REGION FILLING AND HOLE REPAIR OF BIO-MEDICAL MODELS

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Abstract

In this paper, a model repair technique is introduced mainly to handle defective bio-models with triangular surface meshes or in stereo-lithography format [14]. This technique can help in the design of custom implants for “empty regions” on bio-parts with no underlying surfaces. It can subsequently be extended to allow assessment of defects and implants preoperatively, and improve fixation. An element based hole-filling algorithm is developed to fill any complex holes in oriented connected manifold meshes and to ensure water-tightness due to missing surface patches in both 3D surface models and faceted models. We specifically address situations in which the holes are too geometrically and topologically complex to fill using normal triangulation algorithms, and describe a method for filling holes in unstructured triangular meshes. The resulting patching meshes interpolate the shape and density of the surrounding mesh. The steps in filling a hole include boundary identification, two stage hole triangulations using Genetic Algorithm and advancing front technique, and surface approximation based on a Quartic Gregory patch.

Key Words: BioModeling, Custom implants, hole filling, triangular Gregory patch, surface approximation.

1. Introduction

In many cases, faceted (meshed) models may be preferred or may be the only representation available to model a machined part. For example, where natural processes are simulated such as in bio-medical or geo-technical applications, NURBS representations can be difficult or impossible to fit to the prescribed data. Surgeons have reported that using Bio-Modeling software to custom shape implants preoperatively can reduce operating time and risk of infection, and improve implant cosmesis and fit. These softwares import 3D CT scans and export STL files which are similar to that of meshed models. Present design of an implant or prosthesis is not always a straightforward application of CAD and manufacturing and current Bio-Modelling softwares do not allow for the design of implant when there is no underlying surface.

Figure 1 shows a BioModel generated from CT scans showing a defect (hole).

Figure 1: 3D CT scans demonstrate the extent of the defect of a skull

In this paper, we will tackle hole-filling, where missing polygons/surfaces are a common source of holes that awaits implants in bio-models. One of the difficulties of hole-filling is choosing the appropriate topology. Many holes are simple and are nearly planar and can be filled with disc topology; in these cases, triangulation algorithms can be employed [1-2]. However, some holes have convoluted geometry, and such seemingly extreme cases occur frequently, especially when scanning objects that contain curves, joints or crevices. In finite element modeling or scanned images, holes in the meshes/images can be also (and frequently are) highly non-planar and triangulations will not work well [3-4], for example, with holes that require connections between unconnected components, such as a hole with a shape of a step. Mesh-based methods of surface reconstruction [5, 6-7] treat each scan (e.g. one sweep of a laser plane across the surface) as an ordered 2D array of range (i.e. depth) samples, sometimes called a range image. These samples can be triangulated to form a polygonal mesh. These methods can perform hole-filling as a post-process, or they can integrate it with surface reconstruction. The only mesh-based method we know of that integrates hole filling into surface reconstruction is Curless and Levoy's VRIP [8]. Although diffusion methods have not previously been applied to surface reconstruction, they have a long history in the image processing community.
An application of diffusion to filling gaps in sampled data Bertalimio et al. iteratively apply a sequence of operators, one of which is anisotropic diffusion [10], in order to propagate information from known regions of the image into unknown (e.g. scratched) regions. Although these methods have been extended to vector-valued images and parametric domains on 2-manifolds [11], it is not obvious how to extend them to filling holes in surfaces.

Here, we describe a technique for filling holes in erroneous or scanned models or for the design of custom implants without underlying surfaces by processing a surface mesh representation, where we extend the incomplete surface description until it forms a watertight or hole-free model. The method is not only well suited to repair models for mesh generation and analysis applications needs. It can also be used to repair polygonal or faceted models containing holes, produced by data-scanning methods or even to create approximated patches covering the holes that can be used to design custom implants. In some cases the result may not totally match the topology of the original object, but it is always topologically consistent (i.e. manifold), cannot self-intersect, and maintains fidelity to the original data wherever it exists.

2. Methodology

The Main Contribution to this paper is to give a complete account of a geometric method for filling hole in triangular meshes. The main stages of this method are: hole identification, hole triangulation using Genetic algorithm, surface fitting based on a quartic Gregory patch and element-based hole meshing with nodal projection based on customized advancing–front technique. Figure 2 shows the flowchart of the hole-filling process. First, a hole is determined from the meshed model. Next, triangulation of holes is performed based on the boundary edges of the hole and here, no new nodes will be formed. Genetic Algorithm is implemented here in order to obtain a triangulated “patch” with optimal shape. This initial triangulated “patch” will served as a guide to obtain fitting surfaces based on quartic Gregory patch, where the final mesh created using a customized advancing front method will be projected to these surface patches and smoothed with respect to these approximated surfaces.

Identification of Holes

In hole-filling, there are two main types of holes and they are categorized as simple and multi-peripheral holes. A simple hole here is defined as a hole of any shape with only one boundary loop. A multi-peripheral hole is defined as a hole consisting of at least two peripheral loops. Here, we assume that all the meshes are corrected oriented, manifold and connected, and that a given hole will not have islands. Figure 4(a) shows an original mesh model of a sphere and Figure 4(b) shows a hole being is image inpainting [9]. Like our proposed method, created on the surface of the sphere. The identification of holes can be done be merely checking out for connected boundary edges that formed a closed loop.

Figure 2: A Flowchart of the hole-filling algorithm

Hole Triangulation using Genetic Algorithm (GA)

For a hole with a complex peripheral loop, one difficulty of hole-filling is choosing appropriate topology. Using GA at this stage is important as the outcome of this initial triangulation of the hole affects the final shape of the mesh covering the hole. GA will compute the best shape to triangulate the patch with no new nodes to be created. Thus in the chromosome representation, one binary character or bit represents one link. There will be checks to ensure that the created triangles will not overlap one another, i.e. no crossings of the links. The genetic algorithm then works as follows:

1. The initial population is filled with chromosomes that are generally created at random.
   Each chromosome in the current population is evaluated using the fitness measure. The more “fit” solutions reproduce and the less “fit” solutions die off.
   The fitness value is computed taking in consideration of the smoothness of the triangles to be created and the smoothness of the triangulated patch with its surrounding triangles. The overall smoothness of the triangles created is calculated at the links where the angles between the normals of two adjacent triangles are calculated. The smaller this angle is the large the value of \( f_{\text{internal}} \) which denotes smoothness.
   Similarly for \( f_{\text{boundary}} \), we calculate the angles between the normals of the newly created triangles with the surrounding original triangles. The total factor is the summation of all the sub-factors, each multiplies by a weighing factor.
Internal Smoothness fitness factor,
\[ f_{\text{internal}} = \sum_{i=1}^{n-3} (\pi - \alpha_i)^2 \]  
(1)

Boundary Smoothness fitness factor, 
\[ f_{\text{boundary}} = \sum_{i=1}^{6} (\pi - \beta_i)^2 \]  
(2)

Deviation fitness factors 
\[ d = \text{Max}(\alpha_j) - \text{Min}(\alpha_j) \]  
(3)
\[ d = \text{Max}(\beta_l) - \text{Min}(\beta_l) \]  
(4)

Total fitness value, 
\[ f_{\text{total}} = w_d f_{\text{internal}} + w_b f_{\text{boundary}} + w_e (d_\alpha + d_\beta) \]  
(5)

2. If the termination criterion is met, the best solution is returned.
   Once an empty region is re-triangulated and improved as much as possible, the GA moves on to the next worst region. This process of moving to the worst area of the model continues until a global model minimum has been reached. Here a global minimum considers the normal deviations for all elements in the model.

3. Actions starting from step 2 are repeated until the termination criterion is satisfied.
   The crossover can come in a variety of choices. Single, uniform and multi-point crossovers are a few of the types. In this paper multi-point crossover is used. Here crossover is used on two mates, producing two children. Mutation adds diversity to the GA. It is a random walk in the search space; basically perform edge swapping in the empty region. The convergence process may be a good application for fuzzy logic. We are not so lucky as to have a simple "go" or "no go" situation. We have a "maybe" or "maybe not" which is a natural for fuzzy logic. The way the GA works, a gradual relaxation of the convergence is required.

Figure 4(c) shows the optimized triangulation of a hole using GA. This stage is important as it will affect the final shape of the filled region.

Surface fitting based on a Quartic Gregory patch
The computation of a quartic Gregory patch will apply to each of the triangular facets created in the above triangulation process using GA and it includes, firstly, the approximation of the edges of the triangular face using quartic Bézier curves, and, secondly, the determination the interior control points of the quartic Gregory patch. A degree n triangular Bézier patch may be described with the following equation:
\[ X(u, v, w) = \sum_{i+j+k=n} B^n_{i,j,k}(u,v,w) P_{i,j,k} \]  
(6)

\[ B^n_{i,j,k} = \frac{n!}{i!j!k!} u^i v^j w^k \]  
(7)

It has been established that the minimum order of a triangle Bézier patch necessary to model a \( G^1 \) surface is \( n=4 \) [22]. For this case, equation (9) can be written as:
\[ X(u, v, w) = \sum_{i+j+k=n} P_{i,j,k} \frac{4!}{i!j!k!} u^i v^j w^k \]  
(8)

where \( u, v, w \geq 0; u + v + w = 1; i, j, k \geq 0 \)

Walton and Meek [12] describe a procedure where \( P_{i,j,k} \) are the control points of the triangle and \( B_{i,j,k}(u, v, w) \) can be thought of as the weights at polar location \( i, j, k \), where \( i+j+k = n \). Note also that equation (1) can be thought of as a special case of equation (3) where \( n=1 \). In this case \( B_{i,j,k} \) = 1 for all \( i, j, k \). Figure 3 shows the polar values for triangular patches of \( n = 4 \) and equation (11) can be written as:
\[ X(u,v,w) = P_{0,0,0} u^0 v^0 w^0 + 4P_{0,1,0} u^1 v^0 w^0 + 6P_{0,2,0} u^2 v^0 w^0 + 4P_{0,3,0} u^3 v^0 w^0 + 6P_{0,4,0} u^4 v^0 w^0 \]
\[ + 4P_{1,0,0} u^0 v^1 w^0 + 12P_{1,1,0} u^1 v^1 w^0 + 6P_{1,2,0} u^2 v^1 w^0 + 4P_{1,3,0} u^3 v^1 w^0 + \]
\[ + P_{2,0,0} u^0 v^2 w^0 + 6P_{2,1,0} u^1 v^2 w^0 + 12P_{2,2,0} u^2 v^2 w^0 + 6P_{2,3,0} u^3 v^2 w^0 + P_{3,0,0} u^0 v^3 w^0 \]

Owen et al. [13] describes that any two adjacent patches of a composite \( G^1 \) surface should have a common tangent plane as well as a common boundary curve. A triangular quartic Bézier Gregory patch is constructed and positional continuity at the vertices of the domain triangles is ensured since the same sample data point is used as a vertex for each domain triangle that meets there. The control points of the quartic boundary curves are used as control points of the patch boundaries to ensure \( G^1 \) continuity across each
boundary. Making use of the control points on the triangular surface’s edges, the quartic Bézier patch, control points \( P_{1,1,2}, P_{1,2,1} \), and \( P_{2,1,1} \) on the interior of the triangular surface can now be defined. The locations of these points are critical to describing \( G^1 \) continuity between adjoining patches. Walton and Meeks propose a method, whereby two candidate locations for interior control points are defined for each edge, resulting in six vectors \( G_{i,j} \), where \( i=0,1,2 \) and \( j=0,1,2 \). Final locations for interior control points \( P_{1,1,2}, P_{1,2,1} \), and \( P_{2,1,1} \) are a function of the evaluated \((u, v, w)\) parameters and \( G_{i,j} \), and may be uniquely evaluated as:

\[
P_{1,1,2} = \frac{1}{u+v}(uG_{2,2} + vG_{0,1}) \tag{10}
\]

\[
P_{1,2,1} = \frac{1}{w+u}(wG_{0,2} + uG_{1,1}) \tag{11}
\]

\[
P_{2,1,1} = \frac{1}{v+w}(vG_{1,2} + wG_{2,1}) \tag{12}
\]

**Meshing of Holes using Advancing Front Technique**

Using the above surface fitting technique describe by Walton and Owen, we are able to approximate the triangular faces created in hole-triangulation process using GA with quartic Gregory surface patches. An advancing front meshing technique is implemented to mesh the hole with the approximated surfaces as the underlying surfaces for the hole. Figure 4(d) shows the result of the meshing process of the hole in the sphere example using the approximated quartic Gregory patches as the underlying surfaces for the advancing front technique to be carried out. The new nodes created will be projected onto the approximated surfaces. It is followed by a Laplacian process to smooth the newly created mesh, similarly using the approximated Gregory patches as underlying surfaces so that the mesh will not shrink.

**3. Case-Study**

Figure 5 (a) and (b) show the original mesh of a skull and the mutilated mesh of the skull with two large holes respectively. The mesh of the skull after hole-filling in Figure 5(c) can be used to approximate the design of custom implants to cover the holes, assuming the patient underwent a brain surgery. Note that the original mesh is provided for comparison purpose and they are not present in the computation of the hole-filling process.

**4. Conclusion**

The hole-filling technique presented in this paper makes use of surface approximation technique to generate surface patches of high geometric fidelity without any underlying surfaces for the holes. It can be of use in mesh compression or refinement because this hole-filling algorithm enables us to approximately reconstruct a mesh from partial information and this is particularly useful in the areas of bio-modeling and simulation as well as data representation and visualization. Although the intended application is surface fitting, the patch may be used in a CAD environment. Since the patches are defined analytically, the representation is compact and easy to work with and the resulting composite surface can be edited locally in an interactive graphics environment to aid in the design of custom implants. The limitation of using quartic Bézier patch is that the shape of the patched hole tends to shrink more inwards if the size of the hole gets larger which can be spotted in case-studies. One of the future works is therefore to look into handle shapes that may be changed without affecting tangent plane continuity.

**References**

Figure 4: (a) An Original faceted/meshed model of a sphere and (b) a hole is being created by removing a patch of elements/triangles, (c) the outcome of initial triangulation of hole using Genetic Algorithm, and (d) the final mesh of the filled hole fit to the approximated surfaces using Quartic Gregory Bezier patch.
Figure 5 (a) The original mesh of a skull, (b) the mutilated mesh of the skull, and (c) the mesh of the skull after hole-filling.