R_s \end{aligned} \quad (9)$$

Substituting (9) in (8):

$$\frac{I_{sm}}{N_{pm}} = \frac{I_{scm}}{N_{pm}} * \left[1 - \exp\left(\frac{\frac{V_{sm}}{N_{sm}} - \frac{V_{ocm}}{N_{sm}} + R_{sm} * \frac{N_{pm}}{N_{sm}} * \frac{I_{sm}}{N_{pm}}}{V_t} \right)\right] \quad (10)$$

After simplifications:

$$I_{sm} = I_{scm} * \left[1 - \exp\left(\frac{V_{sm} - V_{ocm} + R_{sm} * I_{sm}}{N_{sm} * V_t}\right)\right] \quad (11)$$

where: $N_{sm} * V_t$ - is the curve fitting parameter of the PV module.

To evaluate this parameter, the ideal factor, which has typical values between 1 and 2, must be estimated. The estimation of the ideal factor, also called diode quality factor, is not simple. Walker [7] suggests to make it variable parameter or to use two parallel diodes, where one of them has a quality factor of 1 and the other has a quality factor of 2. González-Longatt [8] proposes an ideal factor, which has an initial value of 1.3. This value may be adjusted through curve fitting to get a more accurate value. Obviously there is a need of curve fitting parameter which can be readily evaluated. Expressions for calculating the curve fitting parameter for a PV module at standard condition (12) and at operating condition (13) are given in Wang [5].

The following *thermal voltage timing completion factor*, which is not function of the ideal factor parameter, is used in this study and its additional benefits are highlighted in the validation section:

$$\alpha_{t,0} = \frac{2 * V_{mp,0} - V_{ocm,0}}{\frac{I_{scm,0}}{I_{scm,0} - I_{mp,0}} + \ln(1 - \frac{I_{mp,0}}{I_{scm,0}})} \quad (12)$$

$$\alpha_t = \alpha_{t,0} * \frac{T_j + 273}{T_{j,0} + 273} \quad (13)$$

Thus, equation (11) becomes:

$$I_{sm} = I_{scm} * \left[1 - \exp\left(\frac{V_{sm} - V_{ocm} + R_{sm} * I_{sm}}{\alpha_t} \right) \right] \quad (14)$$

I_{scm} -is the PV module short circuit current

V_{ocm} -is the PV module open circuit voltage

These two parameters are found in manufacturer's data sheet at standard condition. Other parameters such as rated power (maximum power, i.e., power at maximum power point), voltage at maximum power point, current at maximum power point, number of cells in series, number of cells in parallel can be listed in manufacturer's data sheet as well. Some of these parameters are shown in figure (2). Figure 2 is obtained using figure (3) and specifications from *SPR-90 High Efficiency PV module* [9].

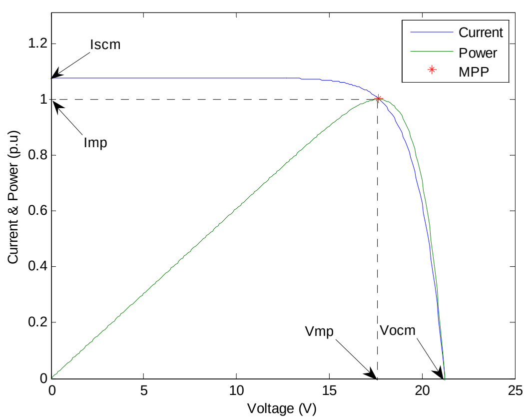


Figure 2. I-V & P-V curves of PV module

The plotting procedure is detailed in the subsequent section. In the next section, an expression for maximum power at operating condition is derived.

4. Maximum Power Operation

The power delivered by the PV module is maximal when the following condition is satisfied:

$$\frac{dP}{dI_{sm}} = 0 \quad (15)$$

where P is given by $P = V_{sm} * I_{sm}$ (16)

That is:

$$\left[V_{sm} + I_{sm} * \frac{dV_{sm}}{dI_{sm}} \right]_{I_{sm} = I_{mp}} = 0 \quad (17)$$

where V_{sm} is derived from (14) as follows:

$$V_{sm} = V_{ocm} - R_{sm} * I_{sm} + \alpha_t * \ln\left(1 - \frac{I_{sm}}{I_{scm}} \right) \quad (18)$$

After mathematical manipulations, the following expression is obtained and used to evaluate the rated current under operating conditions.

$$I_{mp} = \frac{1}{2 * R_{sm}} * \left(V_{ocm} + \alpha_t * \left(\ln\left(1 - \frac{I_{mp}}{I_{scm}} \right) - \frac{I_{mp}}{I_{scm} - I_{mp}} \right) \right) \quad (19)$$

Substituting I_{mp} in (18) and (16), the rated voltage operation (V_{mp}) and the maximum power operation $P_{\max}$, are respectively calculated as follows:

$$V_{mp} = V_{ocm} - R_{sm} * I_{mp} + \alpha_t * \ln\left(1 - \frac{I_{mp}}{I_{scm}} \right) \quad (20)$$

and $P_{\max} = V_{mp} * I_{mp}$ (21)

5. Matlab/Simulink Implementation of a PV Module Model

Matlab/Simulink is an open architecture research tool [10], which offers the following advantages [2]:

- Building hierarchical models with possibility of viewing the system at different levels.
- Possibility of building modular models, i.e., models can be readily connected together.
- Library, which allows picking up components and building a given system.
- Masking feature offering the possibility to simplify the use of the model by replacing many dialog boxes in a subsystem with a single dialog box.

The steps for implementing equation (14) are as follows:

Step 1: PV module specifications (rated power $P_{\max,0}$, rated voltage $V_{mp,0}$, rated current $I_{mp,0}$, open circuit voltage $V_{ocm,0}$, short-circuit current $I_{scm,0}$) at standard conditions are found in manufacturer's data sheet. Standard conditions are defined as: irradiance $G_{a,0} = 1000 W/m^2$ (sun intensity impinging the module), and junction temperature $T_{j,0} = 25^o C$. The number of cells in parallel and series (N_{pm} & N_{sm}) may be given as well.

Step 2: PV module parameters at operating condition are calculated. Since the I-V curves of the PV module vary with irradiance $G_a[W/m^2]$ and junction cell temperature $T_j[^oC]$ [1], the values of V_{ocm}, I_{scm}, and $P_{\max}$ at any combination of G_a and T_j are needed. Metwally [5] recommends the linear relation between T_j - T_a given by:

$$T_j = T_a + A + B * G_a \tag{22}$$

where $A = -2.89^o C$ and $B = 0.034^o Cm^2 / W$ are constants.

The open circuit voltage and the short circuit current at operating point are given by Metwally [5]:

$$V_{ocm} = V_{ocm,0} * \left(1 + \frac{C * \left(T_j - T_{j,0}\right)}{V_{ocm,0}}\right) * \log\left(2.72 + D * \left(G_a - G_{a,0}\right)\right) \tag{23}$$

$$I_{scm} = I_{scm,0} * \frac{G_a}{G_{a,0}} * \left(1 + \frac{E * \left(T_j - T_{j,0}\right)}{I_{scm,0}}\right) \tag{24}$$

where C is temperature coefficient of open-circuit voltage, $D = 0.0005 m^2 / W$ is constant, and E is temperature coefficient of short-circuit current.

An expression for calculating the series resistor, which can be readily derived from equation (14), is given in [3] where this resistor is independent of junction temperature:

$$R_{sm,0} = \frac{\alpha_{t,0} * \log\left(1 - \frac{I_{mp,0}}{I_{scm,0}}\right) - V_{mp,0} + V_{ocm,0}}{I_{mp,0}} \tag{25}$$

where the thermal voltage timing completion factor $\alpha_{t,0}$ is calculated using equation (12). The following assumption is held in this study: $R_{sm} = R_{sm,0}$ = constant.

Step 3: Having all parameters required in equation (14), the module current for operating condition is evaluated at different values of module voltage. It is important to note that in this equation, there is no need for the number of cells in parallel and series as opposed to equation (11) where these numbers are required.

Figure 3 shows the simulink implementation of equations (14), (19), (20), and (21). Figure 4 is the masked representation of figure 3. The variation of maximum current under different irradiance levels is presented in figure 5.

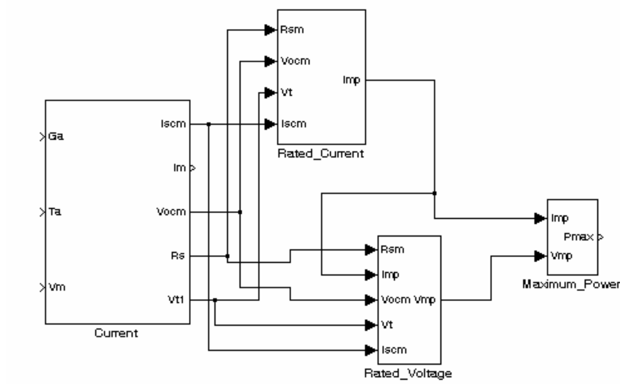
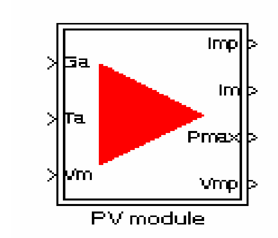
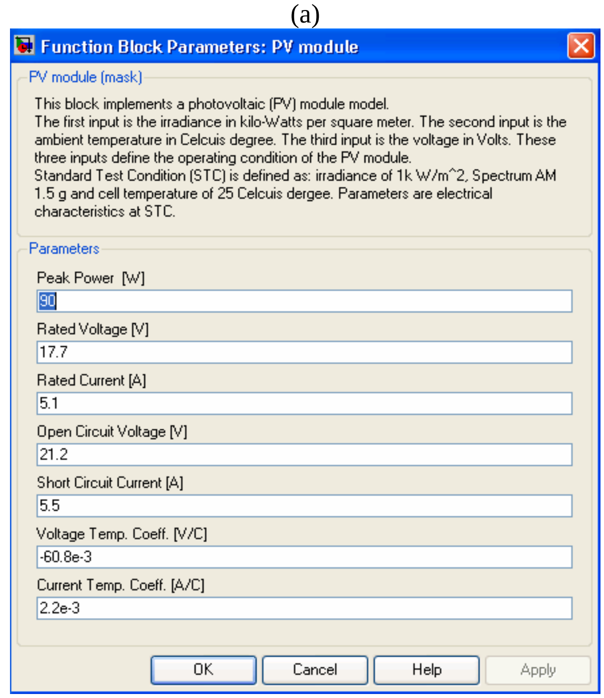


Figure 3. PV module structure in Matlab/Simulink: different subsystems



(a)



(b)

Figure 4. PV module in Matlab/Simulink ((a)-masked structure, (b)-mask parameters)

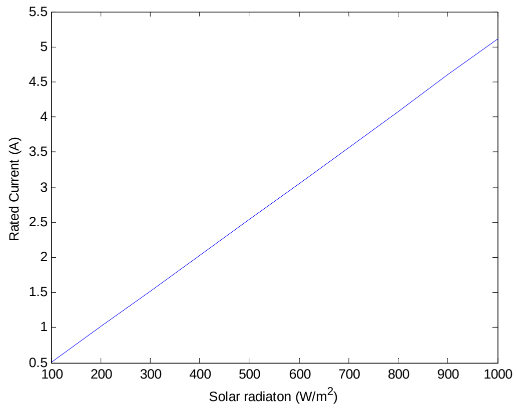


Figure 5. Irradiance vs. Rated Current

6. Validation of the PV Module Model

To validate the PV module model, figure 4 (a), is run using Matlab/simulink. The results (figure 6) of the simulation at standard conditions are compared to specifications and plot (figure 7) given in manufacturer's data sheet. Furthermore, characteristics curves of module found in literatures [1] and [11] are reproduced by simulations (figures 8 and 9) to validate the proposed model as well as its simulink implementation.

As can be seen, figure 6, which is obtained from simulation, is perfectly similar to figure 7, which is given in data sheet [8].

6.1 Curve Fitting Parameter Benefits

In section three, the thermal voltage timing factor was introduced. It exclusively depends on manufacturers' specifications with no need of estimating the ideal factor parameter. In order to investigate its additional benefits, equation (11) was also implemented in Matlab/Simulink then comparison between simulated results from equation (11), equation (14), and parameters from data sheet is made (Table 1). Clearly, simulated results greatly agree with specifications from data sheet when using equation (14) than equation (11), where percentage absolute errors are higher than in the first case. Moreover, the series resistor in equation (11) has the value of 0.67307Ω, whereas its value in equation (14) is 0.0305Ω. These values are obtained using equation (25), where $\alpha_{t,0}$ is replaced by $V_{t,0}$ in case of equation (11). According to Patel [1], for high quality PV module, the series resistor should be low.

Therefore, the value of 0.0305Ω fits the SPR-90 module well, which is a high quality module. Thus, equation (14), which is developed in this work, is a very good model for PV module and figure 4a can be loaded in Matlab/Simulink library for future usage.

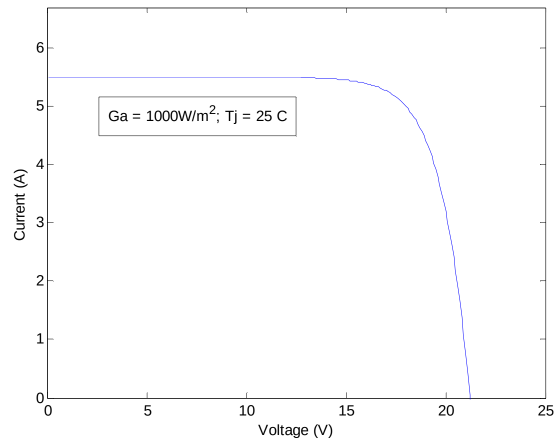


Figure 6. Simulated I-V curve of SPR-90 Module

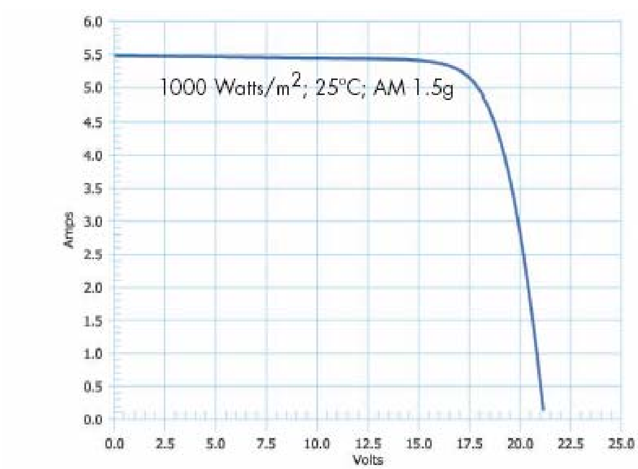


Figure 7. I-V curve of SPR-90 Module

Table 1
Comparison of simulated results and specifications from data sheet

Parameter	Simulated results		Data Sheet	Percentage absolute error (%)	
	Eq. (11)	Eq. (14)		Eq. (11)	Eq. (14)
$I_{sc,0}$ [V]	5.5	5.5	5.5	0	0
$V_{oc,0}$ [V]	21.2	21.2	21.2	0	0
$P_{\max,0}$ [W]	90.63	90.36	90.27	0.4	0.09
$V_{mp,0}$ [V]	16.8	17.63	17.7	5.1	0.4
$I_{mp,0}$ [A]	5.4	5.13	5.1	5.9	0.59

6.2 Temperature Effect

The open circuit voltage of PV module is inversely proportional to temperature, i.e., a rise in temperature produces a decrease in voltage. In reference to short-circuit current, it is relatively steady with temperature variation [1], [11]. Actually, the PV module acts like a constant current source for most parts of its I-V curve.

The above properties are perfectly verified by simulink simulations (figure 8). Figure 8 shows the simulation results on the effect of ambient temperature on P-V and I-V curves. As expected, the maximum power (i.e., value of power at maximum power point) is inversely proportional to temperature and appears at different voltage levels. To extract this maximum power, the module output voltage has to track the voltage at maximum power point as temperatures vary. For instance, from table 2, the PV system must be designed such that the output voltage can be equal to 15.7V for capturing 79.95W at $25\,^{o}C$ and can be decreased to 14.1V for capturing 71.64W at $50\,^{o}C$. This is achieved by the so-called *maximum power point tracker*.

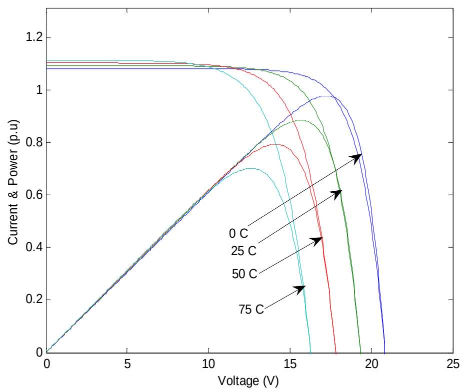


Figure 8. Ambient Temperature effect on P-V & I-V curves ($G_a = 1000 W/m^2$)

Table 2
Influence of irradiance and ambient temperature on Maximum Power

$G_a(W/m^2), T_j = 25^oC$	V_{mp}	I_{mp}	$P_{\max}$
1000	17.7	5.11	90.36
800	17.3	4.10	70.92
600	17	3.06	51.98
500	16.8	2.54	42.74
300	16.4	1.52	24.89
$T_a(^oC), G_a = 1000W/m^2$			
0	17.3	5.11	88.30
25	15.7	5.09	79.95
50	14.1	5.08	71.64
75	12.6	5.03	63.40

6.3 Irradiance Effect

Short-circuit current is directly proportional to irradiance while the open-circuit voltage of the PV module varies little with solar irradiation [1], [9]. As it can be seen in figure 9, the simulation results perfectly agree with this property. Figure 9 and table 2 shows the direct decrease of the maximum output power as sun intensity decreases. In table 2, $1000W/m^2$ represents the solar intensity impinging a normal surface on a bright day, whereas $300W/m^2$ represents the same intensity on an extremely overcast day. Thus, for an extremely overcast day, the PV module produces 24.89W, which is 27.54% of its rated power at standard conditions.

The above properties should be held in case of photovoltaic (PV) array. To investigate this, the next section analyses a PV array consisting of N_{pa} PV modules in parallel and N_{sa} PV modules in series.

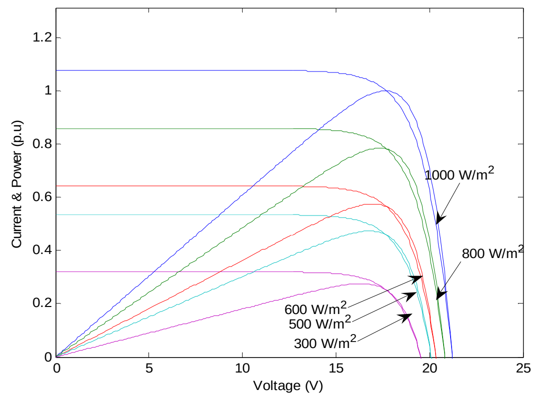


Figure 9. Effect of irradiance on P-V & I-V curves ($T_j = 25^oC$)

7. PV Array

Following the procedure used to derive the PV module model from the PV cell model, the PV array model is readily obtained from PV module. The derivation of equation (26) from equation (14) is straightforward.

$$I_{sa} = N_{pa} * I_{scm} * \left[1 - \exp\left(\frac{V_{sa} - N_{sa} * V_{ocm} + \frac{N_{sa}}{N_{pa}} * R_{sm} * I_{sa}}{N_{sa} * \alpha_t} \right) \right] \quad (26)$$

where N_{pa} is the number of modules in parallel, and N_{sa} is the number of modules in series.

The maximum power point of the PV array is found in the same manner as was developed for PV module. Below are the derived equations for evaluating the PV array maximum power operation.

$$I_{mp} = \frac{N_{pa}}{2 * R_{sm}} \left(V_{ocm} + \alpha_t * \left(\ln\left(1 - \frac{I_{mpa}}{N_{pa} * I_{scm}}\right) - \frac{I_{mpa}}{N_{pa} * Iscm - I_{mpa}} \right) \right) \quad (27)$$

$$V_{mp} = N_{sa} * V_{ocm} - \frac{N_{sa}}{N_{pa}} * R_{sm} * I_{mpa} + N_{sa} * \alpha_t * \ln\left(1 - \frac{I_{mpa}}{N_{pa} * I_{scm}}\right) \quad (28)$$

$$P_{\max} = V_{mpp} * I_{mpp} \quad (29)$$

Figure (10) is the implementation of equations (26), (27), (28), and (29) in Matlab/Simulink.

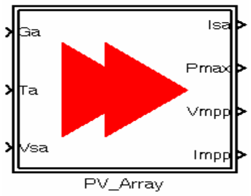


Figure 10. Masked structure of PV array in Matlab/Simulink

Figure 11 shows the simulated results for different combinations of the SPR-90 modules. As can be seen, and as expected; the array voltage is directly proportional to the number of modules in series (Nsa) and the array current is directly proportional to the number of modules in parallel (Npa). In addition, the variation of array (consisting of 7 modules in series and 3 modules in parallel) current under different irradiance levels is shown in figure 12. The two figures (11 & 12) suggest a satisfactory model for PV array.

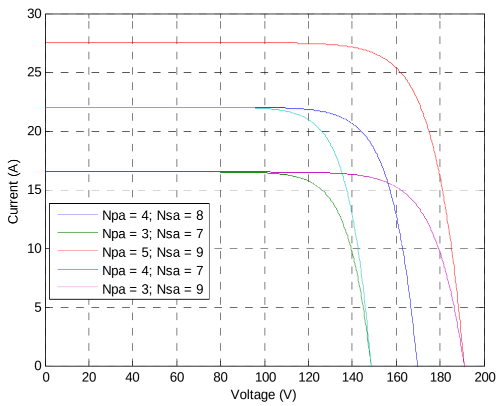


Figure 11. Different combinations (arrays) of SPR-90 modules

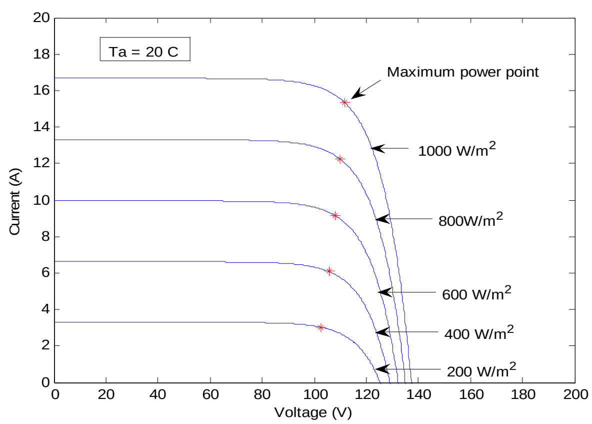


Figure 12. V-I curves of PV array at different irradiances. The asterisks in red are MPP.

8. Conclusion

The aim of this paper was to provide insight into the modelling and simulation of a PV module using Matlab/Simulink environment. The results of the simulation suggest that the model perfectly reflects the influence of irradiance and ambient temperature on I-V and P-V characteristics of PV module. For the simulation presented using specifications from manufacturer's data sheet, the available peak power is about 0.09% higher than the specified maximum power. This was achieved by the selection of a suitable curve fitting parameter called *thermal voltage timing completion factor*, which is exclusively function of specifications given in data sheet and cell temperature. It can be concluded that the proposed model can be used to predict the required PV module parameters in different weather conditions without range restriction.

Future work will investigate the behaviour of the simulation model described here in a realistic integrated PV system where specific load requirements and meteorological conditions at site location are considered.

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