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ANALYSIS OF A PHASE-CONVERTER PERFORMANCE

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ABSTRACT
A phase converter consisting of static reactive elements is described in this paper. It is connected between a three-phase voltage supply and a large single-phase load to enable symmetrical loading of the three phases. The contribution of the paper is the method to determine the parameters of the converter elements so that the source phase-currents are of a symmetrical system. Three phase system with converters is simulated and corresponding measurements are provided at inductive or capacitive single-phase loads. The matching between the calculated and experimental results is of high level.

KEY WORDS
Static phase converter, single-phase load, symmetrical loading, simulation

1. Introduction

A major factor for maintaining the quality of the three-phase voltage systems is the symmetrical loading of all three phases. It is relatively easy to maintain the balance between the phase currents when loads are three phase consumers [1]. But when a powerful single-phase load is connected to the system additional procedures should be taken to enable symmetrical loading of all three phases [2]. Such loads are for example the arc or resistance melting furnaces, single-phase transformers or induction motors etc [3].

This paper describes the arrangement and the operation of a static phase converter built of reactors and capacitors. Contribution of the paper is the development of a method for calculating the converter components by considering the load parameters and the requirement for symmetrical loading of the three phases.

2. Converter Arrangement

The basic circuit of the phase converter PC is presented in Fig.1. The source voltages V_R, V_S and V_T are of a symmetrical three-phase system. The single-phase consumer has an impedance of $\boldsymbol{Z}$ and power factor of $\boldsymbol{\cos\varphi}$. The converter consists of reactive elements with impedances $\boldsymbol{Z_1}$ and $\boldsymbol{Z_2}$ respectively. Each one of them could be a reactor or a capacitor. Coordinating transformers could be used to match the consumer voltage to that of the source if necessary.

The circuit is analyzed by the following assumptions:

- The system operates at steady state conditions;
- The 3-phase source has unlimited power;
- Load and converter elements are linear;
- All alternating quantities are sinusoidal;
- The elements of the PC are ideal and without resistive components.

The types and parameters of the converter elements $\boldsymbol{Z_1}$ and $\boldsymbol{Z_2}$ are to be determined in such away that the source phase-currents $\boldsymbol{I_r}$, $\boldsymbol{I_s}$ and $\boldsymbol{I_t}$ are of a symmetrical 3-phase system.

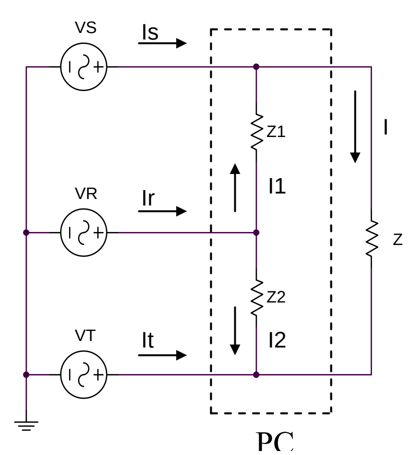

Figure 1. Electrical circuit

In solving this task it is considered that the three-phase source is star connected and the phase voltages are:

$$V_R\,[0^o]\,,\ V_S\,[120^o]\ \text{and}\ V_T\,[240^o].$$

The general solutions are obtained by the use of the symmetrical components method [4]. The voltages of the three-phase symmetrical system are represented as phasors in the following way:

$$V_R = V;\ V_S = a^2 V_R = a^2 V;\ V_T = aV_R = aV \quad (1)$$

where

$$a = (-0.5 + j\frac{\sqrt{3}}{2}) \quad and \quad j = \sqrt{-1}$$

The load impedance is

$$Z = Z(\cos\varphi + j\sin\varphi) \quad (2)$$

The Impedances of the converter elements are

$$Z_1 = jX_1 \ and \ Z_2 = jX_2. \quad (3)$$

The currents of a symmetrical system I_r, I_s and I_t have equal **rms** values I, and are 120° displaced from each other. Represented as phasors the currents are

$$I_r = I, \quad I_s = a^2 I \quad and \quad I_t = aI \quad (4)$$

The following current equations are valid

$$I_r = (\frac{1-a^2}{Z_1} + \frac{1-a}{Z_2})\ V \quad (5.1)$$

$$I_s = (\frac{1-a^2}{Z_1} + \frac{a^2-a}{Z})\ V \quad (5.2)$$

$$I_t = (\frac{a-1}{Z_2} + \frac{a-a^2}{Z})\ V \quad (5.3)$$

From (1), (4) and (5) follows that

$$\frac{a-1}{Z_2} + \frac{a-a^2}{Z} = \frac{a^2(1-a^2)}{Z_1} + \frac{a^2(a^2-a)}{Z} \quad (6)$$

Equations (2), (3) and (6) result in:

$$Z = Z(\cos\varphi + j\sin\varphi) = \\ = \frac{X_1X_2}{2}\{\frac{(X_2-X_1)\sqrt{3} + j(X_1+X_2)}{X_1^2 + X_2^2 - X_1X_2}\} \quad (7)$$

Equation (7) gives

$$Z\cos\varphi = \sqrt{3}\,\frac{X_1X_2}{2}\{\frac{(X_2-X_1)}{X_1^2+X_2^2-X_1X_2}\} \quad (8.1)$$

$$Z\sin\varphi = \frac{X_1X_2}{2}\{\frac{X_1+X_2}{X_1^2+X_2^2-X_1X_2}\} \quad (8.2)$$

The solutions of (8) are

$$X_1 = \delta Z \quad where \quad \delta = \frac{\sqrt{3}}{\sqrt{3}\sin\varphi + \cos\varphi} \quad (9.1)$$

$$X_2 = \beta Z \quad where \quad \beta = \frac{-\sqrt{3}}{\cos\varphi - \sqrt{3}\sin\varphi} \quad (9.2)$$

As seen the reactances X_1 and X_2 are very complex functions of **cosφ** and **sinφ** respectively.

3. Operation with (R-L) Load

The equivalent electrical circuits are presented in Fig 2a and Fig 2b. The impedance of the (R-L) load is given by:

$$\mathbf{Z} = Z_L(\cos\varphi_L + j\sin\varphi_L) \quad (10.1)$$

where

- Index 'L' refers to inductive load;
- $\varphi = \varphi_L$ is the load phase angle;

$$tg\varphi_L = \frac{\omega L}{R};\ \omega = 2\pi f \quad (10.2)$$

R and **L** are the resistance and inductance of the load and f is the frequency respectively.

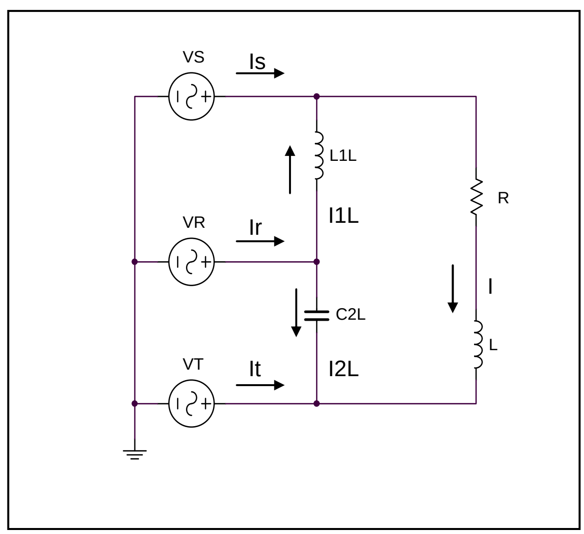

Figure 2a. Electrical circuit at $0<\varphi_L<\pi/6$

Analysis shows that

At ($0<\varphi_L<\pi/2$) the parameter δ is positive (9.1) and the reactance of the converter element is $X_1=X_{1L}>0$. The element $Z_1=Z_{1L}$ is a reactor with inductance L_{1L} (Fig.2a and Fig.2b) (11.1)
At ($0<\varphi_L<\pi/6$) the parameter $\boldsymbol{\beta}$ is negative (9.2) and the reactance of the converter element $X_2=X_{2L}<0$. The element $Z_2=Z_{2L}$ is a capacitor with capacitance C_{2L} (Fig.2a) (11.2)
At ($\pi/6<\varphi_L<\pi/2$) the parameter $\boldsymbol{\beta}$ is positive (9.2) and the reactance of the converter element $X_2=X_{2L}>0$. The element $Z_2=Z_{2L}$ is a reactor with inductance L_{2L} (Fig.2b) (11.3)
At ($\varphi_L=\pi/6$) the parameter $\beta \to \infty$ and the converter consists of element X_{1L} only. (11.4)

The parameters L_{1L}, C_{2L} and L_{2L} are

$$\text{At } (0 < \varphi_L < \pi/2),\quad L_{1L} = \frac{\delta_L Z_{1L}}{\omega} \tag{12.1}$$

$$\text{At } (0<\varphi_L<\pi/6),\quad C_{2L} = \frac{1}{\beta_L\, \omega\, Z_{2L}} \tag{12.2}$$

$$\text{At } (\pi/6 < \varphi_L <\pi/2),\quad L_{2L} = \frac{\beta_L Z_{2L}}{\omega} \tag{12.3}$$

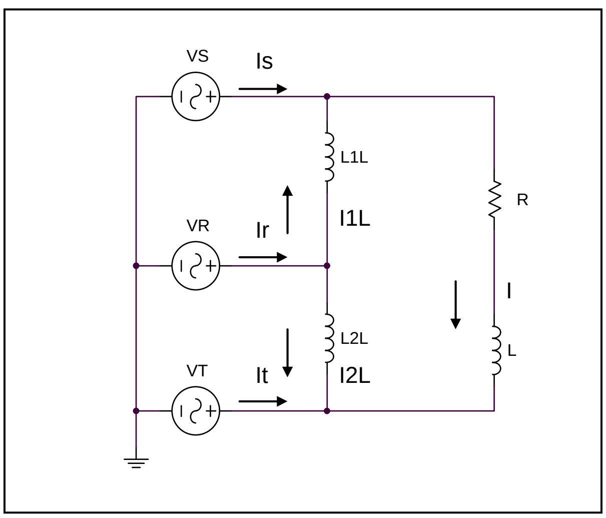

Figure 2b. Electrical circuit at $\pi/6 < \varphi_L <\pi/2$

4. Operation with (R-C) Load

The equivalent electrical circuits are presented in Fig 3a and Fig 3b. The load impedance is given by:

$$Z = Z_C\,(\cos\varphi_C + j\sin\varphi_C) \tag{13.1}$$

$$\varphi = (-\varphi_C) \quad \text{and} \quad tg\varphi_C = \frac{1}{\omega\, CR} \tag{13.2}$$

where

- Index ‘C’ refers to capacitive load;
- C is the capacitance of the load.

Analysis shows that:

At ($-\pi/2 <\varphi_C<0$) the parameter $\boldsymbol{\beta}$ is negative (9.2), the converter reactance $X_2= X_{2C}<0$ and the converter element $Z_2=Z_{2C}$ is a capacitor with capacitance C_{2C} (Fig 3a and Fig 3b) (14.1)
At ($-\pi/6 <\varphi_C <0$), the parameter δ is positive (9.1), the converter reactance $X_1=X_{1C}>0$, and the converter element $Z_1=Z_{1C}$ is a reactor with inductance L_{1C} (Fig.3a) (14.2)
At **($-\pi/2 <\varphi_C <-\pi/6$),** the parameter δ is negative (9.1), the converter reactance $X_{2C}<0$, and the element $Z_1=Z_{1C}$ is a capacitor with capacitance C_{1C}. **(**Fig.3b) (14.3)

At $\varphi_C =-\pi/6$), the parameter $\delta \to \infty$ and the converter is without element Z_{1C}.
The parameters C_{1C}, C_{2C} *and* L_{1C} are:

$$\text{At } (-\pi/2 <\varphi_C <0),\quad C_{2C} = \frac{1}{\beta_C Z_C\, \omega} \tag{15.1}$$

$$\text{At } (-\pi/6 <\varphi_C <0),\quad L_{1C} = \frac{\delta_C Z_C}{\omega} \tag{15.2}$$

$$\text{At } (-\pi/2 <\varphi_C <-\pi/6),\quad C_{1C} = \frac{1}{\delta_C\, \omega\, Z_C} \tag{15.3}$$

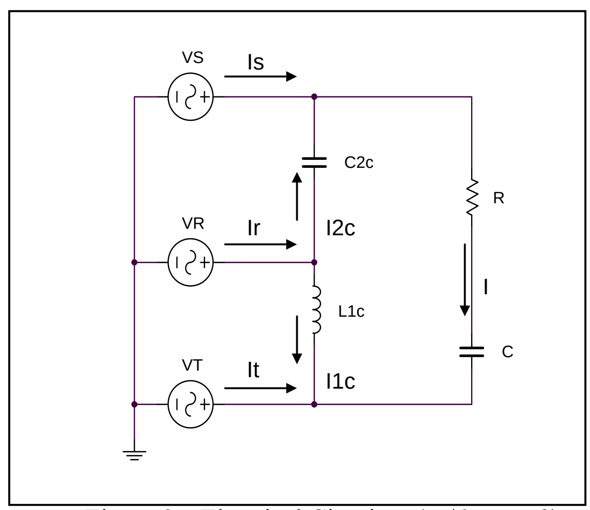

Figure 3a. Electrical Circuit at ($-\pi/6 <\varphi_C <0$)

For single-phase loads with other inductive or capacitive parameters there is need to adjust the converter elements Z_{1L} and Z_{2L} or Z_{1C} and Z_{2C} respectively. The equations (9) and (12) and (15) representing the converter static characteristic should be used.

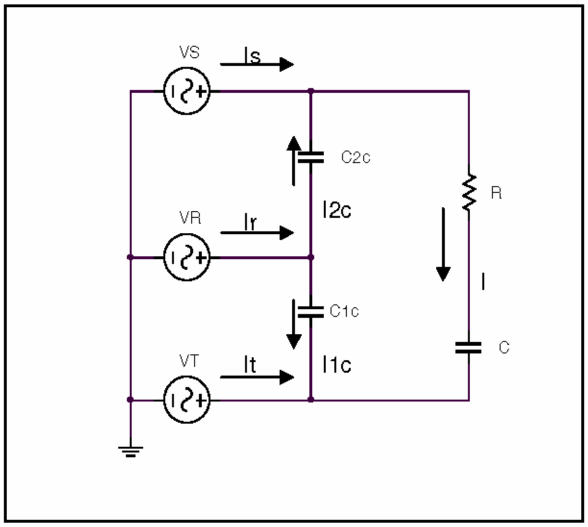

Figure 3b. Electrical circuit at ($-\pi/2 < \varphi_C < -\pi/6$

5. Current Calculations

The load voltage V_Z is the line voltage of the 3-phase system and the load current is I. They are considered as well specified:

$$V_Z = V\sqrt{3}, \quad I = \frac{V_Z}{Z} = \frac{V\sqrt{3}}{Z} \tag{16}$$

5.1 Source phase currents

The rms values of the source phase currents are expressed as complexors in the following way:

5.1.1 Resistive- Inductive Load Z_L

$$I_r = \frac{V}{Z_L}(\cos\varphi_L - j3\sin\varphi_L) \tag{17.1}$$

$$I_s = a^2 I_r = a^2\frac{V}{Z_L}(\cos\varphi_L - j3\sin\varphi_L) \tag{17.2}$$

$$I_t = aI_r = a\frac{V}{Z_L}(\cos\varphi_L - j3\sin\varphi_L) \tag{17.3}$$

The rms value of all phase currents is:

$$I = \frac{V}{Z}\sqrt{9 - 8\cos\varphi_L} \tag{17.4}$$

5.1.2 Resistive- Capacitive Load Z_C

$$I_r = \frac{V}{Z_C}(\cos\varphi_C + j3\sin\varphi_C) \tag{18.1}$$

$$Is = a^2 I_r = a^2\frac{V}{Z_C}(\cos\varphi_C + j3\sin\varphi_C) \tag{18.2}$$

$$I_t = aI_r = a\frac{V}{Z_C}(\cos\varphi_C + j3\sin\varphi_C) \tag{18.3}$$

The rms values of all phase currents is:

$$I = \frac{V}{Z_C}\sqrt{9 - 8\cos^2\varphi_C} \tag{18.4}$$

5.2 RMS current of the element Z_1

5.2.1 Resistive- Inductive Load Z_L

$$V_{Z1} = V\sqrt{3}$$

$$I_{1L} = \frac{V\sqrt{3}}{X_{1L}} = \frac{V}{Z_L}(\cos\varphi_L - \sqrt{3}\sin\varphi_L) \tag{19}$$

5.2.2 Resistive- Capacitive Load Z_C

$$V_{Z1} = V\sqrt{3}$$

$$I_{1C} = \frac{V\sqrt{3}}{X_{1C}} = \frac{V}{Z_C}(\cos\varphi_C + \sqrt{3}\sin\varphi_C) \tag{20}$$

5.3 RMS Currents of the Impedance Z_2

5.3.1 Resistive- Inductive Load

$$V_{Z2} = V\sqrt{3}$$

$$I_{2L} = \frac{V\sqrt{3}}{X_{2L}} = \frac{V}{Z_L}(\cos\varphi_L + \sqrt{3}\sin\varphi_L) \tag{21}$$

5.3.2 Resistive- Capacitive Load

$$V_{Z2} = V\sqrt{3}$$

$$I_{2C} = \frac{V\sqrt{3}}{X_{2C}} = \frac{V}{Z_C}(\cos\varphi_C - \sqrt{3}\sin\varphi_C) \tag{22}$$

5.4 Power Factor $\cos\varphi_S$ of the Three-Phase System

$$\cos\varphi_S = \frac{\cos\varphi_L}{\sqrt{9 - 8\cos^2\varphi_L}} = \frac{\cos\varphi_C}{\sqrt{9 - 8\cos^2\varphi_C}} \tag{23}$$

6. Analysis of the Theoretical Results

The application of a static phase converter makes it possible to load a three-phase voltage system with symmetrical phase currents at any kind of single-phase loading. The analyzed phase converter contains reactive elements only – capacitors and/or inductors.

The power factor of the three-phase source is unity at purely active single-phase load. At inductive-resistive or capacitive-resistive loads the source power factor decreases and remains lower than that of the load.

At inductive-resistive loads the source power factor is capacitive. At capacitive-resistive load the source power factor is inductive.

At unity power factor of the load the source phase currents are $\sqrt{3}$ times less than the load current. At lower power factors the phase currents increase fast and significantly, which may damage the source. In such cases it is advisable to improve the power factor and depending upon the load shunt capacitors or series inductors could be used.

The phase converter can easily be controlled automatically [5, 6]. It is necessary to follow up the symmetry of the source phase currents by the use of filters for symmetrical components or any other suitable method. In case of unbalance the controlling circuit will initiate signals for expedient changes in the parameters of the reactances X_1 and X_2 leading to symmetry of the source phase currents.

7. Measurements

The theoretical and experimental results are shown in Table 1 and Table 2.

Table 1

(R - L) load

Index m=measurement data and th= theoretical data

cosφ	1.0	0.95	0.9	0.87	0.6	0.3
R, Ω	38	36.1	34.2	32.9	22.8	11.4
L,mH	0	37.8	52.7	60.5	96.8	115
L_{1L},mH	210	141	126	121	106	107
R_{1L}, Ω	6.58	4.41	3.98	3.8	3.31	3.37
C_{2L}, μF	48.4	19.8	7.02	-	-	-
R_{2L}, Ω	6.58	16.1	45.4	-	-	-
L_{2L},mH	-	-	-	~	267	155
R_{2L}, Ω	-	-	-	~	8.38	4.87
I_m, A	**10**	**10**	**10**	**10**	**10**	**10**
I_{rth},I_{sth}, I_{tth}, A	5.77	7.72	9.19	10	14.32	16.6
I_{rm}, A	**6.7**	**8.14**	**9.27**	**10**	**14.24**	**16.57**
I_{sm}, A	**6.32**	**8.53**	**10**	**10.8**	**15.05**	**17.24**
I_{tm}, A	**6.32**	**7.81**	**9.20**	**10**	**14.15**	**16.27**
I_{1Lth}, A	5.79	8.63	9.58	10	11.5	11.3
I_{1Lm}, A	**5.76**	**8.60**	**9.52**	**10**	**11.43**	**11.24**
I_{2Lth}, A	5.79	2.36	0.83	0	4.55	7.80
I_{2Lm}, A	**5.76**	**2.36**	**0.83**	**0**	**4.50**	**7.80**
$\cos\varphi_{s\,th}$	1.0	0.71	0.57	0.50	0.24	0.104
$\cos\varphi_{s\,m}$	1.0	0.75	0.61	0.54	0.30	0.17

The voltages are V_R = 220[0°], V_S = 220[120°] V_T = 220[240°]. The load impedance is Z_L = 38Ω. The load current is I_L = 10A at 50 Hz.

The circuits of Fig.2 and Fig.3 were simulated by the use of the software program MULTISIM. Loads with different power factors were examined. The ideal reactive inductive and capacitive elements of the converter were replaced with real elements, containing some small resistive components. It was accepted that the resistive component values were 10% of the corresponding inductive or capacitive impedances of the converter elements. The effects of these resistances over the experimental results are negligible.

Matching between the phase currents determined theoretically and experimentally is of very high level. Typical graphs are given in Fig 4, showing clearly the trends of changes in the source phase currents and in the converter branches. A mismatching of about 10% between the theoretical and measured currents is observed only at load power factors close to unity.

Table 2

(R - C) load

cosφ	1.0	0.95	0.9	0.87	0.6	0.3
R, Ω	38	36.1	34.2	32.9	22.8	11.4
C,μ F	~	268	192	167	104	88
C_{1C},μF	-	-	-	0	38.1	65.6
R_{1C}, Ω	-	-	-	~	8.5	4.9
L_{1C},mH	210	510	1440	~	-	-
R_{2C}, Ω	6.6	16	45	~	-	-
C_{2C},μF	48.4	72.2	80.1	83.8	96.1	94.5
R_{2C}, Ω	6.6	4.4	4.0	3.8	3.3	3.3
I_m, A	**10**	**10**	**10**	**10**	**10**	**10**
I_{rth}, I_{sth} $I_{tth,}$ **A**	5.77	7.72	9.19	10	14.32	16.6
I_{rm}, A	**6.72**	**8.15**	**9.31**	**9.98**	**14.30**	**16.6**
I_{sm}, A	**6.23**	**7.80**	**9.19**	**10**	**14.14**	**16.3**
I_{tm}, A	**6.4**	**8.54**	**10**	**10.8**	**15.06**	**17.3**
I_{1Cth}, A	5.75	2.37	0.83	0	4.54	7.83
I_{1Cm}, A	**5.75**	**2.37**	**0.84**	**0**	**4.54**	**7.81**
I_{2Cth}, A	5.79	8.63	9.55	10	11.45	11.3
I_{2Cm}, A	**5.76**	**8.59**	**9.54**	**9.98**	**11.46**	**11.2**
$\cos\varphi_{s\,th}$	1.0	0.71	0.57	0.50	0.24	0.10
$\cos\varphi_{s\,m}$	1.0	0.75	0.61	0.54	0.30	0.17

8. Conclusion

A static converter consisting of reactive elements, inductors and capacitors, is considered and analyzed in this paper. Such converters could be used successfully for loading three-phase voltage systems with single-phase loads by keeping the symmetry of the supply.

The contribution of the paper is the theoretical analysis of the converter performance at different loads. Another contribution is the method to determine the values of all converter components.

Theoretical and experimental results match very closely to each other as shown in Fig.4 and Fig 5. The source-currents build a three-phase symmetrical system at