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FLATNESS-BASED STABILISATION OF A SINGLE-AXIS SYNCHRONOUS GENERATOR MODEL WITH EMBEDDED TRAJECTORIES OF MOTION

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ABSTRACT
This paper develops on our earlier results about the flatness of a single-axis synchronous generator model. In this paper, the flatness property of the model is used to generate trajectories of motion for the feedback control of the single-axis synchronous generator and to compare the dynamic performance of the generator controlled with and without trajectories of motion. Results presented show that the embedding of trajectories of motion leads to significant improvement in dynamic stability during generator faults that last several cycles.

KEY WORDS
Dynamic feedback linearization, flatness, flat output

1. Introduction

Control strategies based on feedback-linearisation have been used extensively in power systems control with good results [1-10]. More recently, dynamic feedback-linearisation has received a lot of attention and focus as a result of the publications of M. Fliess et al; J. Levine et al; B. Kiss et al; P. Rouchon et al, etc. It has been applied to many mechanical systems like cranes, cars with n-trailers, windshield control, and many theoretical examples [11-17]. M. Fliess et al [11] in their comprehensive paper unifying the theory of flatness and its associated dynamic feedback, formalized the concept that two systems are equivalent if there is an invertible transformation exchanging their trajectories, that is, any variable of one system may be expressed as a function of the variables of the other system and a finite number of its time derivatives. One of the main results of the concept of system flatness is that desired trajectories of plant motion and the input required to drive the plant along these trajectories from one state to another can be generated through interpolation. Thus, once the value of the flat output and its requisite derivatives are known, all the other system variables and the input levels associated to the particular set of values are sufficiently defined [11-17]. It can therefore be shown that if the sets of the flat output and its requisite derivatives exist at two different time instants, the desired trajectories of plant motion and the input required to drive the plant along these trajectories, from one of the states to the other (associated to each set), could be solved as an interpolation problem without integrating the state equations.

While, in general, the derivation of a flat output of a given system may be non-trivial, Levine [18] has given a set of procedures for its computation based on the Smith matrix decomposition algorithm. In [19], we showed the flatness property of a single-axis synchronous generator systems using the method of Levine presented in [18]. Results obtained from the synchronous generator equipped with a flat-based controller compared excellently with those reported from classical feedback-linearised controllers, published in literature [5-10,19]

In the current paper, we explore the flatness property of the third-order synchronous generator to generate trajectories of motion along which the stabilised plant would track to return to equilibrium after a transient. The dynamics of the systems stabilised with trajectories of motion is compared with that of the same system stabilised without trajectories of motion. In section two of the paper, results regarding the flatness of the synchronous machine model is recollected. In section three, gneration of trajectories of motion is discussed. The basic structure of the controller used is retained as in reference [19]. Simulations comparing the effects of the controlled generator with and without trajectories are shown in section four. Results presentations and discussions are also included in this section. The conclusions are presented in section five of the paper. A list of references complete the paper.

2. Flatness of the Single-Axis Synchronous Generator Model

2.1 Description of the Study System

The one-axis model of the synchronous machine is shown in Figure 1. Its dynamics are described by equation (1):

$$\tau_{d0}\dot{e}_q^{'} = e_{fd} - e_q^{'} - (x_d - x_d^{'})i_d$$

$$\frac{2H}{w_R}\frac{d^2\delta}{dt^2} = P_m - D(\omega - \omega_0) - e_d^{'} i_d - e_q^{'} i_q \qquad (1)$$

$\dot{\delta} = \omega - \omega_0$, where: w_R - rated speed of the machine, $e_q^{'}(t)$ Transient emf in the quadrature axis, and

$$\begin{aligned} i_d &= d_{et}(-(r_a + R_e)(e_d^{'} - V_\infty \sin\delta) + (x_q^{'} + x_e)(e_q^{'} - V_\infty \cos\delta)) \\ i_q &= d_{et}(-(x_d^{'} + x_e)(e_d^{'} - V_\infty \sin\delta) + (r_a + R_e)(e_q^{'} - V_\infty \cos\delta)) \\ d_{et} &= \frac{1}{(r_a + R_e)^2 + (x_d^{'} + x_e)(x_q^{'} + x_e)} \end{aligned} \qquad (2)$$

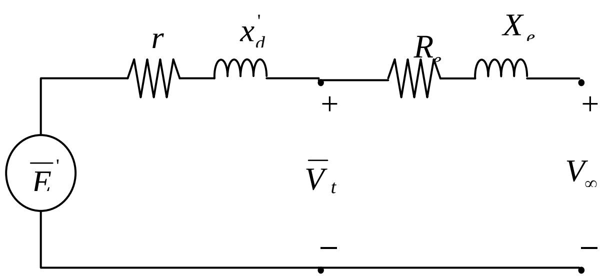

Figure 1. One-machine infinite bus system

For this dynamical systems the flat output, gives us the framework to derive the endogenous dynamic feedback compensators as shown in Figure 2.

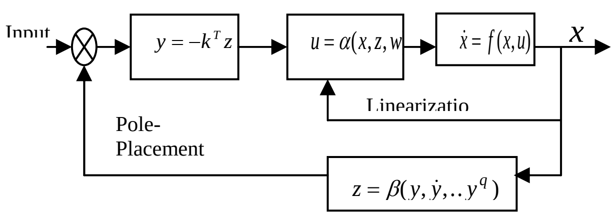

Figure 2. Block structure of feedback-linearisation

It was shown in [19] that in this model of the synchronous generator, the system variables $x = F(\delta, \omega, e_q^{'})$ and e_{fd} can be expressed as real-analytic functions of the component of δ and a finite number of its derivatives

$$x = A(\delta, \dot{\delta}, \ddot{\delta}) \qquad (3)$$

$$e_{fd} = \beta(\delta, \dot{\delta}, \ddot{\delta}) \qquad (4)$$

Thus, the states of the SMIBS are functions of the linearizing output δ and its derivatives up to order $\alpha = 2$. The endogenous feedback system to the following closed loop system is of order $\alpha + 1 = 3$:

So that from the linear system $\dddot{\delta} = v$ the dynamic compensator is obtained from the following state transformations:

$$\begin{aligned} \dot{z}_1 &= z_2 = \dot{y}_1 = \dot{\delta} = \omega - \omega_0 \\ \dot{z}_2 &= z_3 = \ddot{y}_1 = \ddot{\delta} = \dot{\omega} \\ \dot{z}_3 &= z_4 = \dddot{y}_1 = \dddot{\delta} = \ddot{\omega} = v \end{aligned} \qquad (5)$$

yielding the equivalent normal form for the system, and from which we can compute the nonlinear controller by inverting the expressions from $\ddot{\omega}$ and e_{fd}. The state transformations are invertible and exist throughout the domain of stable operation $0 < \delta < 180^o$. The resulting excitation control is given by:

$$\begin{aligned} e_{fd} &= \frac{t_{d0}}{E}\left(\frac{2H(v)}{d_{et}\omega_0} + \frac{D\dot{\omega}}{d_{et}} + A\dot{e}_d^{'} + Be_d^{'} - Ce_q^{'}\right) \\ &+ e_q^{'} + (x_d - x_d^{'})i_d \end{aligned} \qquad (6)$$

where,

$$\begin{aligned} A &= 2R_e T\dot{e}_d^{'} - R_e TV_\infty \sin\delta - x_{qt}V_\infty \cos\delta; \\ B &= x_{qt}V_\infty \sin(\delta)\dot{\delta} - R_e TV_\infty \cos(\delta)\dot{\delta}; \\ C &= (x_{dt} - x_{qt})\dot{e}_d^{'} - x_{dt}V_\infty \cos(\delta)\dot{\delta} - R_e TV_\infty \sin(\delta)\dot{\delta}; \\ E &= (x_{dt} - x_{qt})e_d^{'} - x_{dt}V_\infty \sin\delta - 2R_e Te_q^{'} + R_e TV_\infty \cos\delta; \end{aligned} \qquad (7)$$

The loop closure is then done to stabilize the reference. Equation (6) is used with

$$v = -k_1(\delta - \delta^*) - k_2(\dot{\delta} - \dot{\delta}^*) - k_3(\ddot{\delta} - \ddot{\delta}^*) \qquad (8)$$

and choose k_i such that the linear time invariant error dynamics

$$e^{(3)} = k_1 e + k_2\dot{e} + k_3\ddot{e} \qquad (9)$$

where

$$e^{(j)} = \delta^{(j)} - (\delta^*)^{(j)} \qquad (10)$$

are stable.

3. Flatness and Trajectory Generation

Recall from section (2) that, any component of x and u are functions of the flat output $y = (y_1, y_2, \ldots, y_m)$ and its derivatives up to q and $q+1$ respectively. Thus once any value of the flat output and its requisite derivatives are known, all the other system variables and the input levels associated to the particular set of values are sufficiently defined. It can therefore be shown that if two sets of the flat output and its requisite derivatives exist at two different time slots, the desired trajectories of plant motion and the input required to drive the plant along these trajectories from one state to the other (associated to each set), could be solved as an interpolation problem without integrating the state equations.

3.1 The Interpolation Procedure

By reference [20], consider a variable $x(t)$ whose time evolution is governed by an n^{th} order dynamics. Let the values, and derivatives of the variable $x(t_1), \dot{x}(t_1), \ldots\ldots, x^{(n)}(t_1)$ be known at some instant $t = t_1$ while at another instant $t = t_2$ the values and derivatives of the variable $x(t_2), \dot{x}(t_2), \ldots\ldots, x^{(n)}(t_2)$ are equally known. The problem is to obtain the values of the variable $x(t_1) < x(t) < x(t_2)$.

Formulate an interpolation polynomial given by [19]

$$x(\tau(t)) = \alpha_0 + \alpha_1\tau + \alpha_2\tau^2 + \alpha_3\tau^3 + \ldots\ldots + \alpha_{2n+1}\tau^{2n+1} \quad (10)$$

where $\tau = \dfrac{t - t_1}{t_2 - t_1}$ which on differentiation gives $\dfrac{d\tau}{dt} = \dfrac{1}{t_2 - t_1}$ from which we observe that as

$$t = t_1 \rightarrow \tau = 0 \text{ and as } t = t_2 \rightarrow \tau = 1. \quad (11)$$

From substitutions, differentiations and further substitutions it can be verified that the first half of the constants can be computed from;

$$\alpha_i = \frac{1}{i_{factoria}} x^i(t_1)(t_2 - t_1)^i \quad (12)$$

for $i = 0 \rightarrow n$. While for $i = n \rightarrow 2n+1$,

$$[\alpha_{n+1} \quad \alpha_{n+2} \quad \ldots. \quad \alpha_{2n+1}]^T = \mathrm{A} * \mathrm{B} \quad (13)$$

The matrices A and B are generated for an 3rd order plant dynamics as follows:

The state variable and its derivatives at the given time slots t_1 and t_2 comprise of the set $\{x_1, \dot{x}_1, \ldots.., x_1^{(3)}.\}$ and $\{x_2, \dot{x}_2, \ldots.., x_2^{(3)}.\}$ respectively. Using equation (10) and substituting $\tau = 0$ for $t = t_1$ and $\tau = 1$ for $t = t_2$, and on repeatedly differentiating equation (10) up to the 3^{rd} order, as well as substituting t_1, t_2 into equation (10) respectively the A and B are obtained:

$$\mathrm{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ n+1 & n+2 & \cdots & 2n+1 \\ n.n+1 & n+1.n+2 & \cdots & 2n.2n+1 \\ n-1.n.n+1 & n.n+1.n+2 & \cdots & 2n-1.2n.2n+1 \end{bmatrix}^{-1}$$

and

$$\mathrm{B} = \begin{bmatrix} x(t_2) - x(t_1) - \sum_i \frac{1}{i_{factoria}} \overset{(i)}{x}(t_1)(t_2 - t_1)^i;\ for, i = 1 - n \\ \overset{(i-1)}{x}(t_2) - \overset{(i-1)}{x}(t_1)(t_2 - t_1)^{i-1} - \sum_i \frac{1}{(i-1)_{factoria}} \overset{(i)}{x}(t_1)(t_2 - t_1)^i,\ for, i = 2 - n \\ \overset{(i-1)}{x}(t_2) - \overset{(i-1)}{x}(t_1)(t_2 - t_1)^{i-1} - \sum_i \frac{1}{(i-2)_{factoria}} \overset{(i)}{x}(t_1)(t_2 - t_1)^i;\ for, i = 3 - n \\ \overset{(i)}{x}(t_2) - \overset{(i)}{x}(t_1)(t_2 - t_1)^i;\ for, i = n \end{bmatrix}$$

Having computed the constants, the system trajectories between any two equilibra of the plant corresponding to t_1 and t_2 can be generated from

$$x_{ref}(t) = x_{ref}(t_1) + (x_{ref}(t_2) - x_{ref}(t_1)) \sum_j^{2n+1} \alpha_j \left(\frac{t - t_1}{t_2 - t_1} \right)^j \quad (14)$$

where x_{ref} is the reference trajectory for the state variable of interest.

3.2 Motion Planning

Some obvious results can be inferred:

- The whole trajectory of system motion can be pre-assigned to move the system from the current operating point to a desired equilibrium or steer the system from one equilibrium state to another.
- The time duration of motion from equilibrium state to another can be equally pre-assigned. Furthermore this pre-assignment may be chosen to satisfy physical limitations in the plant such as rate limits or saturation.
- The input required to achieve the specified motion in the plant is determined a-priori.

3.3 Sample Trajectories

The trajectory of the load angle was generated for a -15% step change in the load angle. Figures 3 and 4 also show the generated (planned) trajectory for the 15% fall in steady state load angle value and the system's response in tracking it for a period of 5 seconds. Figures 3 and 4 shows that, the generated trajectories include: displacement, velocity and input. The acceleration component was assumed zero and no faults were induced yet.

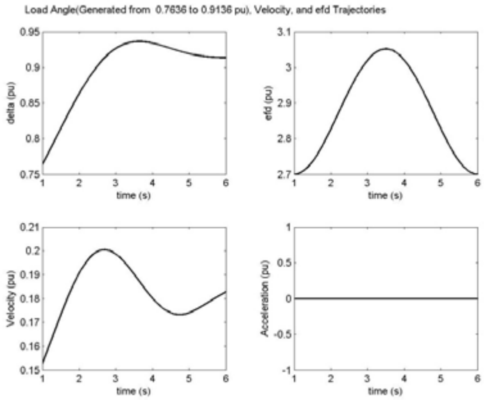

Figure 3. Generated reference trajectories for a step increase in load angle

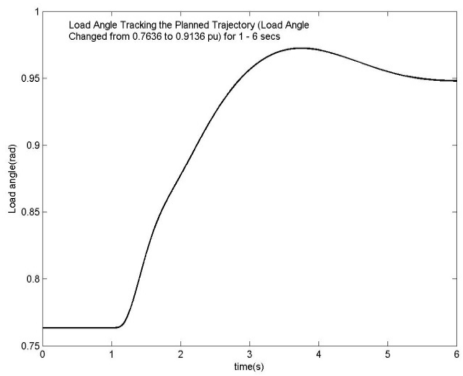

Figure 4. System tracking the generated trajectory for a step increase in load angle

Table 1
Terminal condition and system data of the generator

Parameter	Value
Machine Power [pu]	1.0
Power Factor p_f [pu]	0.85
Infinite Bus Vol. V_∞ [V]	1.0
Machine Speed w_R, ω_0 [$^{rad}/_{s}$]	314.159
D	0.002
t_{d0} [s]	5.9
t_{q0} [s]	0.075
x_d [pu]	1.7
$x_d^{'}$ [pu]	0.245
x_q [pu]	1.64
$x_q^{'}$ [pu]	0.245
r_a [pu]	0.001096

4. Simulations, Results and Discussions

4.1 Simulation Data

The operating point of the system was determined using the data in Tables 1 and 2:

Table 2
Network parameters, control limits and controller gains

Parameter	Value 0.4
Reactance Xe [pu]	0.4
Resistance Re [p	0.02
Field Voltage limits [pu]	$e_{fd\max} = 4.5$ $e_{fd\min} = -4.5$
Controller PID gain [pu]	$k_{11} = 400$ $k_{12} = 95.14$ $k_{13} = 15.86$

In the simulations, it was assumed the generator terminals were connected to the infinite bus via a transformer and a tie line consisting of a resistance R_e and inductance X_e. A three-phase short circuit fault was simulated for: (a) steady state operation from 0.0 seconds to 1.0 seconds. (b) Three phase fault at transformer terminals from 1.0 seconds to 1.06 seconds $V_\infty = 0.0$, $R_e = 0.0$. (c) Post fault stabilization/tracking from 1.06 seconds or more. The infinite bus system under fault condition is as shown in Figure 5 [19].

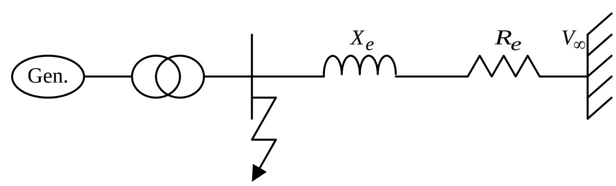

Figure 5. Fault location on the SMIBS

Figure 6 shows trajectories generated for steady state pre-fault values to steady state post-fault values for a 3-cycle short circuit fault. Figures 7-10 show the system's response in tracking the trajectories for a post-fault duration of three seconds to accommodate the fault inception and clearing period (fault duration) for: load angle, speed and electrical power.

Figures 11-12 show the terminal voltage and field voltage tracking response for 11-cycle fault duration compared with the system response to set point stabilization. The results show a better performance of the local trajectory-tracking scheme over the global set point tracking.

Trajectory tracking can also be very useful in reducing the stresses borne by the generator shaft in reacting to restore the machine to post fault equilibrium under the influence of the controller dynamics. The influence of trajectories of motion is particularly pronounced for the longer cycles of system faults.

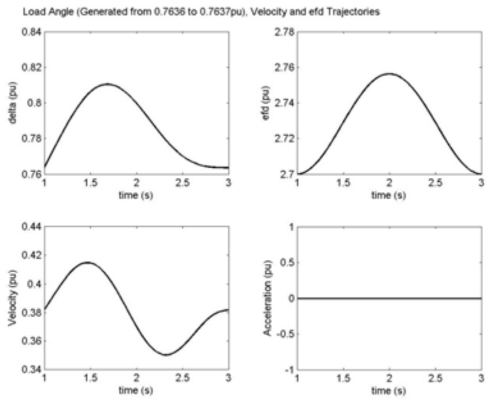

Figure 6. Generated reference trajectories for a 3-cycle fault

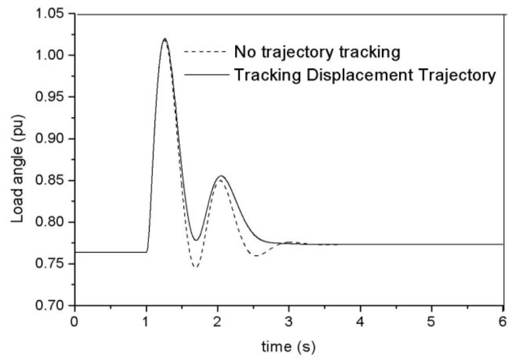

Figure 7. Load Angle Response Tracking due to a 3-cycle fault

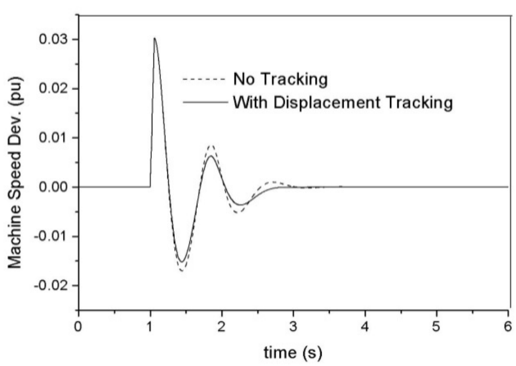

Figure 8. Speed Tracking for a 3-Cycle fault - Response

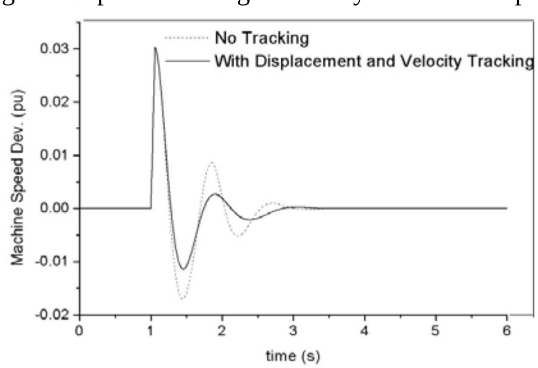

Figure 9. Speed Tracking for a 3-Cycle fault Response

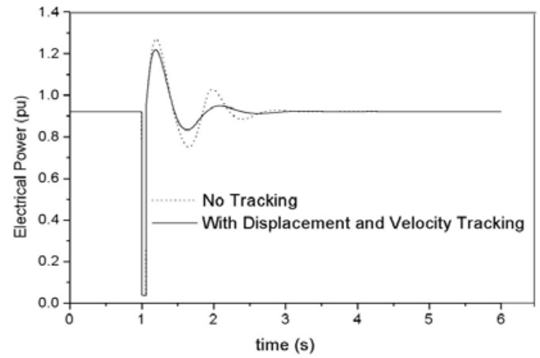

Figure 10. Electrical Power Tracking for a 3-Cycle fault-Response

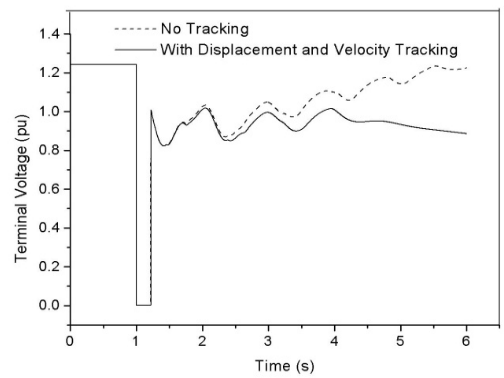

Figure 11. Terminal voltage tracking for an 11-Cycle Fault

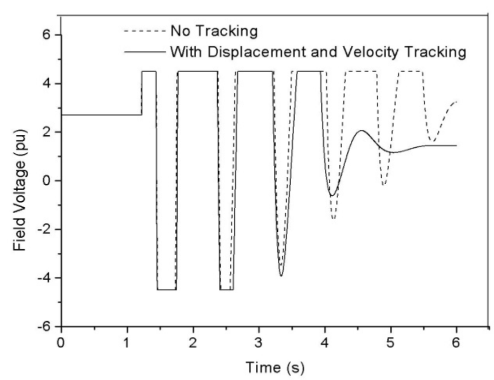

Figure 12. Field voltage tracking for an 11-Cycle Fault

5. Conclusion

The theory of dynamic feedback linearization has been applied on the third order single machine infinite bus system. Simulations have shown that the nonlinear dynamic controller achieves asymptotic stability of the SMIBS in damping and stabilizing oscillations arising from fault induced on the system. The simulations showed the ability of the system to track trajectories generated from a suitable polynomial using stable states of the system. Tracking a generated load angle trajectory and velocity during fault oscillations showed a better system response than the set point stabilization in bringing the system to post fault equilibrium values.

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